EFFECTS OF FIBER LENGTH AND ORIENTATION DISTRIBUTIONS ON THE MECHANICAL PROPERTIES OF SHORT-FIBER-REINFORCED POLYMERS

A Review

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Abstract: Extrusion compounding and injection molding processes are frequently employed to fabricate short-fiber-reinforced polymers. During extrusion compounding and injection molding processing, considerable shear-induced fiber breakage takes place and results in a fiber length distribution (FLD) in final short-fiber-reinforced polymer (SFRP) composites. Also, during compounding and molding processing, progressive and continuous changes in fiber orientation occur and lead to a fiber orientation distribution (FOD) in final composites. Both FLD and FOD are governed by a number of design and fabrication factors including original fiber length, fiber content, mold geometry and processing conditions. The mechanical properties such as strength, stiffness and fracture toughness or specific work of fracture (WOF) of SFRP composites have been shown to depend critically on FLD and FOD. The present paper reviews previous research work on the effects of design/fabrication factors on FLD and FOD and the effects of FLD and FOD on the strength, stiffness and toughness or WOF of SFRP composites. Conclusions which can be drawn from the literature are presented with discussions of areas in which further research is required.

Key words: Short-fiber-reinforced polymer (SFRP) composites, Strength, Stiffness, Toughness/specific work of fracture, Fiber length distribution, Fiber orientation distribution

1. INTRODUCTION

Short-fiber-reinforced polymer (SFRP) composites have found extensive applications in automobiles, business machines, durable consumer items, sporting goods and electrical industries etc. owing to their low cost and easy processing, and their superior mechanical properties over corresponding polymers. Extrusion compounding and injection molding processes are frequently employed to make SFRP composites [1-22]. The use of these conventional fabrication techniques to produce large-scale SFRP composite parts makes the manufacturing of these composites efficient and inexpensive compared with manufacturing of continuous-fiber composites, which are produced by time-consuming processes, rendering them unsuitable for high volume production. In injection molded SFRP composite parts, there are generally a fiber length distribution (FLD) and a fiber orientation distribution (FOD). FLD and FOD are governed by fiber length, fiber content, mold geometry and processing conditions [7-12,17,19-34]. Addition of short fibers to a polymer matrix generally leads to composites that show significant improvements in strength and stiffness [1,3-5,7-11,18,21,32,35]. However, fracture toughness or specific work of fracture (WOF) may either be enhanced [11,18,36,37] or be reduced [6,37,38]. The strength, stiffness and fracture toughness or WOF of short fiber composites have been shown to depend critically on FLD and FOD [3,6,8,37,39-54].

In this review the effects of design/fabrication factors on FLD and FOD are addressed briefly and the effects of FLD and FOD on the strength, stiffness and toughness or WOF of SFRP composites are discussed to a relatively detailed extent. Conclusions are presented with areas in which further research is required.

2. EFFECTS OF THE DESIGN/FABRICATION FACTORS ON FLD AND FOD

2.1. Fiber Length Distribution (FLD)

An extensive research effort has been conducted to evaluate fiber breakage during processing. A number of techniques have been developed for investigating fiber length degradation in compounded and molded materials [5,6,9,11,14,19,23,24,28,29,32,55,56]. In all the techniques employed, fibers must be separated from the polymer matrix. The polymer matrix may be removed by burning off in an oven [5,6,9,11,19,23,28,29,55,56] or by dissolving the polymer in an appropriate solvent [14,24,32]. It has been shown that fiber breakage in processing of reinforced thermoplastics results from fiber-polymer interaction, fiber-fiber interaction, and fiber contact with surfaces of processing equipments [24]. The FLD in final SFRP composites depends on a number of design/fabrication factors including original fiber length, fiber content, mold geometry and processing conditions.

The influence of the design and fabrication factors on fiber length is presented in Table 1. Compounding
accomplished under conditions of low screw speeds and relatively high barrel temperatures minimized fiber breakage [14,22]. During compounding process lower mixing times led to a less damage to fiber length [15,23]. Damage to fibers is a cumulative result of compounding and injection molding. The fiber attrition was observed to be more severe in the injection-molding machine [14,27]. In order to preserve fiber aspect ratio in final molded articles, this generally embraces the use of low injection speed and back pressure, and generous gate and runner dimensions [16,17,31]. Moreover, back pressure was shown to have a more dramatic effect upon fiber length than does injection speed. The increasing injection speed shear component progressively reduces the maximum length retained. In addition to the dependence of fiber breakage on processing, the final fiber length in SFRP composites also depends on fiber content and original fiber length. It was generally observed that the mean fiber length decreases with the increase of fiber volume fraction [7-12,20,21,23], owing to the increased fiber-fiber interaction and fiber-equipment wall contact. It is noticed that longer original glass fibers led to longer fibers in final parts for injection molded SFRG/polyamide 6,6 composites [19]. In another study, the initial fiber length was observed to have little effect on the final fiber length [24]. This may be because the difference in final fiber lengths caused by original fiber length and possible experimental errors are of the same order of magnitude owing to a small difference in initial fiber lengths in this case. Besides, reprocessing of SFRP would further reduce fiber length [57,58].

The fiber length distribution in injection molded polymer composites has an asymmetric character with a tail at the long fiber end [8,18,28,59]. It can be described with a probability density function, \( f(L) \), which is defined so that \( f(L) dL \) and \( F(L) \) are the probability density that the length of a fiber is between \( L \) and \( L+dL \) and the probability that the length of a fiber is less than \( L \), respectively. Then the relationship of \( f(L) \) and \( F(L) \) is

\[
F(L) = \int_0^L f(x) dx \quad \text{and} \quad \int_0^\infty f(x) dx = 1. \quad (1)
\]

The FLD function \( f(L) \) is defined as [19,41-43]

\[
f(L) = a b L^{b-1} \exp(-a L^b) \quad \text{for} \quad L > 0 \quad (2)
\]

where \( a \) and \( b \) are size and shape parameters, respectively. The mean fiber length can be obtained as

\[
L_{\text{mean}} = a^{-1/b} \Gamma(1/b+1) \quad (3)
\]

where \( \Gamma(x) \) is the gamma function. The most probable length (mode length), \( L_{\text{mod}} \), can be got by differentiating Eq. (2) and letting the resultant equation be equal to zero

\[
L_{\text{mod}} = \left[1/a - 1/(ab) \right]^{1/b}. \quad (4)
\]

2.2. Fiber Orientation Distribution (FOD)

Injection molded composites may show a preferential fiber alignment along polymer flow direction or a layered structure with distinct fiber alignments in different layers, which depends on mold geometry employed. Different fiber alignments in composites lead to different FOD.

For dumbbell-shaped SFRP composites, it was shown that fibers align preferentially along the flow direction [1,2,5,6,10,25]. The number of fibers which were oriented along the flow direction increases as fiber content increases, possibly owing to the generation of higher shear stresses along the flow direction [10]. Moreover, the orientation of fibers depends upon their lengths [25].

The fiber orientation in plaque-shaped parts was studied theoretically and experimentally and a skin-core layer structure was recognized [7,29,31,32,36,37,46,59]. In the skin layer the fibers align preferentially along the flow direction; in the core layer the fibers align transverse to the flow direction. Usage of low mold temperature can lead to a comparatively large skin thickness [32]. The FOD in injection molded short-carbon fiber-reinforced composite plaques was shown to be strongly influenced by fiber content [29]. At high fiber contents fiber-fiber interaction plays an important role as a factor affecting
the angular distribution. As the fiber content increases, the skin-core structure gradually disappeared [7].

A skin-intermediate-core layer structure was also observed in SFRP composite plates [30,33,34]. Inside the skin layer, the fibers are oriented at random in the plane of the plate. The fibers in the core layer are oriented perpendicularly to the flow direction. An intermediate area exists between the skin layer and the core layer, where the fiber orientation is parallel to the flow direction. The relative dimensions of the microstructural regions depend on some processing variables (Table 2).

In order to describe FOD, a spatial curvilinear coordinate system is adopted, where the orientation of a fiber can be defined uniquely by a pair of angles ($\Theta, \Phi$) (see Fig. 1). $\Theta$ is defined as the angle a fiber makes with the 1 axis (the mold flow direction), while $\Phi$ is defined as the angle the projection of the fiber onto the 2-3 plane (the 2-3 plane is the one whose normal is parallel to the 1-axis direction) makes with the 2 axis. Provided $\Theta$ is the angle of the one end of the fiber with the 1 axis, then $\pi-\Theta$ is the angle of the other end of the fiber with the 1 axis; similarly, $\Phi$ and $\pi+\Phi$ are the two angles of the projection of the two fiber ends onto the 2-3 plane with the 2 axis, respectively. ($\Theta, \Phi$) is defined as the loading direction and $\delta$ is the angle between ($\Theta, \Phi$) and ($\Theta, \Phi$).

The fiber orientation ($\Theta$) distribution can be described with a two-parameter exponential function [41-43]:

$$g(\Theta) = \frac{(\sin \Theta)^{2p-1} \cos \Theta^{2q-1}}{\int_{\Theta_{\min}}^{\Theta_{\max}} (\sin \Theta)^{2p-1} \cos \Theta^{2q-1} d\Theta}$$

(5)

where $p$ and $q$ are the shape parameters which can be used to determine the shape of the distribution curve, and $p \geq 1/2$ and $q \geq 1/2$. Also, $0 \leq \Theta_{\min} \leq \Theta \leq \Theta_{\max} \leq \pi/2$. The mean fiber orientation angle, $\Theta_{\text{mean}}$, can be derived as:

$$\Theta_{\text{mean}} = \int_{\Theta_{\min}}^{\Theta_{\max}} \Theta g(\Theta) d\Theta$$

(6)

The most probable fiber orientation angle (mode fiber orientation angle) is obtained from Eq. (5):

$$\Theta_{\text{mod}} = \arctan \left( \frac{[(2p-1)(2q-1)]^{1/2}}{} \right)$$

(7)

The fiber orientation coefficient, $f_\Theta$, can be defined as follows:

$$f_\Theta = 2 \int_{\Theta_{\min}}^{\Theta_{\max}} g(\Theta) \cos^2(\Theta) d\Theta - 1$$

(8)

For $f_\Theta = -1$, all fibers lie perpendicular to the normal direction of the 2-3 plane; $f_\Theta = 0$ corresponds to a 2D random distribution or a symmetric distribution about the direction $\Theta = \pi/4$; $f_\Theta = 1$ implies all fibers are aligned parallel to the 1-axis direction.

Similar to the definition of $g(\Theta)$, an orientation probability density function $g(\Phi)$ is defined [45]:

$$g(\Phi) = \frac{\sin \Phi^{2s-1} \cos \Phi^{2t-1}}{\int_{\Phi_{\min}}^{\Phi_{\max}} \sin \Phi^{2s-1} \cos \Phi^{2t-1} d\Phi}$$

(9)

where $s$ and $t$ are shape parameters which determine the shape of the curves of the FOD $g(\Phi)$. $0 \leq \Phi_{\min} \leq \Phi \leq \Phi_{\max} \leq 2\pi$. $g(\Phi)d\Phi$ is the probability density that the orientation of fiber lies between $\Phi$ to $\Phi+d\Phi$.

3. EFFECTS OF FLD AND FOD ON THE MECHANICAL PROPERTIES OF COMPOSITES

3.1. The Strength of SFRP Composites

The modified rule of mixture is often used to predict the tensile strength of short fiber composites [3, 47-52]:

$$\sigma_m = \sigma_f \chi_1 \chi_2 \sigma_m + \sigma_m \sigma_m$$

(10)

where $\chi_1$ and $\chi_2$ are, respectively, the fiber orientation and fiber length factors; their product ($\chi_1 \chi_2$) is the fiber efficiency factor for the composite strength ($\sigma_m$); $\sigma_m$ is the ultimate strength of fibers; $V_f$ and $V_m$ denote the volume fraction of the fibers and matrix and $\sigma_m$ is the matrix stress at the failure of the composite. For unidirectional discontinuous composites $\chi_1 = 1$ and $\chi_2 < 1$. If the fiber length is uniform and equals $L$, then

$$\chi_2 = L/(2L_c) \quad \text{for} \quad L < L_c$$

(11)

$$\chi_2 = 1 - L_c/(2L) \quad \text{for} \quad L \geq L_c$$

(12)

where $L_c$ is the critical fiber length and $L_c = r_f \sigma_{mf}/\tau$, where $\tau$ and $r_f$ are the interfacial shear stress between matrix and fibers and the fiber radius, respectively. If the fiber length is not uniform, Eqs. (11) and (12) must be modified. Kelly and Tyson [51] put forward a model.
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considering the effect of fibers, shorter and longer than the critical fiber length. This model can be given by [48]

\[
\sigma_{cu} = \frac{L}{L_c} \sum_{i} V_i \sigma_{fu} \left( 1 - \frac{L}{2L_c} \right) + V_n \sigma_m. \tag{13}
\]

The first and second terms take into account the contributions of fibers with sub-critical length shorter than \(L_c\) and those of fibers with super-critical length longer than \(L_c\), respectively. In Eq. (13) the contribution of the fiber orientation is not considered. This model is then modified through addition of an orientation factor to the first two terms of Eq. (13) [15, 52].

Fukuda and Chou's critical damage zone model [54] was also used to predict the tensile strength of the SFRP. This model goes one step further by considering the distribution function of both the fiber length and the fiber orientation. However, the generalized formulae of both the FLD function and the FOD function are not given, thus the effects of the fiber length and orientation distributions on the strength of SFRP cannot be studied; the dependences of the ultimate fiber strength and the critical fiber length on the inclination angle \(\theta\), which must be considered [47, 60, 61], are not taken into account.

Fu and Lauke [41] derived the strength of SFRP composites as a function of the fiber length and fiber orientation distributions by taking into account the dependences of the ultimate fiber strength and the critical fiber length on the inclination angle \(\theta\). The expression for \(\chi_1\chi_2\) can be given as follows [41]:

\[
\chi_1\chi_2 = \int_{0}^{\infty} \int_{0}^{\pi} f(L) g(\theta) \left( L/L_{\text{mean}} \right) \exp(\mu \theta) dL d\theta \\
+ \int_{0}^{\infty} \int_{0}^{\pi} f(L) g(\theta) \left( 1 - A \tan(\theta) \right) (1 - L - (1 - A \tan(\theta))(2L_L \exp(\mu \theta))) dL d\theta \tag{14}
\]

where \(\mu\) is the snubbing friction coefficient between fiber and matrix at the crossing point when a fiber is pulled out obliquely [62, 63]. \(A\) is a constant for a given system. \(L_c\) is the critical length of oblique fibers:

\[
L_{c0} = L_c[1 - \tan(\theta)]/\exp(\mu \theta). \tag{15}
\]

The larger the value of \(\chi_1\chi_2\), the higher is the composite strength.

Some results obtained from Eq. (14) are shown below. The following data of the parameters are used, unless noted specially: \(\mu = 0.1, A = 0.4, L_{\text{min}} = 0, L_{\text{max}} = \infty, L_{\text{mod}} = 0.2 \text{ mm}, L_c = 0.2 \text{ mm}, \rho = 0.5 \text{ and } q = 10\).

Figure 2 shows the effect of the mean fiber length on the fiber efficiency factor \(\chi_1\chi_2\). It can be seen from Fig. 2 that the value of \(\chi_1\chi_2\) and hence the strength of composites increases rapidly with the mean fiber length at small mean fiber lengths (in the vicinity of \(L_c\)) and approaches gradually a plateau level at large mean fiber lengths (> about 5\(L_c\)).

Figure 3 represents the effects of the mode fiber length and the critical fiber length on the value of \(\chi_1\chi_2\). It shows that the value of \(\chi_1\chi_2\) and hence the composite strength increases with the decrease of critical fiber length; moreover, the value of \(\chi_1\chi_2\) and hence the composite strength decreases slightly with the increase of mode fiber length.

Table 3. The effect of FOD on the tensile strength of SFRP composites [41].

<table>
<thead>
<tr>
<th>No.</th>
<th>(\theta_{\text{mean}})</th>
<th>(f_\theta)</th>
<th>(\chi_1\chi_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.514</td>
<td>-0.99</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1.351</td>
<td>-0.88</td>
<td>0.013</td>
</tr>
<tr>
<td>3</td>
<td>1.262</td>
<td>-0.78</td>
<td>0.068</td>
</tr>
<tr>
<td>4</td>
<td>1.141</td>
<td>-0.6</td>
<td>0.174</td>
</tr>
<tr>
<td>5</td>
<td>0.982</td>
<td>-0.33</td>
<td>0.308</td>
</tr>
<tr>
<td>6</td>
<td>0.785</td>
<td>0</td>
<td>0.417</td>
</tr>
<tr>
<td>7</td>
<td>0.571</td>
<td>0.33</td>
<td>0.549</td>
</tr>
<tr>
<td>8</td>
<td>0.430</td>
<td>0.6</td>
<td>0.652</td>
</tr>
<tr>
<td>9</td>
<td>0.308</td>
<td>0.78</td>
<td>0.693</td>
</tr>
<tr>
<td>10</td>
<td>0.220</td>
<td>0.88</td>
<td>0.717</td>
</tr>
<tr>
<td>11</td>
<td>0.056</td>
<td>0.99</td>
<td>0.755</td>
</tr>
</tbody>
</table>
Table 3 represents the effect of FOD on the value of \( x_1x_2 \) and hence on the composite strength, where \( L_{\text{mean}} = 0.4 \) and \( L_{\text{mod}} = 0.213 \), showing that the value of \( x_1x_2 \) and hence the composite strength increases with the decrease of mean fiber orientation angle and with the increase of fiber orientation coefficient.

Injection molded SFRP composites are heterogeneous and demonstrate anisotropy in their mechanical properties. The variance of the strength with the loading direction \((\Theta, \Phi)\) can be expressed as follows [44]:

\[
\sigma_{\text{eff}}(\Theta, \Phi) = \lambda V_f \sigma_{\text{in}} + \sigma_m V_m
\]

where \( \lambda \) is the fiber efficiency factor and

\[
\lambda = \frac{\phi_{\text{eff}} \rho_{\text{eff}} L_{\text{mod}}}{\rho_{\text{eff}} L_{\text{mean}}} \int f(L) g(\Phi) g(\Theta/\Phi) \left( L/(2L_c) \right) \exp(\mu \delta) \, dL \, d\delta
\]

where \( L_c \) is the critical length of oblique fibers of an angle \( \delta \) with the loading direction. And \( \delta \) is given by

\[
\cos \delta = \cos \Theta \cos \phi + \sin \Theta \sin \phi \cos (\Theta - \Phi).
\]

The following values of the parameters were used in the calculation unless noted specially: \( A = 0.4, \mu = 0.1, L_c = 0.2 \text{ mm}, L_{\text{mean}} = 3.198 \text{ mm} \) (\( a = 0.15 \) and \( b = 1.5 \)) and \( \Theta_{\text{mean}} = 12.95^\circ \) (\( p = 0.6 \) and \( q = 8 \)). The planar FOD (\( s = 0.5 \) and \( t = \infty \)) for \( g(\Phi) \) is considered.

Fig. 4. The fiber efficiency factor for the strength of SFRP composites versus the direction \((\Theta, \Phi)\) [44].

Figure 4 shows the variation of the fiber efficiency factor (\( \lambda \)) with the direction \((\Theta, \Phi)\). It can be seen from Fig. 4 that the fiber efficiency factor is the maximum at \( \Theta = 0^\circ \) and decreases with the direction angle \( \Theta \) to its minimum at \( \Theta = 90^\circ \) for any value of \( \Phi \).

Figure 5 exhibits the effect of mean fiber length on the variation of the fiber efficiency factor with the direction angle \( \Theta \), showing that the fiber efficiency factor increases with the mean fiber length at a small \( \Theta \), while it is insensitive to the mean fiber length at a large \( \Theta \).

Figure 6 exhibits the effect of mean fiber orientation angle on the variation of the fiber efficiency factor with the direction angle \( \Theta \) for \( \Phi = 0^\circ \). Figure 6 reveals that the fiber efficiency factor decreases with the increase of the mean fiber orientation angle \( \Theta_{\text{mean}} \) when \( \Theta \) is small (e.g. \( \leq 40^\circ \)) and increases with the mean fiber orientation angle when \( \Theta \) is large (\( \geq 50^\circ \)). For the 2D random fiber alignment case the fiber efficiency factor maintains a constant value with changing \( \Theta \).

3.2. The Stiffness of SFRP Composites

Research work on the elastic modulus of short-fiber composites can be categorized according to the fiber orientation: unidirectional, random and partially aligned. The elastic modulus of unidirectional short fiber composites has been studied with various methods [64-67]. Also, for two-dimensional random array of fibers the study on the composite stiffness has been performed with a number of methods [68-71].
Besides the above two limiting cases, another important case is the general one of partially aligned short-fiber composite. Takao et al. [72] presented an analysis of the effective longitudinal Young’s modulus of composites containing misoriented short-fibers based on Eshelby’s equivalent inclusion method [73] and the average induced strain approach of Taya and Chou [74]. Based again on Eshelby’s equivalent inclusion method, Chang and Cheng [75] formulated the elastic modulus of composites reinforced with short-coated fibers whose orientations and aspect ratios are varied. Although Fukuda and Kawata [76] developed a theory for the elastic modulus of short-fiber composites with variable fiber length and orientation, there was a mistake in the calculation of the force sustained by the fibers across the scan line. This mistake was corrected by Jayaraman and Kortschot and the corrected version was called paper physics approach (PPA) [77], and the elastic modulus of short fiber composites can be expressed as follows:

$$E_{11} = k_1 k_2 E_f V_f + E_m (1 - V_f)$$  \tag{19}$$

where $E_f$ and $E_m$ are the Young’s moduli of fibers and matrix, respectively. $k_1$ and $k_2$ are respectively the fiber length and orientation factors for the composite elastic modulus and their expressions are given as follows:

$$k_1 = \frac{1}{L_{\text{mean}}} \int_{\theta_{\text{mn}}}^{\theta_{\text{ma}}} \left[ 1 - \tanh(\theta L/2) \right] f(L) \, dL \tag{20}$$

$$k_2 = \int_{\theta_{\text{mn}}}^{\theta_{\text{ma}}} \left[ \cos^2 \theta - \nu_{12} \sin^2 \theta \right] \cos \theta \, \sin \theta \, f(\theta) \, d\theta \tag{21}$$

where $\beta$ is a function of $E_f$, $E_m$, $r_f$ and $\nu_m$ (Poisson’s ratio of the matrix) [77], $\nu_{12}$ is the longitudinal Poisson’s ratio of a corresponding unidirectional short-fiber composite.

The laminate analogy approach (LAA) has been widely used for the prediction of the stiffness of SFRP composites [8,48,59,78,79]. An assumption that a thin sheet specimen has a geometry such that the thickness dimension is much less than the width and length dimensions was made so that the condition of a planar FOD can be satisfied in order to use the LAA to evaluate the elastic modulus of SFRP composites [8,48,78,79].

Fu and Lauke [43] used the LAA to study the elastic modulus of SFRP composites by taking into account the effects of FLD and FOD. An inspiration obtained from the paper physics approach, that the elastic modulus of SFRP composites with a 3D spatial fiber orientation distribution in a given loading direction is only dependent on the orientation distribution of the angle fibers make with the given loading direction, is applied to the laminate analogy approach in simulating SFRP composites to consist of laminae so that an extension of the LAA to predict the composite elastic modulus with a spatial orientation distribution was carried out. The expression for the composite elastic modulus in the loading direction is expressed as:

$$\overline{E}_{11} = \frac{A_{11} A_{22} - A_{12}^2}{A_{22}}$$ \tag{22}$$

where

$$A_{ij} = \int_{\theta_{\text{mn}}}^{\theta_{\text{ma}}} Q_{ij} (L) g(\theta) \, dL \, d\theta \tag{23}$$

where $\{Q_{ij}\}$ is the stiffness matrix in the off-axis system for a unidirectional short-fiber composite. The transformation equation between the components of $\{Q_{ij}\}$ in the off-axis system and those of $\{Q_{ij}\}$ in the on-axis system can be given by:

$$\{Q_{ij}\} = M \{Q_{ij}\}$$ \tag{24}$$

where

$$M = \begin{pmatrix} m^4 & n^4 & 2m^2 n^2 & 4m^2 n^2 \\ n^4 & m^4 & 2m^2 n^2 & 4m^2 n^2 \\ m^2 n^2 & m^2 n^2 & m^2 + n^2 & -4m^2 n^2 \\ m^2 n^2 & m^2 n^2 & -2m^2 n^2 & (m^2 - n^2)^2 \\ m^3 n^3 & mn^3 & mn^3 & -m^2 n^3 & 2(mn^3 - m^2 n) \\ m^3 n^3 & -mn^3 & mn^3 & -m^2 n^3 & 2(mn^3 - m^2 n) \end{pmatrix} \tag{25}$$

where $m = \cos \theta$ and $n = \sin \theta$. $\{Q_{ij}\}$ are functions of $E_f$, $E_m$, $G_f$ (shear modulus of fibers), $G_m$ (shear modulus of the matrix), $v_f$ (Poisson’s ratio of fibers), $v_m$, $V_f$ and $d_f$ (fiber diameter) [43].

![Fig. 7. The effect of $L_{\text{mean}}$ on the elastic modulus of SFRP composites [43].](image)
increases with the mean fiber length. However, when the mean fiber length is large (e.g. > 1.0 mm and the aspect ratio is > 100), the mean fiber length has nearly no influence on the composite elastic modulus [44].

The predicted results of the composite elastic modulus as a function of mean fiber orientation angle ($\phi = 0.6$ and various $q$) is shown in Fig. 8 for various $V_f$, where $\bar{L}_\text{mean} = 3.198$ mm ($a = 0.15$ and $b = 1.5$) and other parameters are the same as in Fig. 7. Figure 8 shows that both the fiber volume fraction and the mean fiber orientation angle have a significant influence on the elastic modulus of SFRP composites. Higher fiber volume fraction leads to higher elastic modulus. The composite elastic modulus decreases slowly with the increase of mean fiber orientation angle when the fiber volume fraction is small (e.g. $V_f = 0.1$) while decreases dramatically with the increase of mean fiber orientation angle when $V_f$ is large (e.g. = 50%).

![Fig. 8. The effect of $\theta_{\text{mean}}$ on the elastic modulus of SFRP composites [43].](image)

The comparison of the theoretical results of the elastic modulus of SFRP predicted by the LAA and the PPA is made in Fig. 9 for the cases of the mean fiber orientation angle varying from 0 to 48.5°, where the data of the parameters are the same as those in Fig. 8. Figure 9 shows that the elastic modulus predicted by the PPA is slightly higher than that by the LAA and the difference depends on the fiber volume fraction when the mean fiber orientation angle is small. At the intermediate mean fiber orientation angle the results predicted by the two approaches are very close. When the mean fiber orientation angle is large, the results predicted by the PPA are lower than those by the laminate analogy approach. This is because for the PPA the angle $\theta$ must be less than some critical angle $\theta_c (\theta_c = \arcsin([1/(1+\nu_{12})]^{1/2}))$, otherwise the fibers of an orientation angle $\theta$ greater than $\theta_c$ would make a negative contribution to the stiffness of short fiber composites (see Eq. (23)). However, the experimental results [69] showed that the transverse stiffness of unidirectional short- and long-glass-fiber reinforced epoxy composites is higher than that of pure epoxy resin matrix; from this it can be concluded that the fibers of $\theta = \pi/2$ should make a positive contribution to the composite elastic modulus since they constraint the deformation of composites even for the fibers of $\theta = \pi/2$. So, the argument in the PPA that the fibers of an angle with the applied strain direction greater than $\theta_c$ make a negative contribution to the composite elastic modulus is incorrect. Therefore, the contribution of the fibers of a large angle with the applied strain direction to the composite elastic modulus will be underestimated by the PPA, and hence in case the fiber orientation angle between fiber axis direction and applied strain direction is comparatively large, the PPA would be invalid in predicting the elastic modulus of composites.

![Fig. 9. Comparison of the theoretical results of the elastic modulus of SFRP composites predicted by the LAA and the PPA [43].](image)

Recently, theoretical studies on the elastic anisotropy of short-fiber composites are receiving an increasing interest [80-84]. Warner and Stobbs [80] proposed a continuum mechanics model based on the Eshelby method [73] to predict the variation of the elastic modulus of short-fiber composites with loading direction. They studied the model system consisting of a finite volume fraction of fibers, which are all of the same aspect ratio and at the same direction. Dyer et al [81] used again the Eshelby method to develop another theoretical method to calculate bounds on the elastic constants for unidirectional fiber-reinforced composites by considering the fibers and the matrix both show transverse isotropy. Sayers [82] carried out a theoretical analysis on the elastic anisotropy of short-fiber-reinforced composites with fibers of the same length by examining the sensitivity of the elastic stiffness tensor to the fiber orientation. Pan [83] presented a statistical analysis to characterize the modulus anisotropy of short-fiber composites and studied the effect of the fiber volume fraction on the anisotropy of the elastic modulus by considering one planar and harmonic fiber orientation distribution. Dunn et al. [84] proposed a relatively simple micromechanics model to
predict the entire set of elastic constants of short-fiber composites for advancing the understanding of the dependence of the anisotropical elastic stiffness tensor on microstructure including fiber content, shape (aspect ratio) and orientation distribution. The analytical results were given for some special orientation distributions such as unidirectional, two dimensional (2D) random and three dimensional (3D) random fiber alignment etc.

However, in all the above studies the effects of FLD and FOD on the elastic anisotropy of short-fiber composites were not studied in detail. A detailed study on the anisotropy of the elastic modulus of SFRP was performed by Fu and Lauke [45] by taking into account the effects of FLD and FOD. The variation of the elastic modulus of SFRP composites with the loading direction (Θ, Φ) (see Fig. 1) can be presented as follows:

\[
E_{11}(Θ, Φ) = \frac{\overline{A}_{11}A_{22} - \overline{A}_{12}}{A_{22}}
\]  

where

\[
\overline{A}_{ij} = \int_{L_{\text{min}}}^{L_{\text{max}}} \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \int_{\phi_{\text{min}}}^{\phi_{\text{max}}} Q_{ij}(L)g(\theta)g(\phi)dLd\theta d\phi.
\]

The definitions of \( m \) and \( n \) (\( m = \cos \delta \) and \( n = \sin \delta \)) here are different from those (\( m = \cos \Theta \) and \( n = \sin \Theta \)) in the direction (\( \Theta = 0, \Phi \)), because the angle between the fiber axial direction (\( \Theta, \Phi \)) and the direction (\( \Theta, \Phi \)) is now equal to \( \delta \) (see Fig. 1).

The variation of the composite modulus with the direction angle \( \Theta \) is shown in Fig. 10 for various mean fiber lengths (\( a = 0.15 \) and various \( b \)) and \( g(\phi) = 1/(2\pi) \), where other parameters are the same as in Fig. 7. Figure 10 reveals that when the mean fiber length (\( L_{\text{mean}} \)) is relatively small (e.g. < about 1 mm, i.e. the fiber aspect ratio is < about 100), a larger \( L_{\text{mean}} \) leads to a higher composite modulus at \( \Theta \leq 60^\circ \), and the difference in the composite modulus between two cases of different \( L_{\text{mean}} \) decreases as \( \Theta \) increases, while when \( \Theta \) is comparatively large (\( \geq 60^\circ \)), the modulus is insensitive to the mean fiber length. The influence of the mean fiber length on the modulus weakens as \( L_{\text{mean}} \) increases. Moreover, the composite modulus is independent of the direction angle \( \Phi \) because of the assumption of \( g(\phi) = 1/(2\pi) \).

Figure 11 depicts the composite elastic modulus as a function of the direction angle \( \Theta \) and the mean fiber orientation angle \( \theta_{\text{mean}} \) for any value of \( \Phi \). It is shown in Fig. 11 that the composite modulus at a small \( \Theta \) (e. g. ≤ about 30°) decreases dramatically with the increase of the mean fiber orientation angle \( \theta_{\text{mean}} \), while the modulus at a large \( \Theta \) (e. g. ≥ about 50°) increases relatively slowly with the increase of \( \theta_{\text{mean}} \). For the limiting case of the three dimensional (3D) random fiber alignment (see curve (f), \( \theta_{\text{mean}} = 57.32^\circ \), the fibers are oriented randomly without any preferred direction), the composite modulus maintains a constant value for various \( \Theta \) and thus the composite is isotropic in the modulus. In addition, the composite modulus at \( \Theta = 0^\circ \) is the maximum since the fibers when \( \theta_{\text{mean}} < 57.32^\circ \) are preferentially oriented in this direction (\( \Theta = 0^\circ \)).

3.3. The Toughness/WOF of SFRP Composites

For short fiber composites, the failure mechanisms can be concluded from the literature as follows [37,38,42,85-89]: (1) fiber-matrix interfacial debonding, (2) post-debonding friction, (3) matrix plastic deformation, (4) fiber plastic deformation, (5) fiber fracture, (6) matrix fracture, and (7) fiber pull-out. The fracture toughness or specific work of fracture (WOF) of short fiber composites is caused by all the failure mechanisms. In a very simple approach, the composite fracture toughness can be described by an empirical relationship of the form [37]

\[
K_c = (B + C \Psi)K_m
\]

where \( K_m \) is the fracture toughness of the matrix. \( \Psi \) is a 'reinforcing effectiveness parameter' related to the
volume fraction of short fibers and their geometrical arrangement across the composite thickness. \( B \) is a "matrix stress condition factor" which reflects changes in the fracture toughness of matrix material as a function of the composite thickness due to the presence of fibers. \( C \) is a constant related to the energy absorbing mechanisms.

The failure mechanisms can be classified into the three terms: (1) those related to matrix only, including matrix plastic deformation and matrix fracture, (2) those related to fiber only, including fiber plastic deformation and fiber fracture and (3) the interface-related mechanisms including fiber-matrix interfacial debonding, post-debonding friction and fiber pull-out. The total WOF of the composite can then be described as \([6,53,86,88]\)

\[
R_c = (1-V_f)R_m + \sum_i V_i R_i + V_f(R_d+R_{df})(c_0/L) + V_fR_{po}
\]  

(29)

where \( R_m \) is the matrix fracture work caused by the matrix-related mechanisms only in the presence of fibers. \( V_i \) is the subtraction of the fibers which are active in only the fiber-related failure mechanisms and \( R_i \) is the fracture work associated with fiber plastic deformation and fracture. \( c_0 \) is the size of damage zone which corresponds to a critical distance from the tip of the main crack where the local stress is just sufficient to initiate an interface crack. \( R_d \), \( R_{df} \) and \( R_{po} \) are respectively the interfacial debonding energy, the post-debonding friction energy and the fiber pull-out energy. Formulae of \( R_d \) and \( R_{df} \) have been given for unidirectional short fiber composites \([84]\).

For a SFRP composite with a FLD and a FOD, the fiber pull-out energy has been studied in detail \([42]\).

However, the fracture toughness or WOF of SFRP composites is a difficult quantity to predict. When polymer matrices are brittle or in a brittle condition, the addition of short fibers to polymers would result in an increase in the fracture toughness or the WOF. On the contrary, an opposite tendency may occur for the matrices which behave in a highly ductile manner. This is because the fracture behavior of the matrix would be very different between before and after adding short fibers to the matrix. Therefore, unlike in predicting composite tensile strength in which the matrix strength can be used, the matrix fracture toughness may be significantly reduced by incorporating short fibers. The influence of the fiber-matrix interface on the fracture toughness or WOF of short fiber composites is a complicated problem. Friedrich \([37]\) observed that the better fiber-matrix bonding quality led to the higher composite fracture toughness for short-glass-fiber-reinforced polyethylene terephthalate composites. On the other hand, a weaker interface between carbon fibers and epoxy matrix resulted in a higher composite fracture work \([90]\). Moreover, no influence of the interface on the notched Izod impact strength (here it is called the fracture work) was noticed although the authors did not mention this fact \([10]\).

4. CONCLUDING REMARKS

From this review, it is clear how the design/fabrication factors influence FLD and FOD and how FLD and FOD affect the strength and stiffness of SFRP composites. However, the combined effects of the design/fabrication factors have not yet been studied systematically. Also, there has not been a toughness theory suitable for predicting the toughness of SFRP composites by taking into consideration the effects of FLD and FOD. In the total WOF theory, formulae of \( R_m \), \( R_d \), \( R_{df} \) and \( R_{po} \) have not been given for a general case with an FLD and an FOD. Some recommendations can be made on the specific areas in which further research is required.

1. Much work needs to be done on the combined effects of the design/fabrication factors on FLD and FOD.
2. Development of a generalized toughness theory considering the effects of FLD and FOD is necessary for evaluating the toughness of SFRP composites.
3. The relationship of \( R_m \) and \( R_d \) with FLD and FOD needs to be established.
4. Generalized formulae for \( R_d \) and \( R_{df} \) taking into consideration the effects of FLD and FOD need to be proposed.

REFERENCES

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