Review paper

SOLID MECHANICS UNDER MAGNETIC FIELD

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Abstract: This paper reviews the basis of mechanics for deformable electromagnetic materials, their typical behaviors under the magnetic field and applications utilizing coupled phenomena of magnetism and mechanics. Electromagnetic field equations and the conservation laws for moving deformable materials are laid down, on the basis of continuum theory of electromagnetic materials. If such a material is under the magnetic field, it is subjected to several kinds of magnetic forces. Further if the material is electrically conductive and the magnetic field is changing in time, there may also exist the eddy current, the induced magnetic field, the Lorentz force, etc. In the case of ferromagnetic materials, can not be neglected the magnetic nonlinearity, the hysteresis and the magnetomechanical effects such as magnetostriction and stress-dependence of magnetization. In many cases, the magnetic field is static, periodically changing in time, or applied to the demagnetized material. In these cases, the equilibrium theory of deformable magnetic materials can be applied to each magnetization process in which the magnetic field is changing in one-direction including the case of rotating magnetic fields. Mechanical properties under such magnetic fields and the magnetization properties under the stress are discussed. As related topics to the above phenomena, introduced are magnetoelastic problems, nondestructive evaluations of material properties and deformation, and applications of magnetic forces.

Key words: *Continuum mechanics, Electromagnetic material, Electromagnetic force, Magnetic field, Magnetization, Magnetostriction, Maxwell equations, Nondestructive evaluation, Electromagnetic actuator, Intelligent material*

1. INTRODUCTION

According to the development of electromagnetic machinery, its design becomes required high precision from the view points of both electrical and mechanical engineerings. In particular, mechanics of electromagnetic materials is indispensable in the compound regions of both fields: for example, the effects of electromagnetic field on the structural safety or reliability can not be neglected; the electromagnetic performance depends on the mechanical behavior of the materials; the machinery utilizes the electromagnetomechanical interactions; it has functions of actuators or sensors of mechanical fields, etc.

The purpose of this paper is to review continuum mechanics of solids under magnetic field and typical applications of magnetic materials. In the next section, the basic equations are laid down which govern the behaviors of deformable electromagnetic materials. According to the electromagnetic properties of the material, there may occur many kinds of physical phenomena that are not seen in the absence of magnetic field. In order to emphasize the magnetomechanical interactions, basic behaviors of a ferromagnetic material under a uniaxial stress and the magnetic field are discussed in section 3. If a finite dimension of a magnetized material is in the magnetic field, it suffers several kinds of magnetic forces such as Maxwell stress and the forces due to the gradients of the magnetic quantities. When the magnetic field is strong, these magnetic forces may cause serious problems such as damage and fracture in electric coils, cores, yokes and the structural materials of the machinery. Even if the magnetic field is not so strong, the magnetic forces may give rise to fatigue and structural instabilities like buckling and flutter. If the material is electrically conductive and the magnetic field is oscillating, the electric field and hence

the electric current are induced. The phenomena are applied to electromagnetic heating with high-frequency magnetic field and transducers or actuators. These problems in the framework of magnetoelasticity will be reviewed briefly in section 4. In general, non-magnetic properties, stress/strain, inhomogeneity, fracture/damage etc. alter the magnetic or the magnetomechanical responses of the material. In section 5, we shall introduce several nondestructive methods of the above factors by measuring the change of magnetic or magnetomechanical behaviors of the material. In the last section, applications of magnetic forces such as actuators/transducers, magnetic levitation and lubrication, magnetic damping will be discussed, including functional materials utilizing magnetic materials.

2. BASIC EQUATIONS OF DEFORMABLE ELECTROMAGNETIC MATERIALS

2.1. Maxwell Equations

When a static rigid material is in the magnetic field $\mathbf{H}$ or the electric field $\mathbf{E}$, their induction fields are generated in the material, i.e., the magnetic flux $\mathbf{B}$ or the electric displacement $\mathbf{D}$. The deviation from the external magnetic or the electric field is called the magnetization $\mathbf{M}$ or the polarization $\mathbf{P}$, respectively

$$\mathbf{M} = \mathbf{B} - \mathbf{H}, \quad \mathbf{P} = \mathbf{D} - \mathbf{E}. \quad (1)$$

Then the electromagnetic fields are governed by the Maxwell equations for a fixed Galilean coordinate system

$$\nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = 0, \quad \nabla \cdot \mathbf{B} = 0, \quad (2)$$

$$\nabla \times \mathbf{H} - \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} = \frac{1}{c} \mathbf{J}, \nabla \cdot \mathbf{D} = q_f, \quad (3)$$

where $\mathbf{J}$ is the electric current vector, q_f the electric charge, and c the speed of the light, refer to Landau and Lifshitz [1] for classical electromagnetism. The extension of the Maxwell equations to the case of the electromagnetic fields in deformable materials has been studied by many authors. The formulations in this and the next subsections will follow Maugin [2], but we neglect the effects of the spin, the couple stress and the external couples.

When a rigid material is moving at a uniform velocity $\mathbf{v}$ and the above quantities are observed by the coordinate system moving with the material, from the invariance of the Maxwell equations, the transformation laws of the electromagnetic quantities are derived as

$$\begin{aligned} q_f' &= q_f, \mathbf{J}' = \mathbf{J} - q_f \mathbf{v}, \mathbf{P}' = \mathbf{P}, \mathbf{E}' = \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B}, \\ \mathbf{M}' &= \mathbf{M} + \frac{1}{c} \mathbf{v} \times \mathbf{P}, \mathbf{B}' = \mathbf{B} - \frac{1}{c} \mathbf{v} \times \mathbf{E}. \end{aligned} \quad (4)$$

Transformation laws for $\mathbf{D}$ and $\mathbf{H}$ can be derived from the above equations combined with Eq.(1). The time derivatives in Eq.(2) should be replaced with the convected time derivatives defined by for a vector $\mathbf{A}$

$$\dot{\mathbf{A}}^* = \frac{\partial \mathbf{A}}{\partial t} + \nabla \times (\mathbf{A} \times \mathbf{v}) + \mathbf{v}(\nabla \cdot \mathbf{A}). \quad (5)$$

When the material is being deformed, we should specify the coordinate system to describe the field equations, because many scalars or vectors are evaluated in unit volume or surface area. Sometimes, the Lagrangian or the material coordinate system embedded in the material is convenient to describe the material symmetry conspicuous in a undeformed state, and the Eulerian or the spatial (current) coordinate system to describe the boundary conditions of a problem. The material (Bold Italic) and the spatial (Gothic) forms of a vector are related as

$$\mathbf{A} = \mathbf{J} \mathbf{F}^{-1} \mathbf{A}, \quad (6)$$

where $\mathbf{J}$ is the Jacobian of the deformation and $\mathbf{F}$ the deformation gradient. Define the Lagrangian forms $\mathbf{B}$, $\mathbf{D}$ and $\mathbf{J}$ of $\mathbf{B}$, $\mathbf{D}$ and $\mathbf{J}$ by Eq.(6), respectively, and $\mathbf{E}$, $\mathbf{H}$ and Q_f for $\mathbf{E}$, $\mathbf{H}$ and q_f by

$$\begin{aligned} \mathbf{E} &= \mathbf{E} \mathbf{F} + \frac{1}{c} (\mathbf{F}^{-1} \mathbf{v}) \times \mathbf{B}, \mathbf{H} = \mathbf{H} \mathbf{F} - \frac{1}{c} (\mathbf{F}^{-1} \mathbf{v}) \times \mathbf{D}, \\ Q_f &= J q_f. \end{aligned} \quad (7)$$

Then we have the Maxwell equations in the material form

$$\begin{aligned} \nabla_R \times \mathbf{E} + \frac{1}{c} \dot{\mathbf{B}} &= 0, \nabla_R \cdot \mathbf{B} = 0, \\ \nabla_R \times \mathbf{H} - \frac{1}{c} \dot{\mathbf{D}} &= \frac{1}{c} \mathbf{J}, \nabla_R \cdot \mathbf{D} = Q_f \end{aligned} \quad (8)$$

derived by McCarthy [3] and Nelson [4], where $\mathbf{v}$ is the velocity of the material particle and ∇_R denotes the gradient in the material coordinate system.

We see that some of the physical quantities defined in the case of moving or deformed materials contain the non-dimensional coefficients $\mathbf{v}/c$ so that the Maxwell equations are invariant in the form under the Galilean transformation. Such terms indicate a special-relativistic effects and play significant roles in elementary physics and astrophysics. Wang [5] discussed relativistic electromagnetism including general relativity. Although such relativistic effects should be taken into account for intrinsic consistency of the theory, we shall neglect the relativistic terms in what follows, because we are interested in engineering applications. In other words, we restrict ourselves to a non-relativistic theory of electromagnetism. Then any quantity with prime defined for a moving material in Eq.(4) except the free charge, can be identified by the one without prime observed in a fixed reference coordinate system.

As in the mechanical field equations, the boundary conditions for electromagnetic fields are derived from the integral form (volume expression) of the Maxwell equations. For example, at the boundary of two undeformed magnetic insulators in a static magnetic field, we have

$$[\mathbf{B}] \cdot \mathbf{n} = 0, [\mathbf{H}] \times \mathbf{n} = 0, \quad (9)$$

where a bracket denotes the jump of a quantity across the interface of two materials and $\mathbf{n}$ the unit normal to the interface, see [1] and [2] for details.

2.2. Conservation Laws

Another set of equations which govern the dynamic behavior of electromagnetic materials are the conservation laws of the mass, the linear and the angular momentums and the energy expressed in the following forms:

$$\dot{\rho} + \rho \nabla \cdot \mathbf{v} = 0, \quad (10)$$

$$\rho \dot{\mathbf{v}} = \nabla \cdot \mathbf{t} + \mathbf{f} + \mathbf{f}^{\text{EM}}, \quad (11)$$

$$\mathbf{t} \text{ is symmetric}, \quad (12)$$

$$\rho \dot{e} = \text{tr}(\mathbf{t} \mathbf{L}^T) + \mathbf{B} \cdot \dot{\mathbf{M}} + \rho \mathbf{E}' \cdot \dot{\boldsymbol{\pi}} + \mathbf{J}' \cdot \mathbf{E}' - \nabla \cdot \mathbf{q} + \rho h, \quad (13)$$

where ρ is the density, $\mathbf{t}$ the Cauchy stress, $\mathbf{f}$ the mechanical external force, $\mathbf{f}^{\text{EM}}$ the electromagnetic external force, e the internal energy density, $\mathbf{L}$ the velocity gradient, $\boldsymbol{\pi}$ the polarization density, $\mathbf{q}$ the heat flux, and h the heat supply. If we define a symmetric elastic stress tensor $\boldsymbol{\sigma}$:

$$\sigma_{IJ} = t_{(IJ)} + E'_{(I} P_{J)} + B_{(I} M'_{J)}, \quad (14)$$

where a parenthesis indicates the symmetric part of two indices. Then conservation law of the linear momentum reduces to a convenient form

$$\rho \dot{\mathbf{v}} = \nabla \cdot \boldsymbol{\sigma} + \mathbf{f} + \mathbf{f}^{\text{EM}}, \quad (15)$$

where the electromagnetic external force is expressed in the form

$$\begin{aligned} \bar{\mathbf{f}}^{\text{EM}} = & (q_f - \nabla \cdot \mathbf{P})\mathbf{E}' + \frac{1}{c}(\mathbf{J}' + \dot{\mathbf{P}}) \times \mathbf{B} \\ & + (\nabla \mathbf{B}) \cdot \mathbf{M}' - (\nabla \cdot \mathbf{M}')\mathbf{B} - (\mathbf{M}' \cdot \nabla)\mathbf{B}. \end{aligned} \quad (16)$$

The first and the second terms of the right hand of the above equation stand for an effective Lorentz force and the other terms the forces originated by the inhomogeneous distribution of the magnetic quantities. These electromagnetic interactions in deformable solids have been studied by Grot [6] and developed by Maugin [2]. There are many alternative forms of stress tensor (14) and the electromagnetic force (16). Some of them are equivalent by exchanging several terms between the stress tensor and the electromagnetic external force or by taking into account the definitions of the quantities and the reference coordinate systems, while the different formulations reflect the interactions in consideration. Another consistent set of the Maxwell equations and the conservation laws were derived and discussed by Pao et al. [7,8] and Parkus [9]. Four typical formulations of the Maxwell equations in moving media were reviewed in [8].

2.3. Constitutive Equations

The basic equations discussed so far are mathematically completed by assigning the constitutive equations:

$$\begin{aligned} \mathbf{B} = \mathbf{B}(\mathbf{H}, \dots), \quad \mathbf{D} = \mathbf{D}(\mathbf{E}, \dots), \quad \mathbf{J} = \mathbf{J}(\mathbf{E}, \dots), \\ \mathbf{q} = \mathbf{q}(\nabla_R \theta, \dots), \quad \mathbf{t} = \mathbf{t}(\mathbf{e}, \dots), \quad e = e(\mathbf{e}, \theta, \dots) \end{aligned} \quad (17)$$

expressed in the material form, where $\mathbf{t}$ is the convected stress and $\mathbf{e}$ the Lagrangian strain tensor. Constitutive equations (17) are not necessarily functions of the arguments, i.e., they may include any kind of expressions so that the current values of the left hand sides are determined by the past histories, the rates or the gradients of the arguments. Thus electromagnetic materials as well as mechanical ones can be classified, from the constitutive equations, into reversible (non-hysteretic or equilibrium) type, rate type, integral type, abstract functional type, non-local type, etc. The argument explicitly written in each constitutive equation in Eq.(17) except the internal energy denotes the primary variable in the sense that it is the unique argument in a classical theory and its contribution is predominant in a general case. From the extent of dependence on the primary variables, the electromagnetic materials are classified into, magnetic (magnetized), polarized, electrically conductive, heat conductive, etc. A dielectric material is the case of polarized electrical insulators.

When the material is reversible, the constitutive equations except the internal energy are derived from the assumption that the conservation law of energy holds for any admissible process. For example, neglecting thermal effect and electrical conductivity and assuming that $e = e(\mathbf{e}, \mathbf{P}, \mathbf{M})$, we have in the material form

$$\sigma = \rho_0 \frac{\partial e}{\partial \mathbf{e}}, \quad \mathbf{E} = \rho_0 \frac{\partial e}{\partial \mathbf{P}}, \quad \mathbf{B} = \rho_0 \frac{\partial e}{\partial \mathbf{M}}, \quad (18)$$

where ρ_0 is the material density in the reference state. If the material has dissipation, the second law of thermodynamics plays a similar role to derive the constitutive equations. Let us list several works on general constitutive equations for electromagnetic materials. Maugin [10] developed a relativistic deformation theory of electromagnetic materials on the basis of thermodynamics, and derived the electromagnetic forces, couples and powers. Maugin and Daher [11] considered the constitutive equations of elastic semiconductors which exhibit many electromagnetic or coupling effects with mechanical fields. Curtis and Lianis [12] derived relativistic constitutive equations for electromagnetic materials with memory corresponding to Rivlin-Ericksen materials. Coleman and Dill [13] showed thermodynamic reduction of constitutive equations for electromagnetic materials with fading memory. Jordan and Eringen [14] have discussed a general approach to the nonlinear theory of electromagnetic materials including the solutions for typical problems, and derived polynomial tensor expansions of the constitutive equations available to other formulations. Eringen [15] presented nonlocal theory of electromagnetic materials with memory, and applied the theory to infrared dispersion, lattice vibration, natural optical activity, skin effects and superconductivity etc. Ersoy and Kiral [16] formulated a dynamic theory of electromagnetic materials with viscosity, thermal conductivity and anisotropy. Lee [17] derived a variational principle for elastic dielectric crystals.

2.4. Magnetic Materials

The continuum theory of deformable magnetic materials started from the study of magnetoelastic couplings in saturated ferromagnetic materials by Tiersten [18] and Brown [19]. The single crystal or a grain of a polycrystalline ferromagnetic material consists of magnetic domains each of which is magnetically saturated. When the external magnetic field is applied to the body, the domain structure changes so that the total of the magnetic and magnetomechanical energies determined by the magnetic field, the magnetic flux, the deformation, the boundary conditions etc. takes a local minimum. Exchange energy of magnetization defined in terms of the gradient of magnetization field is one of the concept characterizing the averaged distribution of the domain structure. If we pay attention to the local magnetic spin in each domain, the variation of the spin gives rise to the macroscopic behavior of magnetization process, i.e., the nonlinearity and the hysteresis of $M-H$ curve, the magnetic saturation, magnetic relaxation etc. The theory of magnetic materials based on such micro-structure is called micromagnetism, which is important to physical interpretation of the magnetic phenomena, mesoscopic derivation of constitutive equations and a class of dynamic phenomena like magnetoacoustic resonances. There is also a case that 'ferromagnetism' refers to such frame work, especially the theory adopting the local magnetic spin. In this paper, however, the term will be used to express a wide sense of magnetic

materials with large magnetic permeabilities. We do not step into the discussions on micro-magnetism except introducing several articles, because we are interested in global theories applicable to many engineering problems. Maugin [20], taking into account the couplings between the elastic and the magnetic fields such as spin-phonon interactions, presented the foundation of continuum theory of micromagnetism. Van den Bern [21] presented a domain theory of soft-ferromagnetic materials, based on the micromagnetic equilibrium. Veljkovic [22] considered an elastic micropolar material, specially a magnetically-saturated solid. Pao and Yeh [23] proposed a general theory of soft ferromagnetic elastic solids, where the magnetic hysteresis and the exchange energy are neglected. McCarthy [24] presented, not based on micromagnetism, a theory of magnetic materials with heat conduction and with fading memory. Biricikoglu and Kalnins [25] derived the governing equations for an elastic Cosserat surface with interaction with magnetic field. It should be noted that Kaliski derived linear governing equations for many kinds of electromagnetic materials, without separating the constitutive equations, and discussed dynamic magnetic and magnetomechanical problems, e.g., see [26-28]. At the end of this section, let us introduce theories of magnetic hysteresis, not concerned with deformable materials, but promising to further extension to the general case. Jiles and Atherton [29] calculated the magnetization curves of ferromagnetic materials on the basis of the domain wall motion and rotation. Maugin [30] discussed the compatibility of magnetic hysteresis with dissipation inequality of thermodynamics and interpreted several physical concepts on magnetization such as residual magnetization, commutative residual magnetization, magnetic loading function, the first magnetization curve etc.

3. BASIC PROPERTIES OF MAGNETIC MATERIAL

Many structural materials are ferromagnetic, and it is necessary to predict their mechanical properties under the magnetic field for an electromagnetic machinery and the structure subjected to the magnetic field. On the other hand, mechanical effects on the magnetic properties of materials are getting important to obtain high performance of motors, generators, transformers etc. in practical conditions. In this section, let us consider the basic mechanical and magnetic properties of such a material under the magnetic field and the stress. In general, from magnetomechanical interactions, the magnetic properties of a ferromagnetic material depend on the mechanical conditions like deformation or stress, and vice versa. Schneider et al. [31] presented a model of the magnetoelastic effects based on domain structure, and compared the result with experimental data on high strength steels. Sandlund et al. [32] measured the magnetostriction, magnetization and Young's moduli under magnetic field of giant magnetostrictive materials, Terfenol-D. Utsunomiya et al. [33] considered the stress effect on the coefficients which appear in saturation magnetization law of carbon steel. Yamazaki et al. [34] obtained the magnetostriction curve for low carbon steel under parallel and normal uniaxial stresses.

As theoretical analysis on magnetoelastic effects, Sa-

blik et al. [35] extended Jiles and Atherton's theory to explain the stress dependence of hysteretic magnetization curve, which was developed by Sablik and Jiles [36] to interpret hysteresis of magnetostriction and its stress dependence.

In the rest of this section, let us derive several concepts of stress-magnetization effects in deformed magnetized materials from the constitutive equations, cf. Matsumoto [37]. As a magnetoelastic material with hysteresis, we consider the rate-type constitutive equations for the stress and the magnetic field instead of the magnetic flux

$$\begin{aligned}\dot{t}_{ij} &= P(\mathbf{t}, \mathbf{e}, \mathbf{H}, \mathbf{M})_{ijk}^{\pm} \dot{M}_k + Q(\mathbf{t}, \mathbf{e}, \mathbf{H}, \mathbf{M})_{ijkl}^{\pm} \dot{E}_{kl}, \\ \dot{H}_i &= R(\mathbf{t}, \mathbf{e}, \mathbf{H}, \mathbf{M})_{ij}^{\pm} \dot{M}_j + S(\mathbf{t}, \mathbf{e}, \mathbf{H}, \mathbf{M})_{ijk}^{\pm} \dot{E}_{jk},\end{aligned}\quad (19)$$

where dot denotes time derivative, double indices are summed from 1 to 3. Superscripts $\pm$ of the coefficients distinguish the different functional forms corresponding to increasing or decreasing magnetic field. This condition gives rise to the hysteresis in the M - H curve. From an analogy of the loading surface in plasticity, the criterion for increasing or decreasing magnetic field is given by the sign of the rate of a scalar function for the process, i.e., a generalized magnetic loading function

$$\dot{f}(\mathbf{t}, \mathbf{e}, \mathbf{H}, \mathbf{M}) \geq 0. \quad (20)$$

The surface defined by $f=\text{constant}$ in the space of the arguments may be called the neutral magnetization surface, along which both functional forms of the coefficients should be the same from physical consistency. The condition imposes a restriction to the tensorial forms of the coefficients in Eq.(19), which is not discussed here. Suppose that both forms of the constitutive equations satisfy the total differentiation conditions and the sign of Eq.(20) remains unchanged in a time interval. Then for each initial values of $\mathbf{t}$, $\mathbf{e}$, $\mathbf{H}$ and $\mathbf{M}$, Eq.(19) can be integrated as

$$\begin{aligned}t_{ij} &= \Phi(\mathbf{e}, \mathbf{M}; \mathbf{t}_0, \mathbf{e}_0, \mathbf{H}_0, \mathbf{M}_0)_{ij}^{\pm}, \\ H_i &= \Psi(\mathbf{e}, \mathbf{M}; \mathbf{t}_0, \mathbf{e}_0, \mathbf{H}_0, \mathbf{M}_0)_i^{\pm},\end{aligned}\quad (21)$$

where subscript 0 denotes the initial value of the quantity from which the current process started. Equation (21)₁ expresses the stress-strain relation depending on the magnetization, and Eq.(21)₂ the M - H relation depending on the strain. The constitutive equations are rate-independent because Eq.(19) is homogeneous in the time derivatives.

Suppose that the material is homogeneous and isotropic and the stress and the magnetic field are uniform and quasi-static. We further assume that the deformation is small so that the constitutive equations can be linearized with respect to the strain. This is not the case for the magnetization, because we want to take into consideration nonlinearity, saturation, and hysteresis in the magnetization. Then we have the following constitutive equations which is linear in $\mathbf{e}$ (ϵ) but nonlinear in $\mathbf{M}$ (M)

$$\begin{aligned} t_{ij}^{\pm} &= P(M)_{ij}^{\pm} + Q(M)_{ijkl}^{\pm} e_{kl}, \\ H_i^{\pm} &= R(M)_i^{\pm} + S(M)_{ijk}^{\pm} e_{jk}, \end{aligned} \quad (22)$$

where we omit the initial values of the process in the coefficients. Here the Lagrangian and Eulerian expressions can be identified in this case. Let us apply the magnetic field and the uniaxial stress along X_1 -axis:

$$\mathbf{M} = (M, 0, 0), \quad t_{11} = t, \quad t_{\alpha\beta} = 0 \text{ except } t_{11}. \quad (23)$$

Fig.1. Parallel magnetostriction under stress for low carbon steel.

Fig.2. Stress coefficient in parallel magnetostriction.

Substituting Eq.(23) into Eq.(22), solving the result for the strain $e_1 = e_{11}$ and $e_2 = e_{22}$, we have the total strains under the stress and the magnetic field in the form

$$e_1 = e(M)_{10}^{\pm} + e(M)_{11}^{\pm} t, \quad e_2 = e(M)_{20}^{\pm} + e(M)_{21}^{\pm} t. \quad (24)$$

The pure elastic strains by the uniaxial stress t in the demagnetized state are given by putting $M=0$ in Eq.(24) as

$$e_1^E = e(0)_{11} t, \quad e_2^E = e(0)_{21} t, \quad (25)$$

where $e(0)_{10}^{\pm} = e(0)_{20}^{\pm} = 0$. Thus the magnetostrictions

under the uniaxial stress are given by subtracting the pure elastic strains (25) from the total strains (24) as

$$\begin{aligned} e_1^M &= e(M)_{10}^{\pm} + [e(M)_{11}^{\pm} - e(0)_{11}^{\pm}] t, \\ e_2^M &= e(M)_{20}^{\pm} + [e(M)_{21}^{\pm} - e(0)_{21}^{\pm}] t. \end{aligned} \quad (26)$$

Fig.3. M - H relation under stress.

Fig.4. Stress coefficient in magnetization.

Here the first terms on the right hand sides are the magnetostrictions in the parallel and the normal directions to the magnetic field in the absence of the stress. The second terms are the variation of the magnetostrictions by the stress. Figure 1 shows the parallel magnetostriction $(26)_1$ for a cyclic magnetic field with sufficiently low frequency, where the first term on the right hand side of Eq.(26) is given by the curve for $t=0$. By rearranging the data in Fig.1 at each magnetization, we can obtain the coefficients of the stress in Eq.(26)₁ as in Fig. 2. These and the following experimental figures are for low carbon steel, and henceforth the explanations will be omitted in the text.

On the other hand, substitution of Eq.(24) into Eq.(22)₂ yields the M - H relation under the uniaxial stress t

$$H_1 = H(M)_{10}^{\pm} + H(M)_{11}^{\pm} t. \quad (27)$$

If we solve Eq.(27) for M , by linearizing again the result with respect to t , we obtain the alternative form of M - H

relation

$$M_1 = M(H)_{10}^+ + M(H)_{11}^+ t. \quad (28)$$

The first terms of Eqs.(27) and (28) are the M - H relations in the absence of the stress, see Figs.3 and 4.

According to Saito et al. [38], the magnetization under a fixed magnetic field changes in a hysteretic manner by loading and unloading of uniaxial stress for soft magnetic amorphous ribbon. Yamamoto and Sasaki [39] analyzed the magnetization change due to a stress after demagnetization by domain structure observation. Such stress-magnetization effects can be also described by assigning appropriate H and t in the constitutive equations (22).

4. MAGNETOELASTICITY

In this section we consider a magnetoelastic material without magnetic hysteresis, and expand the constitutive equations in the strain and the magnetization around the stress-free and demagnetized state. The deformation is assumed to be small, so that the strain may be regarded as an infinitesimal one, and Lagrangian and Eulerian descriptions are not distinguished. The constitutive equations are the extension of linear elastic ones, but it can describe a large number of problems which appear in devices, instruments, machines and structures associated with magnetism. Since the internal energy is unchanged by reverse of the magnetization, it should be an even function with respect to it. Thus the constitutive equations can be expanded into the second order of M in the form

$$\begin{aligned} t_{ij} &= P_{ijkl} M_k M_l + (Q_{ijmn}^{(0)} + Q_{ijklmn}^{(1)} M_k M_l) e_{mn}, \\ H_i &= R_{ij} M_j + (S_{imn}^{(0)} + S_{ijklmn}^{(1)} M_k M_l) e_{mn}. \end{aligned} \quad (29)$$

As shown in the preceding section, the second term of the coefficients of e gives rise to the magnetoelastic interactions. Since the surface charge and the polarization vanish, the electromagnetic force (16) takes the form

$$\bar{f}^{\text{EM}} = \frac{1}{c} \mathbf{J} \times \mathbf{B} + (\nabla \mathbf{B}) \cdot \mathbf{M} - (\nabla \cdot \mathbf{M}) \mathbf{B} - (\mathbf{M} \cdot \nabla) \mathbf{B}. \quad (30)$$

Magnetomechanical problems based on Eqs.(29) and (30) belongs to the most general case in magnetoelasticity.

If the interactions between the mechanical and the electromagnetic fields are weak, it is convenient to define the deviation of magnetic quantities, h , b , m from the bias fields with suffix 0 in the form

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{h}, \mathbf{B} = \mathbf{B}_0 + \mathbf{b}, \mathbf{M} = \mathbf{M}_0 + \mathbf{m}. \quad (31)$$

The bias fields are given by the solution of the Maxwell equations in the case when the material is a rigid body. The deviations in Eq.(31) generated by the mechanical deformations are small in a similar order to the deformation. In this way, the governing equations can be linearized in the deformation and the deviated magnetic quantities.

Many structural problems in the magnetic field can be formulated in this manner, including dynamic cases such as mechanical vibrations and wave propagations.

Further if the material is uniform and isotropic and the magnetomechanical interactions such as dependence of the elastic coefficients on the magnetic field and that of the magnetic permeability on the strain are small, we have the classical constitutive equations for conductive magnetic materials

$$\begin{aligned} \mathbf{t} - \mathbf{t}_0 &= \lambda (\text{tr} \mathbf{e}) \mathbf{I} + 2\mu_e \mathbf{e}, \\ \mathbf{B} &= \mu \mathbf{H}, \mathbf{J} = \eta \mathbf{E}, \end{aligned} \quad (32)$$

where λ and μ_e are Lamé's constants, η the electric conductivity, and $\mathbf{t}_0$ the Maxwell stress defined by

$$t_{0ij} = \frac{1}{2} (H_i B_j + H_j B_i) - \frac{1}{2} H_k B_k \delta_{ij}. \quad (33)$$

Fig.5. Maxwell Stress under Magnetic Field.

In order to see how the Maxwell stress (33) acts to the material, we apply the magnetic field in X_3 -direction as in Fig 5. When the material is uniformly magnetized, the magnetization and hence the magnetic flux are parallel to H . Then from Eqs.(32) and (33) we have

$$t_{011} = t_{022} = -\frac{\mu}{2} H_3^2, t_{033} = \frac{\mu}{2} H_3^2. \quad (34)$$

Thus the material is subjected to the extension and the compression in the parallel and the perpendicular directions to the magnetic field. When a beam or a plate is subjected to a strong magnetic field in the normal direction to its axis or plane, the Maxwell stress may induce magnetoelastic bucklings, cf. [20].

When the material is not polarized $\mathbf{D}=\mathbf{0}$, by eliminating the electric field from the Maxwell equations, we have a parabolic type of differential equation for H

$$\Delta H = \frac{\eta \mu}{c^2} \frac{\partial H}{\partial t}, \quad (35)$$

where note that the electric conductivity η and the magnetic permeability μ take different values in the interior or the exterior of the material. Since the magnetic permeability of the material is large in the jump relation (9) across the material surface, the normal component of the magnetic field in the material is negligible, and the magnetic field should be parallel to the material surface X_1 - X_2 plane. When the external magnetic field is oscillating, the

magnetic field in the material near the surface can be expressed as

$$H_1 = H_{10} \exp(-i\omega t), H_2 = H_{20} \exp(-i\omega t), H_3 = 0. \quad (36)$$

Fig.6. Eddy current by oscillating magnetic field.

If the frequency is sufficiently large, as in the case of heat conduction, the magnetic field can not permeate into the material. That is, the intensity of the magnetic field decays in the depth direction of the material with attenuation factor $\exp(X_3/\delta)$, where δ is the skin depth proportional to $1/\sqrt{\omega}$. In this case, the electric field with the same frequency, but with $\pi/4$ phase lag, is induced such that

$$E = \sqrt{\omega\mu/2\eta}(1-i)\mathbf{n} \times \mathbf{H}, \quad (37)$$

where $\mathbf{n}$ is the unit normal to the material surface. When the problem is axisymmetric around X_3 axis, the counter lines of the electric field intensity constitute concentric circles. If the material is an electric conductor, the electric current along the concentric circles, an eddy current, is induced which is also localized near the material surface as shown in Fig.6. Then the material is subjected to the Lorentz force in addition to the Maxwell stress (33)

$$\mathbf{f}^{\text{EM}} = \frac{1}{c} \mathbf{J} \times \mathbf{B} = \frac{1}{c} \sqrt{\omega\mu\eta/2}(1-i)\mathbf{n} \times \mathbf{H}. \quad (38)$$

In view of Eqs.(37) and (38), the eddy current and the Lorentz force are large for materials with large magnetic permeability. In the case of a pulsed magnetic field, we can obtain the induced electric field, eddy current and the Lorentz force by applying the above discussion to each frequency component of the magnetic field.

Many problems in magnetoelasticity are within the frame work discussed so far. Moon [40] summarized several static and dynamic magnetoelastic problems, in particular for conducting materials under a dynamic magnetic field. Parkus [9] considered magnetoelastic materials with defects like dislocations and applied the result to inelastic deformation in the magnetic field.

Vibrations of structures and wave propagations in magnetic materials have been studied from several points of view. In a basic study of constitutive equations, magnetomechanical properties of the material are sometimes conspicuously observed in wave motion coupled with

electromagnetic fields. Other aspects are interests in the dynamic structural vibrations in magnetic field and applications to nondestructive measurement. Instead of introducing many studies done on dynamic magnetoelasticity, refer to the review articles, i.e., Kaliski [41] for wave motion in classical linear magnetoelasticity, Ambartsu-mian [42, 43] for vibrations of thin plates, strips and shells, and Shido [44] for behavior of cracks under static and dynamic magnetic fields.

Nonlinear wave motions in magnetoelastic materials are interesting as mathematical physics and applications to magnetoacoustic devices. For example, the magnetization propagates along a wire as a magnetomechanical coupled wave with a speed near the one of elastic waves, which enables us to constitute magnetic delay lines in electric circuits. Bazer and Ericson [45] formulated nonlinear waves in general magnetoelastic materials. Maugin and Miled [46] showed existence of solitary waves in ferromagnetic materials, and Domanski [47] studies resonant phenomena in magnetoelastic materials.

5. NDE BY MAGNETISM

We have seen that the magnetomechanical behaviors of magnetic materials depend not only on the material properties but on their distribution, the geometry of the body and the bias fields like the external magnetic field, the initial magnetization, the initial strain or the initial stress. In other words, by observing the behavior of the material under the magnetic field, we can obtain a certain information on such factors. In this section, we shall consider typical Non-Destructive Evaluation methods associated with magnetic properties and magnetomechanical interactions. Jiles [48] summarized several advantages of magnetic methods for NDE compared with ultrasonics and eddy currents. Bakker and Van den Berg [49] reviewed the eddy current method and magnetostriction method for inspection of cracks in pipeline.

5.1. Eddy Current and Leak Flux

Eddy current by oscillating or pulsed external magnetic field induces the magnetic field change in the opposite direction to the bias one. Since the eddy current is localized near the material surface, the intensity of the current is affected by the geometrical conditions, the local electric conductivity and magnetic permeability near there. When the bias magnetic field is excited by electric coil, the induced magnetic field changes the impedance of the coil, which is the base of Eddy Current Testing of flaws, damages and electromagnetic properties of the material. In experimental NDE techniques, evaluation is achieved by referring to the correlation between the field change and the object obtained by experiences. Spal and Kahn [50] considered detection of cracks in the cylinder by eddy current method. Remote field eddy current technique was presented by Schmidt [51] for the problems such that far electromagnetic fields are more sensitive to defects such as cracks inside the pipe. Dover and Zhou [52] obtained a theoretical solution for alternative current by magnetic anisotropy induced by stress and presented several applications for stress evaluation in tension and bending, pipe stress etc. In general, analytical solutions

can not be expected for practical problems, so that numerical analysis using FEM and BEM has been developed. In many problems of eddy currents, it is convenient to convert the Maxwell equations by means of a magnetic vector potential and an electric scalar potential. Miya et al. [53] proposed the current vector potential method, T-method for analysis of eddy current.

Even when the external magnetic field is static or slowly changing according to scanning the probe, the magnetic field or flux near the material is influenced by defects, damage, fatigue, deformation or stress and inhomogeneity of the material properties. Magnetic Leakage Flux Testing is the method for nondestructive evaluation by measuring the magnetic flux change outside the material. Sekiguchi et al. [54] obtained the relations between uniaxial stress and magnetic flux density outside of the material. Gottvald [55] formulated the inverse problem for reconstruction of the original image of the object from altered magnetic field. As a sensing technique, magnetic particle method is used to visualize the distribution of magnetization or magnetic flux, which is available on site. Lewis [56] presented a theoretical model for detection and measurement of metal fatigue cracks by this method.

5.2. Change of Magnetization Curve

The magnetization curve depends on many material factors as well as the deformation or the stress discussed in section 3. For quantitative measurement, the magnetization curve should be characterized by a parameter sensitive to the factors to be measured, e.g., the magnetic permeability or the susceptibility for the initial, cyclic or anhysteretic magnetization, the coercivity, the remanence (residual magnetization), the hysteresis loss, the saturation magnetization, etc. Several NDE methods and their applications using such parameters of the magnetic hysteresis curves were reviewed in [48]. Dobranski et al. [57] showed dependence of the anhysteretic magnetization curve on the uniaxial stress. Sekiguchi et al. [58] obtained the stress-permeability relations for ferromagnetic steel by measuring the magnetic flux outside of the material. Endo et al. [59] applied the wave analysis of magnetically induced current for NDE of heat treatment condition of carbon steel. Kashiwaya [60] utilized the magnetization in a weak magnetic field for stress measurement in ferromagnetic materials and explained the related magnetomechanical interactions from domain structure. Nakata et al. [61] measured the local magnetic flux density by setting the specimen itself as search coil to obtain the residual stress distribution due to cutting. Wakiwaka et al. [62] applied the stress-induced anisotropy in the magnetic permeability to measure the distribution of residual stress.

5.3. Magnetostriction

Similarly to the magnetization curve, the magnetostriction curve depends on the material properties and the deformation. The advantage of the method is that the local or the surface stress or the strain can be measured by conventional strain gages even when the applied magnetic field is not localized. Atherton and Szpunar [63] measured the magnetostriction for NDE of stress in pipeline steel. Yamasaki et al. [64] aimed at the stress dependence

of the magnetostriction to measure the residual stress in steel. Iino [65] used foil and film of magnetostrictive materials to measure the surface stress/strain in non-magnetized materials. Wun-Fogle et al. [66] proved large sensitivity of magnetostrictive strain gages.

5.4. Barkhausen Noise and Magneto-Acoustic Emission

During magnetization process of a ferromagnetic material, domain wall movement intermittently occurs due to the pinning effects by dislocations, grain boundaries, inclusions, voids, local stress distribution etc. The sudden change of the magnetization induces electromagnetic waves, so called Barkhausen Noise, which is sensitive to the above factors. Parakka et al. [67] studied the change of BN due to the residual stress and surface conditions. Furuya et al. [68] applied BN to evaluation of low cycle fatigue in steel. Several NDE techniques for steel by BN was reviewed by Nishihara [69].

Fig.7. MAE distribution for half cycle of magnetization.

Fig.8. Stress dependence of MAE.

Each domain of the ferromagnetic material has spontaneous magnetostriction whose principal axis coincides with the direction of the magnetization of the domain. Thus the abrupt movement of a domain wall during magnetization process excites also elastic waves, so called Magneto-Acoustic Emission, see Figs.7 and 8. In addition to a similar feature to BN method, MAE has advantage that conventional AE sensors and data processing technique are available. Kim and Kim [70] studied the effect of magnetizing frequency and stress on MAE for silicon steel. Namkung et al. [71] discussed the difference in the

dependence of the MAE characteristics to tensile and compressive stresses. Ng et al. analyzed the dependence of MAE on uniaxial stress [72] and on the biaxial stress [73]. Dhar et al. [74] found the magnetizing frequency dependence of the number of MAE signals and discussed the result based on the existing models of domain wall motion. Shinke et al. [75], by developing automatic MAE analyzer, presented the dependence of the number and the amplitudes of MAE signals on stress and hardness of SS400 steel. Tsukada [76] reviewed the application of several magnetic NDE methods to maintenance of wire ropes.

5.5. Magnetoacoustoelasticity

Acoustoelastic effects indicate the dependence of the speed of an acoustic wave on the bias strain or the stress due to the mechanical nonlinearity. Such speed change is small in a similar order to the one by the material anisotropy induced by processing, the temperature effect and the experimental looseness errors, which brings difficulties in practical use of acoustoelasticity. For ferromagnetic materials such as steel, from magnetomechanical interactions the speed shift of a wave due to the stress depends also on the magnetic state of the material. Magnetoacoustoelasticity was originally proposed to avoid the above difficulties by taking the difference of the speeds in the demagnetized and the magnetized states. For example, Kwun et al. [77-79] measured and discussed stress dependence of the speed shift of shear, longitudinal and surface waves by magnetic field. Namkung et al. [80] also examined the stress dependence of magnetically induced speed change in bulk compressive wave and Layleigh waves. Yamasaki et al. [81] measured the stress dependence of acoustoelastic constants on the magnetic field for mild steel. Matsumoto et al. [82-84] theoretically derived the magnetoacoustoelastic formulas from nonlinear constitutive equations. They [85] also proposed a new technique to evaluate the plane stress components from the speeds of a single wave in different magnetized states.

In order to derive the magnetoacoustoelasticity laws, we start from the nonlinear constitutive equations for the Kirchhoff stress and the magnetic field. We consider small perturbation excited by ultrasonic waves from a static and uniform state subjected to the stress and the magnetic field. Then the constitutive equations combined with the other basic equations (2), (3), (11) and (12) yield the governing equations for the small displacement $\mathbf{u}$ and the deviation of the magnetization $\mathbf{m}$ in this form

$$\begin{aligned} \mathcal{E}_{IJKL} \frac{\partial^2 u_K}{\partial X_J \partial X_L} + \mathcal{A}_{IJK} \frac{\partial m_K}{\partial X_J} &= \rho_0 \frac{\partial^2 u_I}{\partial t^2}, \\ (\mathcal{A}_{IJKL} + \delta_{IJ}) \frac{\partial m_J}{\partial X_I} + \mathcal{B}_{IJKL} \frac{\partial^2 u_J}{\partial X_I \partial X_K} &= 0, \\ \varepsilon_{IJK} (\mathcal{A}_{KL} \frac{\partial m_L}{\partial X_J} - \mathcal{L}_{KLM} \frac{\partial^2 u_L}{\partial X_J \partial X_M}) &= 0, \end{aligned} \quad (39)$$

where ε_{IJK} is the permutation symbol and we omit the exact expression of the coefficients, see [85]. Note that

they depend nonlinearly on the magnetization and linearly on the small initial displacement gradient. We consider an ultrasonic wave propagating in the direction $\mathbf{n}$.

$$\mathbf{u} = \bar{\mathbf{u}} \exp[i(\mathbf{k}\mathbf{n} \cdot \mathbf{X} - \omega t)], \quad \mathbf{m} = \bar{\mathbf{m}} \exp[i(\mathbf{k}\mathbf{n} \cdot \mathbf{X} - \omega t)] \quad (40)$$

Fig.9. Speed of longitudinal waves under stress and magnetic field.

Substituting Eq.(40) into Eq.(39), we have the propagation condition for the amplitudes of the wave

$$\begin{aligned} (\Lambda_{IK} - \rho v^2 \delta_{IK}) \bar{u}_K - \Gamma_{IK} \bar{m}_K' &= 0, \\ \Phi_{IK} \bar{u}_K - \Psi_{IK} \bar{m}_K' &= 0, \end{aligned} \quad (41)$$

where $\bar{\mathbf{m}}' = i\bar{\mathbf{m}}/k$ and the exact forms of the coefficient matrices are omitted. It is easy to see that the propagation condition (41) reduces to an eigenvalue problem with the eigenvalue ρv^2 by eliminating $\bar{\mathbf{m}}'$. In general, there exist three kinds of magnetoelastic waves whose speeds and the polarization directions depend on the bias deformation and the magnetization. As an example, we consider the plane stress in X_1 - X_2 plane and apply the magnetic field in a direction of the plane. Then the speed of a wave can be expressed in a linear form of the stress components

$$v = v_0(\mathbf{M}) + c_{11}(\mathbf{M})T_{11} + c_{22}(\mathbf{M})T_{22} + 2c_{12}(\mathbf{M})T_{12}. \quad (42)$$

The first term on the right hand side of Eq.(42) is the speed in the absence of the stress, and the other terms are the speed change due to the acoustoelastic effect, see Fig.9. The coefficients of the stress components, the acoustoelastic coefficients, depend on the magnetization $\mathbf{M}$. The uniaxial stress can be estimated from Eq.(42) by measuring v . In the case of plane stress, we can obtain three independent equations sufficient to determine unknown stress components, by applying three different magnetizations.

6. APPLICATION OF MAGNETIC FORCES

6.1. Electromagnetic Actuators

We know that when the external magnetic field is changing in time, the induced eddy current in a conductive material results in the Lorentz force (38), which is

applied to Electro-Magnetic Acoustic Transducers. If the mechanical oscillation is exited in this way at the surface of the specimen, an acoustic wave propagates into the body, reflects from the boundary or the inside object and comes back to the original surface. The reflected wave generates the electric current by moving the free charges, so that the eddy current and the induced magnetic field are altered. The change of the magnetic field can be detected by the coil of the EMAT, so that it can be also used as a sensor. Although this type of transducers are available only to conductive materials, they have the advantage such that many oscillating modes can be generated without contact to specimens. Maxfield et al. [86] summarized theoretical bases of many types of EMATs. If a similar dynamic magnetic field is applied to a ferromagnetic material, the magnetostriction also generates the dynamic deformation of the specimen. On the other hand, an oscillation or a wave motion of the magnetized material changes the magnetization and hence the magnetic field outside of the material, which can be detected by the exciting coil. Mechanism of acoustic wave generation by this type of EMAT was studied by Lanotte et al. [87]. Kleinke and Uras [88] presented a mathematical model for magnetostrictive sensors based on an equivalent magnetic circuit. Bakker and Van den Berg [49] reviewed the development of the above two types of EMATs.

As well as eddy current, magnetostriction is used as actuators except acoustic transducers. In comparison with piezoelectric materials, the magnetostrictive actuators have several advantages, e.g., large driving force due to high stiffness, stability in high frequency, capability of remote energy supply without electric connector, etc. Since the magnetostriction is an even function of the magnetic field, a bias magnetic field is necessary to obtain linear correspondence between the electrical input and the mechanical output. Giant magnetostrictive materials are expected as actuators with large output in mechanical motion or force, cf. Clark et al. [89] and Fahlander and Richardson [90]. Lord [91] discussed giant magnetostrictive materials as transducers and actuators, including amorphous ribbons and wires. Eda [92] reviewed the development of giant magnetostrictive materials and their applications to actuators and sensors.

6.2. Magnetic Levitation and Magnetic Lubrication

A typical magnetic levitation system utilizes the resistive or the controlled attractive magnetic force between the exciting coil (superconducting magnet) and the induction coil or conducting plate which generates the resistive magnetic induction. For general introduction of magnetic levitation, refer to Borcherts et al. [93], Rhodes and B.E.Mulhall [94] and Jayawant [95]. Kriezis et al. [96] reviewed the applications of eddy current including magnetic levitation, magnetic launching and nondestructive testing. In such a levitation system, the conductive plate (guide of vehicles) is subjected to a moving force resistive to the levitation. Takagi et al. [97] considered the dynamic behavior of a plate under the moving magnetic force. Magnetic lubrication or bearing is another application of magnetic levitation. Krodkiwski [98] gave a mathematical model of magnetic bearing rotor system for a flywheel energy storage system. In order to avoid the

instability of rotating axis, the conventional magnetic bearing using permanent magnets or magnetic coils requires active control. Superconducting magnetic bearing is expected to compose a passive stable system, see Coombs [99].

6.3. Magnetic Damping

If a magnet or a magnetic coil is moving over a conducting surface in any direction, the magnetic field by eddy current causes a damping force. Since such an effect is taken into account in the articles discussed in dynamic problems of magnetoelasticity in section 4, we shall only introduce several works in the light of magnetic damping effects. In general, this type of problems except simple cases do not have analytical solutions. Nagaya and Kojima [100] analyzed rectangular magnetic dampers with finite dimension by Fourier expansion. Kanamori and Ishihara [101] analyzed the damping using conductor slab by finite element method. Yoshida et al. [102] investigated vibrations of fusion reactor first wall, taking into account magnetic damping effect in magnetoelastic materials. Weinberger and Kanehira [103] obtained the expression for drag force exerted by spacecraft by means of successive approximation technique.

When a ferromagnetic material with multi-domain structure is dynamically deformed, each domain plays a role of magnet which induces micro-eddy current. Coronel and Beshers [104] studied the damping in iron due to both macro- and micro- eddy currents. In the case of magnetostrictive materials with hysteresis, the local magnetization change by mechanical vibration also contributes to the damping effect. Motogi [105] considered such hysteresis damping effect in torsional vibration of ferromagnetic wires.

6.4. Magnetic Forming and Processing of Materials

Magnetic forming and processing of materials utilize Lorentz force for conductors and additionally the Maxwell stress for magnetized materials, see Moon [40]. In many cases, the eddy current is also used as an induction heating in such magnetic processing. The material behaviors in the magnetic processing will be described in magnetoplasticity, or specially in magnetohydrodynamics for conducting molten metal, which are beyond the framework of this paper. Refer to review article by Asai [106] for techniques in magnetic forming and processing.

6.5. Magnetic Intelligent Materials

Several attempts have been done to create more functional or intelligent materials/structures utilizing magnetic materials, as well as improvement of ferromagnetic properties. For example, ferromagnetic shape memory alloys are promising as new functional materials whose phase transitions can be controlled or caused by the magnetic field, cf. Vasil'ev et al. [107]. Kanada and Enokizono [108] attempted to provide Fe-Mn-Si shape memory alloy with ferromagnetic property by oxidized surface by annealing. Kinoshita et al. [109] examined the stress dependence of the magnetic impedance in Fe-based shape memory alloy to add a function of sensor. Chiriac and Ovari [110] reviewed the electromagnetic and mechanical material properties of glass-covered magnetic wires and

their applications as functional materials.

Composite materials using magnetic materials are also expected as intelligent structures with functions which can not be exhibited by the matrix materials. For example, if a beam or a plate consists of layers of magnetostrictive and non-magnetic materials, the membrane behaves as an actuator and its deflection or vibration can be controlled by applying the magnetic field. Modelling of such laminated structures was studied by Chandra and Chopra [111]. Many studies have been done in the experiment on this kinds of intelligent structures. For example, Ohmura et al. [112] analyzed the deflection of bimorph type of plate and compared with experimental results. Chaudhry and Rogers [113] considered dynamic behavior of structures in which actuator materials are attached discretely. Eda et al. [114] reviewed actuators/smart structures using giant magnetostrictive materials. If we embed magnetostrictive particles in a structural material, we can dynamically control mechanical properties of the material by the magnetic field. Duenas et al. [115] analyzed the behavior of the composite containing giant magnetostrictive particles. Anjanappa and Yuefei [116] studied the performance of such magnetostrictive particle actuators under several mechanical conditions. Kannan and Dasgupta [117] presented the finite-element method suitable for designing the above active or adaptive structure taking into account the strong nonlinearity in the constitutive equations and nonlinear terms in the governing equations. Instead of attaching or embedding the magnetostrictive materials, Quandt et al. [118] prepared a layered magnetostrictive composite by sputtering. Watanabe et al. [119] prepared magnetically gradient materials by means of partial martensite transition in rolling process of wedge samples.

7. CONCLUDING REMARKS

We have described the governing equations for solid mechanics under magnetic field, but we could not enter the details of the formulations. Furthermore, several important topics in this area could not be mentioned from the space of the paper. For example, the formulation of micromagnetism and related applications, associated thermodynamics, superconductivity closely related to ferromagnetism, inelastic deformations under magnetic field like magnetoplasticity, typical solutions and numerical methods for magnetoelastic problems, and many interesting physical phenomena in magnetism coupled with electricity, acousticity and optics. Missing of other important subjects and articles owes to the limit of the author's ability. In order to establish the fundamental theory and enlarge the range of its applications, there remain several subjects, i.e., description of magnetic hysteresis and magnetic rate effects, formulation of 3-dimensional magnetization, magnetic phase transitions, etc. It would be pleasant if this article could stimulate interests in this area.

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