General paper

MACRO-MICRO UNCOUPLED HOMOGENIZATION PROCEDURE FOR MICROSCOPIC NONLINEAR BEHAVIOR ANALYSIS OF COMPOSITES

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Abstract: For easy and practical simulation of microscopic nonlinear mechanical behavior of composite materials, new homogenization analysis procedure named macro-micro uncoupled method is proposed. In this procedure, microscopic simulation of material strength test is carried out first, then nonlinear homogenized material properties are obtained. Both the homogenization method and RVE (Representative Volume Element) approach are shown for this microscopic simulation. Macroscopic nonlinear behavior of a structure made of composite materials can be analyzed by only referring the pre-calculated homogenized material nonlinearity. As an example, microscopic fracture propagation for textile composite materials and microscopic large deformation for porous elastic materials are analyzed.

Key words: *Composites, Micromechanics, Nonlinear analysis, Homogenization method, RVE method*

1 INTRODUCTION

Because classical theories on the mechanics of composites regard heterogeneous media as macroscopically homogeneous one considering microscopic behaviors, many studies on the prediction of homogenized material properties have been presented[1-4]. For continuous straight fiber reinforcements, the Halpin-Tsai equation[1] is still most commonly used. For more complicated reinforcements such as woven fabrics, RVE (Representative Volume Element) method[3,4] has long been used. Recently, the homogenization method[5-8] is becoming a matter of interest, as the need for understanding of the microscopic behaviors grows. We can find many studies showing the accuracy of the homogenization method comparing with the Halpin-Tsai equation[5], RVE[6] and experiments[7]. Although they can be applied to linear elastic problems, nonlinear micromechanics of composites has not been accomplished.

One topic in this study is the computational scheme to predict the nonlinear homogenized material properties due to microscopic architecture and/or behaviors. Two methods, RVE approach with microscopic large deformation analysis and enhanced homogenization method for microscopic fracture propagation, are shown in this study.

Final goal of the micromechanics of composite materials is the strength prediction of a structure made of composite materials. Therefore, nonlinear structural behavior must be analyzed considering microscopic nonlinear behaviors. However, we can find only few literature on nonlinear micromechanics of composites such as the application of the homogenization method to elasto-plastic problems described by rate form based on Total and/or Updated Lagrangian formulation[8,9]. The nonlinear homogenization method,

however, takes a very large amount of computational cost as pointed out in Refs.[8,9]. Therefore, we present easier and more practical scheme to solve macro-micro coupling nonlinear problems in this paper.

2 MICROSCOPIC NONLINEAR PROBLEMS

Nonlinear mechanical behaviors of composite materials arise from their microscopic behaviors. Microscopic nonlinearity includes material nonlinearity such as matrix yielding, geometrical nonlinearity such as fiber reorientation which is due to the microstructural architecture as well as microscopic large deformation, and microscopic fracture propagation. Up to date, above all nonlinear problems were linearized in numerical simulations by means of step-wised linear homogenized material properties, because classical theories on the mechanics of composite materials was for only linear elastic problems. However, the application of elasto-plastic theory[8-11], for instance, is more reasonable for problems with material nonlinearity, because linearized elastic theory cannot consider residual plastic strain after unloading. Recently, macroscopic elasto-plastic theory was applied also to microscopic fracture propagation[10,11]. Microscopically, however, both alive matrix and fiber behave as elastic body. Also, microscopic geometrical nonlinearity should be treated as elastic problem. Figure 1 illustrates an example of microscopic geometrical nonlinear problem. Fiber bundle which consists of straight fibers in tension behaves linearly (Fig. 1(a)). On the other hand, when fiber bundle which consists of wavy fibers is stretched, nonlinear stress-strain relation is observed macroscopically due to microscopic large deformation (Fig. 1(b)). From above consideration, we can conclude that appropriate theory of me-

Received June 30, 1995

Original papers in Japanese have been published in Journal of the Society of Materials Science, Japan, Vol.44, No.505 (1995) pp.1231-1237 and Vol.45, No.2 (1996) pp.163-168.

chanics must be employed to describe the nonlinear behaviors of composite materials, especially considering their microscopic behaviors.

In this paper, we present two typical nonlinear problems. The one is microscopic fracture propagation, and the other is microscopic large deformation. As mentioned above, we describe these two problems by the theory of elasticity microscopically.

Fig. 1. Dependence of macroscopic behavior on the microstructure.

3 MACRO-MICRO UNCOUPLED HOMOGENIZATION PROCEDURE

Now we discuss the simulation algorithm for macro-micro coupling nonlinear problems. Generally, in order to solve nonlinear problems, we have to analyze incrementally as shown in Fig. 2(a). In this case, if we want to consider macro-micro coupling effect, we have to solve both macroscopic and microscopic problems in each increment as shown in Fig. 2(b). Here, microscopic states differ in the macroscopic domain due to macroscopic stress and/or strain distribution. Therefore, we have to solve infinitely many microscopic problems everywhere in the domain. This conventional scheme requires very large amount of computational cost[8,9]. The contribution of this paper is to propose easier and more practical analysis procedure for nonlinear problems of composite materials. To reduce the computational cost, we propose to uncouple the simulation for macro- and microscopic behaviors as shown in Fig. 2(c). First we calculate nonlinear material properties by precise macro-micro coupling analyses simulating material strength tests such as uniform tension test, uniform compression test, shearing test and so on. Note that only several representative problems mentioned above have to be analyzed in order to determine the nonlinear material properties. This is one of the advantages of the pro-

posed procedure as compared with the conventional procedure shown in Fig. 2(b). For the simulation of material strength tests, we use conventional homogenization analysis procedure as shown in Fig. 2(b). However, in the simulation of uniform tension test, for instance, we have only to solve one microscopic problem because microscopic states are uniform everywhere in the domain. Therefore, the proposed procedure shown in Fig. 2(c) can reduce the computational cost remarkably. Calculated nonlinear material properties are stored as material database. Then we can analyze macroscopic problems by only referring this pre-calculated nonlinear material database without solving many microscopic problems simultaneously. This obviously leads to the reduction of computational cost. Another advantage of the proposed method is that standard existing finite element analysis codes are available to solve macroscopic problem in Fig. 2(c).

In order to refer the nonlinear material database, we use macroscopic principal stress and/or strain. Figure

(a) Nonlinear macroscopic response.

(b) Conventional homogenization analysis procedure.

(c) Proposed macro-micro uncoupled analysis procedure.

Fig. 2. Schematic flowchart of macro-micro uncoupled analysis procedure.

(a) Example of nonlinear material database.

(b) Distribution of principal stress.

Fig. 3. Example of analysis of a structure made of composite materials.

3 shows a typical example. Figure 3(a) shows an example of database of material properties with respect to the stress level and the angle between stress direction and anisotropic material principal axis. If principal stress is obtained like Fig. 3(b), we can predict the material property everywhere in the domain by means of the level and direction of principal stress. In chapters 4 and 5, we show two methods for nonlinear material test simulation to calculate the material database.

4 MICROSCOPIC FRACTURE PROPAGATION ANALYSIS FOR TEXTILE COMPOSITES BY THE HOMOGENIZATION METHOD

The first application is microscopic fracture propagation for textile composites. In this case, microscopic stresses affect the nonlinear behavior of textile composites. Therefore, the homogenization method is applied to calculating the microscopic stresses precisely.

On the application of the homogenization method

Fig. 4. A plate of woven fabric composites with hole.

Fig. 5. A flat plate of woven fabric composite materials for material tension test simulation.

Fig. 6. Microscopic unit cell model.

to nonlinear problems, Kikuchi and his colleagues[8,9] have presented homogenization formulation based on rate form of Total and/or Updated Lagrangian formulation. The authors[12] have also presented simplified step-wised linear homogenization method. Although fast and handy simulation was found to be possible by the simplified algorithm for uniform macroscopic stress and/or strain field in Ref.[12], its application to practical problems has not been presented. We show in this paper that the proposed macro-micro uncoupled analysis procedure can be a very effective tool.

As a typical problem, we consider a plate of woven fabric composite materials with open hole as shown in Fig. 4[13-15]. Figure 4 illustrates the orientation of woven structure just schematically. The real dimension of the microstructure is very small compared to the dimension of the plate and hole. Hence, the number of periodicity of microscopic unit woven structures is supposed to be large enough to apply the homogenization method. We have to consider macroscopic

stress distribution and especially stress concentration around the hole edge, which results in different microscopic states in whole domain. In the finite element discretization, the size of mesh around the hole must be larger than approximately 6 periods of microscopic unit cell to assure the assumption of periodicity. If we use such macroscopic finite element mesh, homogenized material properties are applicable in the proposed procedure.

Therefore, a flat plate of woven fabric composite materials without hole was analyzed first by the simplified step-wise linear homogenization method[12] considering microscopic stresses and microscopic fracture propagation carefully. Only uniform tension tests are needed in this analysis because the principal stress in the macroscopic problem is tensile everywhere. See Refs.[12-15] on its formulation and detail of this analysis. Figure 5 shows the object analyzed for material test simulation, which is a uniform tension test. Here again, Fig. 5 schematically illustrates the orientation of woven structure, and the real dimension of the microstructure is supposed to be very small compared to

Fig. 7. Reduction of homogenized elastic constant E_x for database with respect to the level and direction of macroscopic principal stress.

Fig. 8. Macroscopic tensile stress versus displacement curve.

the dimension of the plate. Some representative problems were analyzed with different θ , which is the angle between load direction (i.e., stress direction) and microscopic woven architecture. Figure 6 shows the microscopic unit cell model. Using the step-wise linear homogenization method, we can calculate the reduction of the homogenized material constants caused by microscopic fracture[13-15]. Figure 7 shows the calculated homogenized elastic constant E_x . From Fig. 7, we can predict the instantaneous homogenized elastic constant for given principal stress by means of its level and direction. In other words, we express the microscopically fractured state by the reduction of homogenized material properties in the homogenized (averaged) sense. In this analysis, fracture at fiber bundles is considered[12-15], whereas debonding between fiber and matrix is not considered. Bridging is not also considered because we used the microscopic unit cell model shown in Fig. 6. Referring the nonlinear material properties as database, a plate with hole under tension was solved just macroscopically. Macroscopic tensile stress versus displacement curve is shown in Fig. 8. Knee point appears at around 0.06mm of displacement, and then the stiffness decreases slightly. At the knee point, microscopic fracture near the hole edge causes the decrease of the stiffness. Evolution of microscopic fracture is recognized in this portion after the knee point. Just before breaking of the structure, this portion can not sustain the load any more, and then the fractured zone propagates rapidly throughout the ligament.

5 MICROSCOPIC LARGE DEFORMATION ANALYSIS FOR POROUS MEDIA BY RVE METHOD

As one of the unsolved microscopic nonlinear problems, microscopic large deformation analysis is presented in this chapter. The authors have presented so far the application of the simplified step-wise linear homogenization method to large deformation analysis of rat tendon in biomechanical field[12]. However, considered problem was macroscopic uniform tensile problem in Ref.[12], and practical application has not been presented. We can find similar problem in the analysis of tire with cord reinforcement considering cord reorientation under large deformation. Here, reorientation is considered just geometrically and the homogenization is done by the rule of mixture[16]. Because microscopic stresses obviously affect the homogenized material properties, however, they must be taken into account in the homogenization process.

In the previous chapter, we applied the simplified step-wise linear homogenization method. To consider microscopic large deformation, however, simplified algorithm seems not to be adequate. One of the solution might be given by Ref.[9]. In this chapter, we adopt conventional RVE approach by using existing large deformation analysis codes.

Problem setting is shown in Fig. 9, where laminated cylinder consisting of porous elastic material is con-

sidered. Obviously the homogenized material property gets stiffer under large deformation. To calculate this nonlinear homogenized material property, RVE approach has been taken as shown in Fig. 10. Only uniform compression test is needed in this analysis by considering the objective model, geometry and boundary condition. Note here that 25 RVEs were considered to assure the periodic arrangement of RVE. This important caution is also pointed out in Refs.[4,6]. We investigated the appropriate number of RVE, then we

Fig. 9. Composite laminated cylinder made of porous media.

Fig. 10. 25 unit cell RVE model under compression for material test simulation.

Fig. 11. Relation between averaged stress and averaged strain.

came to the conclusion that more than 25 RVEs are necessary[17]. This is because the definition of boundary conditions are not strict in RVE method different from the homogenization method.

Two types of microstructures, i.e., 30% and 50% volume fraction of circular hole, are considered. Figure 11 shows the relation between averaged stress and averaged strain obtained by large deformation analysis. We can observe that the homogenized material property becomes slightly stiffer than linear analysis. Figure 12 shows the tangent modulus obtained from Fig. 11 regarding this compression test as uniaxial problem[17]. Because the large deformation analysis was carried out incrementally, obtained tangent modulus is step-wise.

According to the proposed macro-micro uncoupled procedure, macroscopic problem was solved by only referring this material database (Fig. 12). In this example, database is referred by means of macroscopic strain. Here we show two cases as shown in Fig. 13. Calculated relation between the applied pressure and macroscopic strain is also shown in Fig. 13. For both cases (a) and (b), nonlinear response is obtained. What is more interesting is that we can find the sequence of strain distribution as pressure increases for the case (b); that is, the increase of strain at outermost point in softer layer is more outstanding than innermost point when the applied pressure is over around 2MPa.

6 CONCLUDING REMARKS

For the numerical simulation of microscopic nonlinear mechanical behaviors of composite materials, new homogenization procedure has been developed. Through two practical examples, we can remark the following conclusions.

- (1) Proposed macro-micro uncoupled homogenization method is very useful and practical for evaluation and design of advanced composite materials.
- (2) As material test simulation, both the homogenization method and RVE approach are useful.

Fig. 12. Nonlinear relation between tangent modulus and averaged strain for material database.

As is pointed out in chapter 2, many other nonlinear mechanical behaviors of composite materials must be analyzed such as matrix yielding, although only nonlinear elastic problems have been shown in this paper.

Another issue to be discussed in future works is to consider the effect of load history. Enhancing the macro-micro uncoupled method is expected to solve these problems.

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Fig.13. Nonlinear macroscopic behavior due to microscopic large deformation.