General paper

STIFFNESS IDENTIFICATION OF SYMMETRICALLY LAMINATED PLATES USING BENDING DEFLECTION

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Abstract: The present paper shows an identification method of bending stiffnesses of symmetrically laminated plates using bending deflections under static loading. Firstly, three bending stiffnesses of simply-supported orthotropic plates are identified by using an analytical expression. Secondly, four or six bending stiffnesses of symmetrically laminated plates are identified by using the finite element method and the nonlinear optimization technique. Effects of the number of measured deflections on the identified bending stiffnesses are examined numerically.

Key words: Identification, Bending stiffnesses, Symmetric laminate, Orthotropic laminate, Static deflection

1 INTRODUCTION

It is important to develop a nondestructive evaluation on material degradation or damage for composite structures in aircrafts and space structures. Damage inspection methods such as x-ray and ultrasonic testing which have often used for composite structures can be time consuming and are local assessments. They also require the exposure of structural elements to the inspector and equipment for detecting damage, and thus they may not be appropriate as a real-time nondestructive evaluation in flight.

An alternative approach making use of vibration test data or static test data can be useful as the structural health monitoring system. Several approaches have been suggested for the structural health monitoring. Optimal matrix update algorithms have been developed in system identification to refine a finite element model using measured frequencies and mode shapes [1,2]. Nonlinear optimization techniques have also been applied to identify material properties and damage using the static or vibration test data [3,4]. When material properties such as Young's modulus are identified, damage can be detected from the comparison between initial material properties and those after damage. The use of vibration test data has been attempted to locate structural damage in composite laminates [5] and to identify bending stiffnesses of composite laminates [6]. Static test data have also used to identify material properties of isotropic structures [7] and bending stiffnesses of composite laminates [8,9].

As a fundamental study on the structural health monitoring system, the present paper treats an identification problem on bending stiffnesses of symmetrically laminated plates using bending deflection

data under static loading. Firstly, the identification method is shown for three bending stiffnesses of orthotropic plates. Secondly, four or six bending stiffnesses of a general symmetric laminated plate are identified based on the nonlinear optimization technique.

2 BENDING STIFFNESS IDENTIFICATION OF ORTHOTROPIC PLATES

2.1 Fundamental Equation

We consider an orthotropic plate with principal direction parallel to the sides subject to a concentrated force as shown in Fig.1. The fundamental equation for bending is given by

$$D_1 w_{,xxxx} + 2D_3 w_{,xxyy} + D_2 w_{,yyyy} = P \delta(x - x_0) \delta(y - y_0), \quad (1)$$

where D_1 , w and P , respectively, denote bending stiffnesses, a bending deflection and a concentrated force, and δ is the Dirac delta function.

The normalized bending deflection for simply-supported plates is given as follows [10]:

$$w^*(X, Y, \xi, \eta) \equiv \frac{\pi^4}{4b^3} \frac{w}{(P/(aD_0))} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} f_{mn}(X, Y) f_{mn}(\xi, \eta) c_{mn}(\bar{D}_1, \bar{D}_2, \bar{D}_3), \quad (2)$$

where

$$f_{mn}(X, Y) = \sin(m\pi X) \sin(n\pi Y) \\ f_{mn}(\xi, \eta) = \sin(m\pi \xi) \sin(n\pi \eta) \quad (3)$$

$$c_{mn}(\bar{D}_1, \bar{D}_2, \bar{D}_3) = 1 / \left\{ \bar{D}_1 (m/c)^4 + 2\bar{D}_3 (m/c)^2 n^2 + \bar{D}_2 n^4 \right\}$$

and $X=x/a$, $Y=y/b$, $\xi=x_0/a$, $\eta=y_0/b$, $c=a/b$, $\bar{D}_i = D_i / D_0$,

Fig. 1. Orthotropic plate under a concentrated force.

Fig. 2. Measurement points of deflection.

with an adequate value D_0 used for normalization.

From Eq.(2), we can obtain the following relation:

$$c_{ij} = \frac{4}{\sin(i\pi\xi)\sin(j\pi\eta)} \int_0^1 \int_0^1 w^*(X, Y, \xi, \eta) \sin(i\pi X) \sin(j\pi Y) dX dY. \quad (4)$$

When three terms in c_{ij} , for example, c_{11} , c_{12} and c_{21} can be obtained, the normalized bending stiffnesses are determined as follows:

$$\begin{pmatrix} 1 & c^4 & 2c^2 \\ 1 & 16c^4 & 8c^2 \\ 16 & c^4 & 8c^2 \end{pmatrix} \begin{Bmatrix} \bar{D}_1 \\ \bar{D}_2 \\ \bar{D}_3 \end{Bmatrix} = c^4 \begin{Bmatrix} 1/c_{11} \\ 1/c_{12} \\ 1/c_{21} \end{Bmatrix}. \quad (5)$$

When deflections at any location can be measured without any error, c_{ij} and the bending stiffnesses can be identified exactly. Usually we have only a limited number of measured deflection data as shown in Fig.2 where the measurement points are indicated by the grid points. In the example of Fig.2, the total number of measurement points is four and the deflection on the boundary is zero. The values of c_{ij} can be evaluated through the numerical integral of Eq.(4) using deflection data measured at the finite number of location. In the present paper, it is assumed that the exact deflection data can be measured without any error such as a measurement error although the effect of the error on the identification results is an important subject.

Fig. 3. Flow chart for improvement of bending stiffnesses.

2.2 Improvement of Estimated Bending Stiffnesses

In the evaluation of c_{ij} in Eq.(4), the numerical error is involved through the numerical integral. Thus the estimated bending stiffnesses have also the numerical error and thus the deflection $\tilde{w}^*$ corresponding to the estimated bending stiffnesses is different from the measured deflection w^* .

We can improve the estimated bending stiffnesses $\tilde{D}_i$ by minimizing the difference between the measured deflection and that calculated using the estimated bending stiffnesses. The flow chart for improvement is shown in Fig.3. First the bending stiffnesses $\tilde{D}_i$ are evaluated from Eq.(5) using the measured deflection w^* . Then the deflection $\tilde{w}^*$ is calculated from the estimated bending stiffnesses $\tilde{D}_i$. Using Eq.(4), the variation of c_{ij} denoted by Δc_{ij} is evaluated corresponding to the difference of deflections ($w^* - \tilde{w}^*$) and the value of c_{ij} is updated to $c_{ij} + \Delta c_{ij}$. These procedures are continued until convergence is reached.

Figure 4(a) shows the convergence history of three bending stiffnesses for $(\bar{D}_1, \bar{D}_2, \bar{D}_3) = (5.0, 0.5, 1.0)$, where the number of measurement data is four, the aspect ratio of the plate $c = a/b = 1.0$ and the loading point $(\xi, \eta) = (0.2, 0.3)$. Figure 4(b) shows the convergence history of the deflection error between the measured deflection and the calculated deflection which is defined by

(a) Bending stiffnesses.

(b) Deflection error.

 Fig.4. Convergence histories of bending stiffnesses and deflection error for $(\bar{D}_1, \bar{D}_2, \bar{D}_3) = (5.0, 0.5, 1.0)$.

$$\Delta w = \sum_{k=1}^4 (1 - \tilde{w}_k^* / w_k^*) . \quad (6)$$

Figure 5 shows the convergence histories for $(\bar{D}_1, \bar{D}_2, \bar{D}_3) = (0.8, 0.7, 1.2)$, $c = 1.0$ and $(\xi, \eta) = (0.4, 0.3)$. It can be seen from Figs.4 and 5 that both the bending stiffnesses and the deflection converge to the exact ones. It was found from many other examples that four-point measurement is enough to identify three bending stiffnesses of orthotropic laminates if the precise measurement data on bending deflections are available.

3 BENDING STIFFNESS IDENTIFICATION OF GENERAL LAMINATED PLATES

3.1 Fundamental Equation

In this chapter, we treat more general symmetric laminates than orthotropic ones. Boundary conditions are also more general than simply supported conditions. We use the finite element method for the bending deflection analysis of symmetrically laminated plates.

The fundamental equation is given as follows:

(a) Bending stiffnesses.

(b) Deflection error.

 Fig.5. Convergence histories of bending stiffnesses and deflection error for $(\bar{D}_1, \bar{D}_2, \bar{D}_3) = (0.8, 0.7, 1.2)$.

$$[K]\{u\} = \{P\} , \quad (7)$$

where

$$K = K_1 D_1 + K_2 D_2 + K_3 D_3 + K_4 D_4 + K_5 D_5 + K_6 D_6 \quad (8)$$

and

$$\begin{aligned} D_1 &= D_{11}, D_2 = D_{12}, D_3 = D_{66}, \\ D_4 &= D_{22}, D_5 = D_{16}, D_6 = D_{26}. \end{aligned} \quad (9)$$

In Eq.(7), K , u and P , respectively, denote the stiffness matrix, the displacement vector and the force vector. The stiffness matrix is divided to six submatrices corresponding to six bending stiffnesses as shown in Eq.(8). In Eq.(9), D_{11} , D_{12} , D_{22} and D_{66} represent the on-axis bending stiffnesses, and D_{16} and D_{26} represent the bending-twisting coupling terms [10]. The measurement of all displacements involving the angle of deflection is, in fact, difficult and also not all deflections need to be measured.

Therefore, Eq.(7) is partitioned into u_1 and u_2 vectors, corresponding to the measured and unmeasured displacements.

Fig.6. Geometry of laminated plate.

(a) 75-points

(b) 35-points

(c) 21-points

(d) 9-points

Fig.7. Measurement points of deflection.

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{12}^T & K_{22} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} \quad (10)$$

From the second row of Eq.(10), we can obtain

$$u_2 = K_{22}^{-1} (P_2 - K_{12}^T u_1) \quad (11)$$

Substituting Eq.(11) into the first row of Eq.(10), the following equation is obtained:

$$K^* u_1 = P^* \quad (12)$$

where

$$\begin{aligned} K^* &= K_{11} - K_{12} K_{22}^{-1} K_{12}^T, \\ P^* &= P_1 - K_{12} K_{22}^{-1} P_2. \end{aligned} \quad (13)$$

Table 1. Bending stiffnesses of carbon/epoxy laminates.

[75/75/90] _s laminate		[90/45/0] _s laminate	
D_1	3.98×10^{-1}	D_1	7.91×10^{-1}
D_2	3.11×10^{-1}	D_2	3.27×10^{-1}
D_3	4.59×10^{-1}	D_3	4.76×10^{-1}
D_4	3.89×10^0	D_4	3.46×10^0
D_5	1.15×10^{-1}	D_5	2.60×10^{-1}
D_6	8.50×10^{-1}	D_6	2.60×10^{-1}

(unit: N·m)

Vectors u_1 , P_1 and P_2 are obtained from the test data. The analytical stiffness matrices, K_{11} , K_{12} and K_{22} , are functions of six bending stiffnesses. Equation (12) is a nonlinear function of the bending stiffnesses due to inversion of K_{22} .

In order to identify the bending stiffnesses, an optimization problem to minimize an error norm is defined:

$$\text{minimize } E(x)^T E(x), \quad (14)$$

where $E(x) = K^* u_1 - P^*$ and $x^T = (D_1, D_2, D_3, D_4, D_5, D_6)$.

As a nonlinear optimization technique, the Davidon-Fletcher-Powell method was adopted with the golden section method in the ADS program [11].

3.2 Numerical Results

As numerical examples, we consider a laminated composite plate clamped at two edges and free at two edges as shown in Fig.6. The plate is divided into 4x16 rectangular bending elements. The loading point of a concentrated force is also indicated in the figure. The laminate consists of carbon/epoxy composites and two types of laminates $[75/75/90]_s$ and $[90/45/0]_s$ are treated where the bending stiffness properties are shown in Table 1. Figure 7 shows the measurement points of bending deflections from 75-points measurement to 9 points. Only the deflection at the measurement point is measured and the angle of deflection is unmeasured. It is assumed that the exact deflection data can be measured without any error.

(a) Orthotropic laminates without bending-twisting coupling

We treat orthotropic laminates without bending-twisting coupling terms. In Table 1, D_5 and D_6 are set to zero and four bending stiffnesses $D_1 \sim D_4$ are non-zero. The four bending stiffnesses are identified from the measured deflection data where the exact deflection data

corresponding to the orthotropic laminates without bending-twisting coupling are used.

Figure 8 shows the relation between the identified bending stiffnesses and the number of measurement data for $[\pm 75/90]_s$ orthotropic laminate. As the initial bending stiffnesses in the optimization, the values about 10% and 30% deviated from the exact values are used. Figure 9 shows the identification results for $[90/(\pm 45)_{0.5}/0]_s$ orthotropic laminate where $(\pm 45)_{0.5}$ represents ± 45 layers with one ply thickness. It is seen from Figs.8 and 9 that the deflection data more than 21-points are enough to identify the four bending stiffnesses precisely. On the contrary, the 9-points measurement is not reliable to identify the four bending stiffnesses especially for $[\pm 75/90]_s$ laminate using about 30% deviated initial values. This seems to be due to the difficulty of finding the global minimum in Eq.(14) because of the existence of many local minima. In general, less deviated initial values give the better identification results although the

results in Fig.9 are not such a case.

(b) Symmetric laminates

Figure 10 shows the identification results for $[75/75/90]_s$ laminate where six bending stiffnesses involving the bending-twisting coupling terms are identified. About 10% and 30% deviated values are used as initial bending stiffnesses. It is seen that the 35-points measurement is necessary in order to identify the six bending stiffnesses precisely. In the case of small number of measurement data less than 21, it is difficult to identify the small coupling term D_5 although the identification error in the other bending stiffnesses is comparatively small. This seems to be due to the low sensitivity of D_5 on the error norm in Eq.(14). Figure 11 shows the identification results for $[90/45/0]_s$ laminate. The identification results are better as compared with those for $[75/75/90]_s$ laminate. The 21-points measurement gives the identified bending stiffnesses with errors less than several percents.

(a) Initial values with about 10% deviation.

(b) Initial values with about 30% deviation.

Fig.8. Identified bending stiffnesses for $[\pm 75/90]_s$ laminate.

(a) Initial values with about 10% deviation.

(b) Initial values with about 30% deviation.

Fig.9. Identified bending stiffnesses for $[90/(\pm 45)_{0.5}/0]_s$ laminate.

(a) Initial values with about 10% deviation.

(a) Initial values with about 10% deviation.

(b) Initial values with about 30% deviation.

(b) Initial values with about 30% deviation.

Fig.10. Identified bending stiffnesses for [75/75/90]_s laminate.

Fig.11. Identified bending stiffnesses for [90/45/0]_s laminate.

4 CONCLUSIONS

The present paper treated an identification method of bending stiffnesses of symmetrically laminated plates using bending deflection data. The rough estimation and the improvement was suggested for identifying three bending stiffnesses of orthotropic plates. It was shown that the 4-points measurement gives the precise identification results. Next, four bending stiffnesses for orthotropic laminates and six for general symmetric laminates were identified using the nonlinear optimization technique. The number of measured deflection data required for the identification was 21 for orthotropic laminates and 35 for symmetric laminates.

In the present study, we neglect the error involved in the measured deflections. It will be the next important subject to take into consideration the effect of the measurement error on the identification of the bending stiffnesses.

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