NONLINEAR BEHAVIORS OF THICK COMPOSITES WITH FIBER WAVINESS UNDER PURE BENDING

Heoung-Jae CHUN and Jai-Yoon SHIN
School of Electrical and Mechanical Engineering, Yonsei University, 134 Shinchon-Dong, Seodaemun-Ku, Seoul 120-749, Korea

Abstract: The effects of fiber waviness on the nonlinear behavior of unidirectional composites under pure bending were studied theoretically and experimentally. Constitutive models were proposed for the predictions of the flexural properties and nonlinear behavior of composites with fiber waviness. Three types of wavy patterns (uniform, graded and localized fiber waviness) were considered. The material nonlinearity and geometrical nonlinearity due to fiber waviness were incorporated into the analyses. Specimens with various degrees of fiber waviness were fabricated. Bending tests were conducted to compare the experimental results with the predictions for the uniform fiber waviness model. It was found that the experimental results were in good agreement with the predictions.

Key words: Nonlinear behavior, Fiber waviness, Thick composites, Pure bending

1. INTRODUCTION

There has been a growing interest in thick composite materials, especially for primary structures. Fiber waviness is one of the manufacturing defects frequently encountered in thick composite structures. It occurs from local buckling of prepregs or from wet hoop-wound filament strands under the pressure exerted by the overwrapped layers during the filament winding process or from the lamination residual stress build up during curing. Its characteristics can be represented by the through the thickness undulation of fiber within a thick composite laminate.

Although a number of studies have been conducted on the behavior of thick composites with fiber waviness, there are no studies related to flexural behavior of composites with fiber waviness. Shuart [1] considered both in-plane and out-of-plane waviness into a micro-buckling model for multidirectional laminates. Bogetti et al. [2] applied laminated plate theory to a model to predict stiffness and strength reduction due to layer waviness. Telegadas and Hyer [3] conducted finite element analyses on composites with fiber waviness. Chou and Takahashi [4] predicted the tensile stress-strain response of flexible wavy fiber composites. Hsiao and Daniel [5-7] investigated the effect of fiber waviness on compressive behavior of thick composites.

In this study, the nonlinear behavior of composites with fiber waviness under pure bending was investigated theoretically and experimentally. The effects of material and geometrical nonlinearities were incorporated into the theoretical analyses by employing complementary energy density and incremental method.

2. ANALYSIS

2.1. Uniform Fiber Waviness Model

A representative volume element, encompassing one

\[ \theta_r = \tan^{-1}\left( \frac{2\pi l}{\lambda_r \cos \left( \frac{2\pi x}{\lambda_r} \right)} \right) . \]
Heoung-Jae Chun and Jai-Yoon Shin

where $a_l$ and $\lambda_l$ are the amplitude and wavelength of fiber waviness in the $l$th thin slice element, respectively.

Complementary energy density ($W^*$) is used to incorporate the material nonlinearity to the model. For nonlinear elastic composite material, the fourth order expansion of $W^*$ is considered:

$$W^* = \frac{1}{2} r_{11} \sigma_{11}^2 + \frac{1}{2} S_{22} \sigma_{22}^2 + \frac{1}{2} S_{33} \sigma_{33}^2 + \frac{1}{2} S_{44} \tau_{23}^2 + \frac{1}{2} S_{55} \tau_{13}^2$$

$$+ \frac{1}{2} S_{66} \tau_{12}^2 + \frac{1}{4} S_{11} \sigma_{11}^4 + \frac{1}{4} S_{22} \sigma_{22}^4 + \frac{1}{4} S_{33} \sigma_{33}^4 + \frac{1}{4} S_{44} \tau_{23}^4$$

$$+ \frac{1}{4} S_{55} \tau_{13}^4 + \frac{1}{4} S_{66} \tau_{12}^4 + S_{12} \sigma_{11} \sigma_{22} + S_{13} \sigma_{11} \sigma_{33}$$

$$+ S_{23} \sigma_{22} \sigma_{33},$$

(2)

where 1, 2 and 3 are direction numbers in material coordinates. The strain-stress relations can be derived from the complementary energy density as follows:

$$\varepsilon_{ij} = \frac{\partial W^*}{\partial \sigma_{ij}}.$$  

(3)

If the nonlinear coupling terms between normal and shear stresses are neglected, the strain-stress relations referred to material coordinates can be expressed in the following matrix form:

$$[\varepsilon_{ij}] = [S^*][\sigma_{ij}],$$

(4)

where the corresponding compliances are expressed as follows:

$$S_{11}^* = S_{11} + S_{111} \sigma_{11} + S_{1111} \sigma_{11}^2,$$

$$S_{22}^* = S_{22} + S_{222} \sigma_{22} + S_{2222} \sigma_{22}^2,$$

$$S_{33}^* = S_{33} + S_{333} \sigma_{33} + S_{3333} \sigma_{33}^2,$$

$$S_{44}^* = S_{44} + S_{444} \tau_{23} + S_{55} = S_{55} + S_{555} \tau_{13},$$

$$S_{66}^* = S_{66} + S_{666} \tau_{12}^2, S_{12}^* = S_{12}, S_{13}^* = S_{13}, S_{23}^* = S_{23}.$$

For composite material with off axes fiber orientation, on the $x-z$ plane, the strain-stress relations referred to the loading coordinates can be obtained by using transformation relation:

$$[\varepsilon_{ij}]_{k,x,y} = [R][T]^{-1}[R]^{-1} [S^*][\sigma_{ij}]_{x,y},$$

(5)

where

$$[T] = \begin{bmatrix}
  m^2 & 0 & 0 & 2mn & 0 \\
  0 & 1 & 0 & 0 & 0 \\
  n^2 & 0 & m^2 & 0 & -2mn \\
  0 & 0 & m & 0 & -n \\
  -mn & 0 & mn & 0 & -n^2 \\
  0 & 0 & 0 & m & 0
\end{bmatrix}, 

[R] = \begin{bmatrix}
  0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0
\end{bmatrix}.$$
NONLINEAR BEHAVIORS OF THICK COMPOSITES

expressed in terms of the middle surface extensional stra-
ins \( [e^0] \), and overall laminate curvature \( [\kappa] \), as follows:

\[
\begin{bmatrix}
(\sigma_{xx})_l \\
(\sigma_{yy})_l \\
(\tau_{xy})_l
\end{bmatrix} =
\begin{bmatrix}
(\epsilon_{xx})_l \\
(\epsilon_{yy})_l \\
(\tau_{xy})_l
\end{bmatrix} =
\begin{bmatrix}
\epsilon^0_{xx} \\
\epsilon^0_{yy} \\
\tau_{xy}^0
\end{bmatrix}
\begin{bmatrix}
Q_{xx}'''
\\
Q_{yy}'''
\\
Q_{xy}'''
\end{bmatrix} =
\begin{bmatrix}
\epsilon^0_{xx} \\
\epsilon^0_{yy} \\
\tau_{xy}^0
\end{bmatrix}
\begin{bmatrix}
\kappa_{xx} \\
\kappa_{yy} \\
\kappa_{xy}
\end{bmatrix},
\]

(9)

where \( Q_{ij}''' \) are plane-stress moduli calculated from
off-axis stiffnesses \( (C_{pq}')_l \) and \( [Q_{ij}']^* = [S_{ij}']^{-1} \).

The resultant forces \( (N) \) and moments \( (M) \) for the
representative volume are obtained by integrating the
effects for all the short length subelements and stiffness
over a period of wavelength. Thus, the resultants are
obtained as follows:

\[
\begin{bmatrix}
N \\
M
\end{bmatrix} =
\begin{bmatrix}
| A | B \\
| -B | D
\end{bmatrix}
\begin{bmatrix}
e^0 \\
\kappa
\end{bmatrix}.
\]

(10)

where

\[
\begin{align*}
A_y &= \frac{1}{\lambda_i} \int_0^h \frac{h}{2} Q_{yy}''' \, dx, \\
B_y &= \frac{1}{\lambda_i} \int_0^h \frac{h}{2} z Q_{xy}''' \, dx, \\
D_y &= \frac{1}{\lambda_i} \int_0^h \frac{h}{2} z^2 Q_{xx}''' \, dx,
\end{align*}
\]

where \( e^0 \) is effective middle surface extensional strains in
the representative volume and \( \kappa \) is effective curvature in
the representative volume. Equation (10) can be expre-
ssed in the alternative form through the matrix inversion:

\[
\begin{bmatrix}
e^0 \\
\kappa
\end{bmatrix} =
\frac{1}{\lambda_i} \begin{bmatrix}
A \\
B
\end{bmatrix}
\begin{bmatrix}
N \\
M
\end{bmatrix}.
\]

(11)

The effective flexural properties for a composite with
fiber waviness are obtained from the following relations:

\[
\overline{D}_y = \frac{1}{\lambda_i} \int_0^h \frac{h}{2} z^2 Q_{yy}''' \, dx,
\]

(12)

where \( [Q_{ij}']^* \) are obtained from \( [Q_{ij}']^* \) in Eq. (9) by dr-
oping the compliances of higher order.

Geometrical nonlinearity is introduced when the sha-
pe of fiber waviness in the representative volume is chan-
gen under flexural loading.

The fiber length \( (dS_y) \) in a infinitesimal short sub-
element with the length of \( dx \) in the \( l \)th thin slice element is obtained as

\[
dS_y = \sqrt{dx^2 + dx^2 \tan^2 \theta_i}.
\]

(13)

The total length of fiber in one period of wavelength of
the thin slice element, using the elliptic integral of the
second kind, is obtained as follows:

\[
S_l = \int_0^1 dS_y = \frac{\lambda_i}{2\pi} \frac{1}{\sqrt{1-k_l^2}} \int_0^{2\pi} \sqrt{1-k_l \sin^2 \beta} \, d\beta
\]

\[
= \lambda_i (1 + 2 \left( k_l^2 + \frac{1}{8} \right) + 9 \left( \frac{k_l^2}{8} \right)^2 + \ldots),
\]

(14)

where

\[
c_i = \frac{2\pi a_i}{\lambda_i} \quad \text{and} \quad k_l^2 = \frac{c_i}{c_i + 1}
\]

Since \( k_l \) is smaller than 1, this series converges.

The strain in the fiber direction of a infinitesimal
short subelement is given in terms of strains of the
loading coordinates by

\[
(\epsilon_f)_l = \cos^2 \theta_i (\epsilon_{xx})_l + \sin^2 \theta_i (\epsilon_{yy})_l + \sin \theta_i \cos \theta_i (\epsilon_{xy})_l.
\]

(15)

The average strain of each thin slice element along the
fiber direction is obtained as

\[
(\overline{\epsilon}_f)_l = \frac{1}{S_l} \int_0^h \cos^2 \theta_i (\epsilon_{xx})_l + \sin^2 \theta_i (\epsilon_{yy})_l
\]

\[
+ \sin \theta_i \cos \theta_i (\epsilon_{xy})_l \, dS_l,
\]

\[
= \left( (S_{13}''' - \overline{S}_{13}''') \right)_l + \left( (S_{11}''' - \overline{S}_{11}''') \right)_l
\]

\[
\times \frac{1}{S_l} \int_0^h \cos^2 \theta_i \, dS_l \\sigma_{\alpha l},
\]

(16)

where
F(kₙ) = \frac{1}{S_i} \int_{0}^{S} \cos^2 \theta dS_i = \frac{1}{4} - k_i^2 + \frac{9}{64} k_i^4
+ \frac{25}{512} k_i^6 + \frac{1225}{16384} k_i^8 + \frac{3969}{65536} k_i^{10} + \cdots \times (1 - \frac{1}{4} k_i^2)
- \frac{3}{64} k_i^{10} - \frac{1}{512} k_i^{16} - \frac{175}{16384} k_i^{20} - \frac{441}{65536} k_i^{22} + \cdots .

The incremental method is adopted together with the change of strains in the fiber direction to examine the geometrical nonlinearity associated with deformation under pure bending. If we assumed that only bending moment component in the x-z plane (M_xz) is applied to the composite material, the incremental strain (\(\varepsilon_{xx}\)) and curvature (\(\kappa_{xx}\)) for the given incremental bending moment in the lth thin slice element are obtained as

\[
(\varepsilon_{xx})_l = (b_{xx}^{\varepsilon} + d_{xx}^{\varepsilon} \varepsilon_t) M_{xx}, \quad (17.a)
\]
\[
\kappa_{xx} = d_{xx}^{\kappa} \kappa_{xx}, \quad (17.b)
\]

where \(b_{xx}^{\varepsilon}\) and \(d_{xx}^{\varepsilon}\) are differential compliances obtained by substituting \(2\sigma_{xx}\) for \(\sigma_{xx}\) and \(3\sigma_{xx}^2\) for \(\sigma_{xx}^2\) in Eq. (11). The corresponding incremental stress in the loading direction for the thin slice element is obtained from

\[
(\varepsilon_{xx})_l = (E_x^{\varepsilon})_l (\varepsilon_{xx})_l, \quad (18)
\]

where

\[
(E_x^{\varepsilon})_l = \frac{1}{(S_{11}^{\varepsilon})_l}
= \frac{1}{S_{11}^{\varepsilon}} \left[ (S_{11}^{\sigma})_i + (2S_{13} + S_{33}) I_3 + S_{33} I_3 \right]
+ 2(\sigma_{xx})_i (S_{111} I_{15} + S_{333} I_{13})
+ 3(\sigma_{xx})^2 (S_{111} I_{15} + S_{333} I_{17} - S_{555} I_{16})
\]

\[
I_1 = \frac{1 + c_i / 2}{(c_i + 1)^{3/2}}, \quad I_2 = \frac{c_i / 2}{(c_i + 1)^{3/2}}, \quad I_3 = 1 - \frac{1 + 3c_i / 2}{(c_i + 1)^{3/2}},
\]
\[
I_5 = \frac{1 + c_i + 3c_i^2 / 8}{(c_i + 1)^{5/2}}, \quad I_{13} = 1 - \frac{1 + 5c_i / 2 + 15c_i^2 / 8}{(c_i + 1)^{5/2}},
\]
\[
I_{16} = \frac{3c_i^2 / 8 + c_i^2 / 16}{(c_i + 1)^{5/2}}, \quad I_{18} = 1 - \frac{1 + 7c_i / 2 + 35c_i^2 / 8 + 35c_i^3 / 16}{(c_i + 1)^{5/2}}.
\]

The average strain increment of the fiber in the lth thin slice element by uniaxial stress in the x direction is given by

\[
(\Delta \varepsilon_f)_l = \left( \frac{(S_{11}^{\varepsilon})_l - (S_{11}^{\varepsilon})_i}{(S_{11}^{\varepsilon})_l} \right) F(k_i) + (S_{11}^{\varepsilon})_l (\Delta \sigma_{xx})_l, \quad (19)
\]

where

\[
(S_{11}^{\varepsilon})_l = (S_{11} + S_{13} - S_{55}) I_1 + S_{13} I_2
+ 2(\sigma_{xx})_i (S_{111} I_{15} + S_{333} I_{13}),
+ 3(\sigma_{xx})^2 (S_{111} I_{15} + S_{333} I_{17} - S_{555} I_{16}),
\]

\[
I_1 = \frac{c_i}{(c_i + 1)^{3/2}}, \quad I_6 = \frac{c_i / 2 + c_i^2 / 8}{(c_i + 1)^{5/2}}, \quad I_{12} = \frac{3c_i^2 / 8}{(c_i + 1)^{5/2}}.
\]

The current stress and strains are determined when the rth increments are added to the previous state of stress and strains as follows:

\[
(\sigma_{xx})_i = \sum_{i=1}^{r} (\Delta \sigma_{xx})_l, \quad (20.a)
\]
\[
(\varepsilon_{xx})_i = \exp \left( \sum_{i=1}^{r} ((\Delta \varepsilon_{xx})_l) - 1 \right), \quad (20.b)
\]
\[
(\varepsilon_f)_i = \exp \left( \sum_{i=1}^{r} ((\Delta \varepsilon_f)_l) - 1 \right). \quad (20.c)
\]

The changed wavelength and length of fiber in the lth thin slice element are calculated from these strains as follows:

\[
(\lambda_i)_l = (\lambda_i)_0 (1 + ((\varepsilon_{xx})_i)_l), \quad (21.a)
\]
\[
(S_i)_l = (S_i)_0 (1 + ((\varepsilon_f)_i)_l). \quad (21.b)
\]

In order to determine the change of shape of fiber waviness during load increment, it is assumed that the fibers in each thin slice element maintain a similar shape of waviness while varying only their amplitude (a₁) and wavelength (λ). The amplitude of waviness in the thin element for the rth step loading is determined by Eqs. (14) and (21) with Newton-Raphson Method. From these newly determined values of amplitudes (a₁) and changed wavelength (λ), the changed angle ((θ₁)) between the tangent to fiber and the x axis is obtained from Eq. (1). With this changed angle, calculations from Eq. (14) to Eq. (21) are repeated for the (r+1)th step of loading. These repeated procedures continue till the final value of loading is reached.
2.2. Graded Fiber Waviness Model

Figure 2 shows a representative volume of graded fiber waviness. The representative volume consists of thicknesswise volume fraction \( V_{ct} \) of graded fiber waviness and volume fraction without fiber waviness. It is assumed that amplitude of fiber waviness decays linearly from a maximum at the mid-surface to zero at the certain surfaces parallel to the outer surfaces.

The angle \( \theta_l \) between the tangent to fiber and the \( x \) axis in each slice is now a function of both \( x \) and \( z \). For the \( l \)th thin slice element, the angle is given by

\[
\theta_l = \tan^{-1}\left( \frac{2\pi a_l(z_l)}{\lambda_l} \cos \left( \frac{2\pi x}{\lambda_l} \right) \right),
\]

and

\[
a_l = a_0 \left( 1 - \frac{2|z_l|}{h} \right) \left( 1 - u(|z_l| - \frac{h}{2V_{ct}}) \right),
\]

where \( a_0, z_l, h, u \) and \( V_{ct} = \frac{h_w}{h} \) are the maximum amplitude of fiber waviness, distance from mid-plane to the \( l \)th thin slice element, thickness of the composite material, unit step function and thicknesswise volume fraction, respectively.

![Fig. 2. Schematic drawing of a representative volume and coordinates for unidirectional composite with graded fiber waviness.](image)

2.3. Localized Fiber Waviness Model

Figure 3 shows a representative volume element containing a single period of localized fiber waviness. The average transformed stiffnesses and compliances for the \( l \)th thin slice model are obtained as follows:

\[
\begin{align*}
\overline{A_y} &= V_d \overline{A_y} \text{ (graded)} + (1 - V_d) \overline{A_y} \text{ (no-waviness)}, \\
\overline{B_y} &= V_d \overline{B_y} \text{ (graded)} + (1 - V_d) \overline{B_y} \text{ (no-waviness)}, \\
\overline{D_y} &= V_d \overline{D_y} \text{ (graded)} + (1 - V_d) \overline{D_y} \text{ (no-waviness)},
\end{align*}
\]

where \( \overline{A_y} \text{ (graded)}, \overline{B_y} \text{ (graded)}, \overline{D_y} \text{ (graded)} \), \( \overline{A_y} \text{ (no-waviness)} \), \( \overline{B_y} \text{ (no-waviness)} \) and \( \overline{D_y} \text{ (no-waviness)} \) are average stiffnesses of composites for graded and without fiber waviness from Eq. (10), respectively. \( \overline{S_{yy}} \text{ (graded)} \) and \( \overline{S_{yy}} \text{ (no-waviness)} \) are average compliances of composites for graded and without fiber waviness from Eq. (8) and \( V_d = \frac{L_w}{L} \) is lengthwise volume fraction.

![Fig. 3. Schematic drawing of a representative volume and coordinates for unidirectional composite with localized fiber waviness.](image)

3. EXPERIMENTAL PROCEDURES

The material investigated in this study was DMS 2224 graphite/epoxy composite material (Hexcel, Inc.). Mechanical characterizations were conducted on composites to obtain the elastic properties and higher order compliances.

The standard composite coupons without fiber waviness were used for the mechanical characterization to obtain the elastic properties and behaviors under tensile and compressive loadings. The IITRI compression test fixture was used for the compression tests. The characterizations were conducted in a servo-hydraulic testing system (MTS 810.23). The test coupons were instrumented with commercially available strain gages (EA and CA gages from Measurements, Inc.) for measuring longitudinal and transverse strains. The strain gages were bonded on the surfaces of specimens with M-Bond 200 (Measurements, Inc.). The strain gage outputs were conditioned and amplified through a dynamic strain bridge conditioner and amplifier (Shinkoh, Inc.) and recorded by a data acquisition system installed in the control system (MTS Test Star II). The acquired data were transferred to a personal computer and stored on a hard disk. A special fabrication technique was developed to produce thick composite plates with the controlled uniform fiber waviness. The fiber waviness ratios \((a/\lambda)\) for the fabricated composite plates were 0.011, 0.034 and 0.059.
Four point bending test fixture was designed and manufactured to conduct flexural tests. Geometries of four point bending coupons are shown in Fig. 4. Strain gages were mounted in the longitudinal (x) and transverse (y) directions on both top and bottom of each test coupon in the gage section encompassing half of fiber wavelength to measure strains on both surfaces. Four point bending tests were conducted also in the servo-hydraulic testing machine (MTS) while monitoring the deflection, strains and applied load. The deflection of the composite specimen was measured with the house-made deflection meter. In this study, the deflection at the center of load span section was considered, since only in that section the composite material was loaded under pure bending.

![Fig. 4. Schematic drawing of four point bending test geometries (Ls=35 mm, L=70 mm, thickness=5.2 mm and width=10 mm).](image)

### 4. RESULTS AND DISCUSSION

The elastic properties and higher order compliances of DMS2224 graphite/epoxy composite materials were obtained from mechanical characterizations and are listed in Tables 1 and 2, respectively.

Figures 5 and 6 show the predicted and experimentally obtained normalized elastic flexural moduli as a function of fiber waviness ratio for three different fiber waviness models. The figures show the predictions are in good agreement with the experimental results for the uniform fiber waviness model. The changes of flexural moduli for graded fiber waviness model with the increment of fiber waviness ratio are smaller than those for uniform fiber waviness model. The localized fiber waviness model shows the least changes of flexural moduli as a function of fiber waviness.

Figures 7-9 show the comparisons between the predicted and experimentally obtained bending moment-deflection curves for the three types of models. It is observed in the figures that the difference of flexural behavior of composites among different models enlarges as the fiber waviness ratio increases and the flexural behaviors of composites are strongly influenced by the degree of fiber waviness ratio. The flexural behavior of the composite material is affected less by the types of fiber waviness if the fiber waviness ratio is less than 0.011. However, it starts to show appreciable differences when the fiber waviness ratio is greater than 0.034. It is concluded that the flexural behavior of composite materials with the uniform fiber waviness model is affected most by the degree of fiber waviness whereas those of the graded and localized fiber waviness models are not strongly influenced. The predicted curves are in good agreement with the experiments for the case of uniform fiber waviness model.

Figure 10 shows the change of fiber waviness ratio along the thickness with the increment of bending moment. Under pure bending, the longitudinal strain on the upper portion is compressive and that in the lower portion is tensile. The portion under tensile strain tends to stretch the waved fiber while the portion under compressive strain tends to undulate the waved fiber further more. The figure shows how the fiber waviness ratio in composites changes with deformation along the thickness direction.

<table>
<thead>
<tr>
<th>Table 1. Mechanical properties of DMS 2224 graphite/epoxy composite.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Properties</strong></td>
</tr>
<tr>
<td>Tensile Longitudinal Modulus $E_{1l}$, Gpa</td>
</tr>
<tr>
<td>Tensile Transverse Modulus $E_{1t}$, Gpa</td>
</tr>
<tr>
<td>In-Plane Shear Modulus $G_{12}$, Gpa</td>
</tr>
<tr>
<td>Tensile Major Poisson's Ratio $\nu_{12}$</td>
</tr>
<tr>
<td>Tensile Minor Poisson's Ratio $\nu_{2l}$</td>
</tr>
<tr>
<td>Compressive Longitudinal Modulus $E_{1c}$, Gpa</td>
</tr>
<tr>
<td>Compressive Transverse Modulus $E_{1c}$, Gpa</td>
</tr>
<tr>
<td>Compressive Major Poisson's Ratio $\nu_{1c}$</td>
</tr>
<tr>
<td>Compressive Minor Poisson's Ratio $\nu_{2c}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2. Compliances of DMS 2224 graphite/epoxy composite.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Compliances</strong></td>
</tr>
<tr>
<td>$S_{11e}$, GPa$^1$</td>
</tr>
<tr>
<td>$S_{12e}$, GPa$^2$</td>
</tr>
<tr>
<td>$S_{11e}$, GPa$^3$</td>
</tr>
<tr>
<td>$S_{12e}(S_{22e})$, GPa$^1$</td>
</tr>
<tr>
<td>$S_{12e}(S_{22e})$, GPa$^2$</td>
</tr>
<tr>
<td>$S_{12e}(S_{22e})$, GPa$^3$</td>
</tr>
<tr>
<td>$S_{12e}(S_{22e})$, GPa$^1$</td>
</tr>
<tr>
<td>$S_{12e}(S_{22e})$, GPa$^2$</td>
</tr>
<tr>
<td>$S_{12e}(S_{22e})$, GPa$^3$</td>
</tr>
<tr>
<td>$S_{12e}(S_{22e})$, GPa$^3$</td>
</tr>
</tbody>
</table>
NONLINEAR BEHAVIORS OF THICK COMPOSITES

Fig. 5. Comparison between the predictions and experiments of normalized flexural modulus \( \frac{D_{xx}}{D_{11}} \) as a function of fiber waviness ratio \( \frac{a}{\lambda} \).

Fig. 6. Comparison between the predictions and experiments of normalized flexural modulus \( \frac{D_{xy}}{D_{12}} \) as a function of fiber waviness ratio \( \frac{a}{\lambda} \).

Fig. 7. Comparison between the predictions and experiment of bending moment-deflection curves for three types of fiber waviness model \( \frac{a}{\lambda} = 0.011, \ V^c_t = 1.0 \) and \( V^a = 0.6 \).

Fig. 8. Comparison between the predictions and experiment of bending moment-deflection curves for three types of fiber waviness model \( \frac{a}{\lambda} = 0.034, \ V^c_t = 1.0 \) and \( V^a = 0.6 \).

Fig. 9. Comparison between the predictions and experiment of bending moment-deflection curves for three types of fiber waviness model \( \frac{a}{\lambda} = 0.059, \ V^c_t = 1.0 \) and \( V^a = 0.6 \).

Fig. 10. Changes of fiber waviness ratio along the thickness with the increment of bending moment for uniform fiber waviness model \( \frac{a}{\lambda} = 0.034 \).
5. SUMMARY AND CONCLUSIONS

The nonlinear behavior of composites under flexural loading was studied by means of three constitutive models. Elastic flexural properties and behaviors of the composite materials were predicted. The material and geometrical nonlinearities were incorporated in the models.

A special fabrication technique was developed to produce thick composite specimens with the controlled uniform fiber waviness ratios of 0.011, 0.034 and 0.059. Four point flexural tests were conducted in a servo-hydraulic testing machine. The experimental results were in good agreement with the predictions based on one of the constitutive models. It was found that the fiber waviness in the composite material caused notable changes in flexural moduli as well as flexural behaviors. They tended to degrade as the degree of fiber waviness increased. The flexural behavior of uniform fiber waviness model was affected most whereas those of the graded and localized fiber waviness models were not strongly affected by the degree of fiber waviness.

The changes of the fiber waviness ratio during deformation were also computed. The results confirmed that the geometrical nonlinearity should be considered for the analyses of composites with fiber waviness.

Acknowledgements—This study is supported by the academic research fund of Ministry of Education, Republic of Korea.

REFERENCES