General paper

Tunneling Algorithm Applied to Reliability Analysis of Laminated Composite Plate

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Abstract: In this study, the tunneling algorithm is applied to the first order reliability method (FORM) to find a global design point. In the FORM, the structural reliability is evaluated as the minimum distance from the origin to the limit state surface in the standard normal distribution space. Therefore, the FORM is formulated as a nonlinear optimization problem. However, it is difficult to find the design point, when the limit state surface has several local minimum points such as a case of the laminated composite plate subject to the first ply failure. In general, the original tunneling method is not worked well for the constraint problem. In this study, a new tunneling function suitable for the FORM is proposed to find the other design point which has a lower objective function value and satisfies the equality constraint. In the tunneling function, the limit state function is treated as a kind of a penalty term with a pole. Through the reliability analysis of a laminated composite plate subject to in-plane loads, the efficiency of the proposed tunneling function is demonstrated. Also, the selection of the penalty parameter is suggested.

Key words: *First order reliability method, Structural reliability, Tunneling method, Global optimization method, Laminated composite plate, Tsai-Wu criterion, First ply failure*

1. INTRODUCTION

Laminated plates are widely used in structural applications because of their high specific strength and stiffness. Therefore, many studies have been conducted on the optimum design of composite plates [1].

However, most of them yield the optimum fiber orientation angles under deterministic conditions, where the material properties and the load conditions are assumed to have no variations. It has been known that such a deterministic optimum design is strongly anisotropic and sensitive to the change in loading conditions [1]. Therefore, it is necessary to consider the effect of such variations by applying the structural reliability theory [2]. Especially, the reliability-based design which maximizes the structural reliability is important [2, 3].

The reliability-based design under the in-plane strength by the first ply failure criterion have already been studied [4–7]. The studies have shown that the reliability increases as the number of fiber axes is increased, and that the reliability-based design approaches a quasi-isotropic configuration as the load variations are increased. The design is very different from the deterministic optimum design of which the orientation angle runs along the loading direction.

For the reliability-based design, the first order reliability method (FORM) is commonly used to evaluate a failure probability [2]. Since the reliability index is defined as a minimum distance from the origin to the limit state surface in the standardized normal distribution space (U -space), the FORM is formulated as an optimization prob-

lem with an equality constraint. Therefore, a mathematical programming method such as the sequential quadratic programming (SQP) method [8] is commonly used.

The conventional optimization algorithm may converge to a local design point, when the limit state function is strongly nonlinear such as the laminated composite design problem. In that case, the reliability would be overestimated and hence the obtained reliability-based design would be wrong. Especially, near the reliability-based optimum design, more than two design points have almost the same distance from the origin [9]. Therefore, during the reliability-based optimization run, the misleading will often occur around the real optimum design.

Several studies are proposed to search for the multiple design points. Shao and Murotsu [10] proposed a FOS-MUOP3 algorithm. After finding local design points, the other design point searching will be started from the point which is located at different directions from all of the detected design points and lies in the failure region. If the design point will be found outside the hypercone region around any local design point with given angle of direction, the design point is adopted. Repeating above process, multiple design points are effectively selected. The algorithm corresponds to narrow the searching region in every repetition. On the other hand, Der Kiureghian and Dakessian [11] proposed a simple heuristic method. After finding a local design point, a bulge is added to the limit state function around the design point. Because the objective function in the FORM is a distance from the origin to the limit surface, the modification makes the design point move away from the origin. Then, the next optimization

Fig. 1. Symmetric laminated composite plate.

run will find the other design point. All of the local design points will be found by repeating above process. When the searching is converged to the edge of bulges, it is judged that no other design points will exist.

In this study, a tunneling algorithm as one of global optimization methods is applied to the FORM for searching for a global design point. The tunneling algorithm is known as one of global optimization methods [12–16]. In the minimization phase, a local optimum is found by a conventional optimization algorithm. Then, in the tunneling phase, another starting point which has the same value of the objective function as that of the local minimum is searched by using a tunneling function which introduces a pole at the local optimum.

However, the tunneling method has seldom applied to engineering design problems because it is difficult to solve a constraint problem [13]. The hybrid strategy combining the conventional optimization and the tunneling methods is proposed recently [17]. But only the initial point selection method is utilized after the finding local optimum. Applying the tunneling method to the constraint problem is still under development.

In this study, a new tunneling function suitable to the FORM is proposed, where the equality constraint is treated as a penalty function with a pole at the local design point obtained at the minimization phase. Through the reliability analysis of a laminated composite plate subject to in-plane loads [6], the efficiency of the proposed tunneling function is demonstrated. Also, the selection of the penalty parameter is suggested.

2. STRENGTH ANALYSIS OF FIRST PLY FAILURE CRITERION

Consider a symmetric laminated composite plate subjected to in-plane load as shown in Fig. 1. The subscripts 1, 2 and 6 represent a major axis, the axis perpendicular to the 1-axis, and the shear with respect to 1-2 plane, respectively. Each ply has its own fiber orientation angle and stiffness. The stresses in a laminate system are formulated as follows [18]:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = 2 \sum_{i=1}^{N/2} v_i [Q_i] \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_6 \end{Bmatrix}, \quad (1)$$

where N is the number of plies, v_i denotes the content of the i -th ply. Q_i is the stiffness matrix of the i -th ply, which

is defined by the material invariant U_j ($j = 1, \dots, 5$) and the ply orientation angle θ_i as follows:

$$\begin{Bmatrix} Q_{11} \\ Q_{22} \\ Q_{12} \\ Q_{66} \\ Q_{16} \\ Q_{26} \end{Bmatrix}_i = \begin{bmatrix} U_1 & \cos 2\theta_i & \cos 4\theta_i \\ U_1 & -\cos 2\theta_i & \cos 4\theta_i \\ U_4 & 0 & -\cos 4\theta_i \\ U_5 & 0 & -\cos 4\theta_i \\ 0 & 1/2 \sin 2\theta_i & \sin 4\theta_i \\ 0 & 1/2 \sin 2\theta_i & -\sin 4\theta_i \end{bmatrix} \begin{Bmatrix} 1 \\ U_2 \\ U_6 \end{Bmatrix}. \quad (2)$$

The material invariants U_j ($j = 1, \dots, 5$) are defined as follows:

$$\begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{Bmatrix} = \frac{1}{8} \begin{Bmatrix} 3Q_{xx} + 3Q_{yy} + 2Q_{xy} + 4Q_{ss} \\ 4Q_{xx} - 4Q_{yy} \\ Q_{xx} + Q_{yy} - 2Q_{xy} - 4Q_{ss} \\ Q_{xx} + Q_{yy} + 6Q_{xy} - 4Q_{ss} \\ Q_{xx} + Q_{yy} - 2Q_{xy} + 4Q_{ss} \end{Bmatrix}, \quad (3)$$

where Q_{xx} , Q_{yy} , Q_{xy} and Q_{ss} are defined by the Young's moduli along the material and the perpendicular axes, the in-plane shear modulus and Poisson's ratio:

$$\begin{aligned} Q_{xx} &= \frac{E_x^2}{E_x - \nu_x^2 E_y}, & Q_{yy} &= \frac{E_x E_y}{E_x - \nu_x^2 E_y}, \\ Q_{xy} &= \frac{\nu_x E_x E_y}{E_x - \nu_x^2 E_y}, & Q_{ss} &= E_s. \end{aligned} \quad (4)$$

As a ply failure criterion, Tsai-Wu criterion [18] is adopted. The failure envelope is expressed as an ellipsoid in stress space as follows:

$$\begin{aligned} &F_{xx}\sigma_x^2 + 2F_{xy}\sigma_x\sigma_y + F_{yy}\sigma_y^2 \\ &+ F_{ss}\sigma_s^2 + F_x\sigma_x + F_y\sigma_y \begin{cases} < 1 & \text{safety} \\ = 1 & \text{limit state} \\ > 1 & \text{failure} \end{cases} \end{aligned} \quad (5)$$

where F_{ij} s and F_i s are the strength parameters as follows:

$$\begin{aligned} F_{xx} &= \frac{1}{X_c X_t}, & F_x &= \frac{1}{X_t} - \frac{1}{X_c}, \\ F_{yy} &= \frac{1}{Y_c Y_t}, & F_y &= \frac{1}{Y_t} - \frac{1}{Y_c}, \\ F_{ss} &= \frac{1}{S^2}, & F_{xy} &= F_{xy}^* (F_{xx} F_{yy})^{\frac{1}{2}}, \end{aligned} \quad (6)$$

where X_t and X_c are the tensile and the compressive strengths along the fiber direction, respectively, Y_t and Y_c are the tensile and the compressive strengths perpendicular to the fiber direction, respectively, and S is a shear strength. F_{xy}^* is set to -0.5 [18].

The i -th Ply stresses $(\sigma_x, \sigma_y, \sigma_s)_i$ are obtained by the stress-strain relationship:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix}_i = \begin{bmatrix} Q_{xx} & Q_{xy} & 0 \\ Q_{xy} & Q_{yy} & 0 \\ 0 & 0 & Q_{ss} \end{bmatrix}_i \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_s \end{Bmatrix}, \quad (7)$$

Fig. 2. Failure envelope of Tsai-wu criterion.

where the ply strains $\epsilon_i = (\epsilon_x, \epsilon_y, \epsilon_s)_i$ are obtained by converting the following coordinate transformation from the plate strains $\epsilon_{\text{plate}} = (\epsilon_1, \epsilon_2, \epsilon_6)$:

$$\epsilon_i = T \epsilon_{\text{plate}}, \quad (8)$$

$$T = \begin{bmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & mn \\ -2mn & 2mn & m^2 - n^2 \end{bmatrix},$$

where $m = \cos \theta_i$, $n = \sin \theta_i$.

The ply strength ratio R is defined as the ratio between the ply failure stress σ_F and the applied stress σ_0 under the proportional loading assumption as shown in Fig. 2, which is obtained by substituting $\sigma_F = R\sigma_0$ into Eq. (5). In this study, the first ply failure criterion is adopted. Thus, the plate will fail when the minimum of the ply strength ratio is less than unity. Therefore, the limit state function is defined as follows:

$$g(x) = R - 1 \begin{cases} > 0 & \text{safety} \\ = 0 & \text{limit state} \\ < 0 & \text{failure} \end{cases}, \quad (9)$$

where x denotes the random variables. The material stiffness properties (E_x, E_y, E_s, ν_x) and the strength properties (X_t, X_c, Y_t, Y_c, S) are treated as random variables as well as the applied loads ($\sigma_x, \sigma_y, \sigma_s$).

3. TUNNELING ALGORITHM

The tunneling algorithm is known as one of global optimization methods [12–16]. The algorithm will find the global optimum point by iterating two phases: the minimization and the tunneling phases as shown in Fig. 3. In the minimization phase, a local minimum x^* is searched by a conventional optimization method. The tunneling phase finds another point x^0 that has the same value of the objective function as that of the local optimum but differs from the local point; $f(x^0) = f(x^*)$, $x^0 \neq x^*$. In order to move from x^* , the tunneling function is defined

Fig. 3. Geometric interpretation of the tunneling algorithm and tunneling function.

by introducing a pole at the local optimum:

$$T(x) = \frac{f(x) - f(x^*)}{[(x - x^*)^T(x - x^*)]^\lambda}, \quad (10)$$

where λ is a pole strength parameter. The another point x^0 satisfying $T(x^0) = 0$, but $x^0 \neq x^*$ is searched by a kind of Newton method.

The tunneling phase is started from the perturbed point from the local optimum:

$$x = x^* + \epsilon, \quad (11)$$

where ϵ is a random vector with $|\epsilon| \ll 1$. The pole strength λ is determined so that the tunneling function will decrease, as the point is moved away. When the point x^0 satisfying $T(x^0) = 0$ is found, the point is used as a starting point at the next minimization phase.

On the other hand, the minimization phase is converged to the point $x_{T_{opt}}$ with $T(x_{T_{opt}}) > 0$, the tunneling function is regarded as having a waving property [12–15], as shown in Fig. 4. That is, the tunneling function has local minima with positive value. In that case, the tunneling function is modified by introducing the movable pole at the local optimum points. Then, the tunneling searching is repeated until the zero-point is found. The modified tunneling function is written in the following form:

$$T(x) = \frac{f(x) - f(x^*)}{[(x - x^*)^T(x - x^*)]^\lambda [(x - x^m)^T(x - x^m)]^\eta}, \quad (12)$$

where x^m is called a movable pole located at $x_{T_{opt}}$ and η is called a movable pole strength. Then, the searching

Fig. 4. Waving property of tunneling function.

process is repeated to find the point $T(x) < 0$. The starting point is randomly selected as the following equation instead of Eq. (11).

$$x = x^m + \epsilon. \quad (13)$$

If the point x^0 satisfying $T(x^0) = 0$ is found, the tunneling phase is finished and the point is used as a starting point for the next minimization phase. Otherwise, the tunneling searching with the movable pole is repeated with a different initial point.

When the number of the searching reaches the allowable limit, the tunneling phase is restarted from a new randomly selected point in Eq. (11).

After several successive numbers of the whole tunneling phase fail to find the better solution, the local optimum found in the previous minimization phase is regarded as the global optimum. The details are described in the reference [14, 15].

In this study, an optimization algorithm is adopted to find the minimum point of $T(x)$ instead of finding the zero-point $T(x) = 0$. When the point which satisfies $T(x) < 0$ is found during the optimization run, the minimization is broken off to proceed the next local minimization phase.

This algorithm is easily comprehensible. However, the method is not widely applied to engineering design problems, because of the difficulty of treating constraint conditions [17].

4. TUNNELING METHOD APPLIED TO FORM

4.1. First Order Reliability Method

The failure probability is determined to integrate the joint probability density function $f_X(x)$ over the failure region:

$$P_f = \int_{h(x) < 0} f_X(x) dx, \quad (14)$$

where $x = (x_1, x_2, \dots, x_n)^T$ is a random vector consisting of n random variables. However, it is difficult to process the integration by even a numerical integration. Because the limit state function $h(x)$ and the joint probability function is usually complicated.

Fig. 5. First order reliability method.

Therefore, the FORM is used in the reliability-based design. At first, the random variables x is transformed into the standardized normal vector u . Where, the mean value is moved to the origin in the standardized normal variable space which is commonly called U -space. In U -space, the coordinate value is normalized by the standard variation of each random variable. Then, the reliability is evaluated by solving a nonlinear optimization problem:

$$\begin{aligned} \text{Minimize : } & \beta = \sqrt{u^T u} \\ \text{subject to : } & h(u) = 0, \end{aligned} \quad (15)$$

where u and $h(u)$ are a random vector and the limit state function transformed into the standardized normal distribution, respectively.

The solution $u^* = (u_1^*, u_2^*, \dots, u_n^*)^T$ is called a design point and the minimum distance from the origin to the limit state surface $\beta = |u^*|$ is called a reliability index as shown in Fig. 5. Each coordinate value u_i^* of the design point implies the relative sensitivity on the reliability. That is, the component with large absolute value at the design point has large effect on the reliability. On the other hand, the component with small absolute value at the design point has little effect on the reliability.

Then, the limit state function is linearized at the design point u^* :

$$\begin{aligned} h(u) & \approx -\frac{1}{\beta} u^{*T} u + \beta \\ & = -\frac{1}{|\nabla_u h(u^*)|} \nabla_u h(u^*)^T u + \beta, \end{aligned} \quad (16)$$

where $\nabla_u h(u^*)$ is the gradient of the limit state function $h(u)$ at the design point u^* . Upon this linearization, the failure probability is evaluated by using the standardized normal distribution function:

$$P_f = \Phi(-\beta) = \int_{-\infty}^{-\beta} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx. \quad (17)$$

This integration is shown by the shaded area schematically in Fig. 5.

4.2. Tunneling Function for the FORM

Since a limit state function is treated as an equality constraint in the FORM, it is difficult to directly use the tunneling function, Eq. (3.). Accordingly, the following tunneling function suitable for FORM is proposed:

$$T(u) = f(u) - f(u^*) + \frac{\alpha \{h(u)\}^2}{[(u - u^*)^T(u - u^*)]^\lambda}, \quad (18)$$

where u^* denotes a local design point, $f(u)$ is the objective function defined as square of the distance from the origin $f(u) = \beta^2 = u^T u$, and α is a penalty parameter.

The second term of Eq. (18) is incorporated as the penalty term in order that the tunneling function value will be increased as the random vector apart from the limit state. Additionally, it is required to avoid that the tunneling function will have multiple local points, even if the function value takes large positive number. Especially, the original objective function of FORM in Eq. (15) have an absolute minimum at the origin, though the limit state constraint will be violated. In order that the tunneling function will not have a local minimum at the origin, the pole of the tunneling function is imposed only for the limit state function.

Additionally, the penalty parameter is set as a multiplier of the limit state function value at the origin:

$$\alpha = k \cdot h(0), \quad (19)$$

where k is a positive number which is determined after some numerical experiments. Because the original objective function $f(u)$ has an absolute minimum at the origin $u = 0$, the tunneling function should be enough large at the origin.

In the tunneling phase, an optimization algorithm is adopted to find the minimum point of $T(u)$ instead of searching the zero-point $T(u) = 0$, ($u \neq u^*$). Because the purpose of the tunneling phase is to find the better point apart from the previous local point rather than the exact zero point. Additionally, a computational effort is not so different between the minimization and the zero-finding. Therefore, the problem is formulated as follows:

Minimize : $T(u)$

subject to : $g_{in}(u) = f(u) - f(u^*) + \epsilon_g \leq 0$, (20)

where the inequality constraint g_{in} is introduced to enforce the the searching region smaller and ϵ_g is a small positive number. The constraint g_{in} forces the searching space to limit to the region where the distance from the origin is smaller than the reliability index β obtained at the previous minimization phase. The multiplier method [1] is adopted to solve this optimization problem.

However, the purpose is not to find the exact optimum point of the problem (20). Therefore, as soon as the point satisfying the constraint and $T(u) < 0$ is found, the algorithm will jump out from the tunneling phase and go to the next minimization phase.

When the searching is failed to find the negative value of the tunneling function, the tunneling function is modi-

Fig. 6. Convergence history of global optimization.

fied to introduce the movable pole as follows:

$$T(u) = f(u) - f(u^*) + \frac{\alpha \{h(u)\}^2}{[(u - u^*)^T(u - u^*)]^\lambda [(u - u^m)^T(u - u^m)]^\eta}. \quad (21)$$

This equation corresponds to the normal tunneling function with movable pole, Eq. (12).

5. NUMERICAL EXAMPLES

5.1. Simple Two-Dimensional Problem

The feasibility of the proposed method is demonstrated by a following simple problem:

Minimize : $f(u) = u_1^2 + u_2^2$

subject to : $h(u) = -(u_1 + 4)^2 - 3(u_2 - 0.1)^2 + 60 = 0$. (22)

The problem has two local optima, (2.11, 2.85) and (1.90, -2.80). The latter is the global optimum point with $f = 11.44$.

The optimization path is shown in Fig. 6, where “SQP” means the minimization phase and “Tunnel” means the tunneling phase. The sequential quadratic programming (SQP) method [8] is used in the minimization phase. Starting from (-5, 5), the minimization phase finds the local optimum point ($u_{local} = (2.11, 2.85)$).

Then, the process moves to the tunneling phase. In this example, the penalty parameter α in Eq. (18) is set to 100,

Fig. 7. Contour of the tunneling function.

where the contour plot is illustrated in Fig. 7. The multiplier method [1] is used in the tunneling phase Eq. (22). The starting perturbation vector ϵ in Eq. (11) is randomly selected, where the magnitude $|\epsilon|$ is set to 0.1. The tunneling function value at the point is very large 1.6×10^5 , because the point is closed to the pole. The line searching process in the tunneling phase is shown in Fig. 6. At first, the line searching converges to (2.92, 2.11) whose tunneling function value is 0.405. The next line searching finds the point (2.62, -2.33) of which value of the tunneling function is negative, $T = -0.733$.

Then, the minimization phase again starts from the point. The minimization phase finds the other optimum point (1.90, -2.80); $f = 11.44$. The next tunneling phase never finds the better solution. Therefore, the point is regarded as the global optimum point.

5.2. Laminated Composite Plate

5.2.1. Comparison with conventional method

The proposed method is applied to the reliability analysis of the laminated plate subject to in-plane loads [6].

The in-plane loads are assumed to be normally distributed with mean of $(\sigma_1, \sigma_2, \sigma_6) = (0.1, 0.1, 0.01)$ (GPa) and coefficient of variations of 0.2. The material is a carbon/epoxy (T300/5208) whose properties are assumed to be normally distributed, where the means and coefficients of variation are listed in Table 1.

The reliability-based designs for the three kind of laminates; $[\theta/\theta]_s$, $[\theta/\theta/0^\circ]_s$ and $[\theta/\theta/0^\circ/90^\circ]_s$ are listed in Table 2. The tri-axial and the tetra-axial laminates have higher reliability than the bi-axial laminates. For the tri-axial and the tetra-axial laminate, a quasi-isotropic laminate is selected as a reliability-based optimum design. These results are cited from the reference [6].

The difference between the SQP and the tunneling methods is investigated for the typical cases. The design points obtained by both methods in case of $[+41^\circ/-41^\circ]_s$ are compared in Table 3. For simplicity, only the coordinates

Table 1. Material properties.

		Mean	COV
Stiffness	E_x (GPa)	181	0.1
	E_y (GPa)	10.3	0.1
	E_s (GPa)	7.17	0.1
	ν_x	0.28	0.1
Strength	X_T (MPa)	1500	0.1
	X_C (MPa)	1500	0.1
	Y_T (MPa)	40.0	0.1
	Y_C (MPa)	246.0	0.1
	S (MPa)	68.0	0.1

Table 2. Reliability-based optimum designs.

Laminate construction	Reliability index
$[+45^\circ/-45^\circ]_s$	3.64
$[+60^\circ/-60^\circ/0^\circ]_s$	4.70
$[+45^\circ/-45^\circ/0^\circ/90^\circ]_s$	4.70

 Table 3. Design point coordinates of $[+41^\circ/-41^\circ]_s$.

Method	u_{σ_1}	u_{σ_2}
SQP	2.982	-2.850
Tunnel	-1.095	1.889

 Table 4. Change in the significant reliability mode, $[+61^\circ/-61^\circ/0^\circ]_s$.

Method	Reliability index	Failure mode
SQP	4.797	0°-ply
Tunnel	4.697	-61°-ply

of the axial load component (σ_1, σ_2) are listed. Where, u_{σ_1} and u_{σ_2} indicate the coordinate values of the design points in U -space of the corresponding components to the subscripts. The two design points locate on the other sides with respect to the origin in the U -space.

The significant failure mode of $[+61^\circ/-61^\circ/0^\circ]_s$ is compared in Table 4. The SQP finds the 0°-ply failure as the local design point. However, the tunneling method finds the global design point -61°-ply failure of which the reliability index is slightly smaller than 0°-ply failure.

For the three kinds of laminated construction, the variations of the reliability index on the ply orientation angle are shown in Fig. 8. The curves of "Tunnel" correspond to the reliability indices obtained by the proposed method. On the other hand, points of "SQP" indicates the reliability index obtained by the SQP method. The reliability seems to reach the maximum at the cross point of the two curves, where the two dominant failure modes have almost the same reliability. As shown in Fig. 8, SQP sometimes fails to find the global design point around the reliability-based design. It means that only the conventional optimization

Fig. 8. Comparison between SQP and tunneling method for reliability analysis.

Fig. 9. Variation of reliability index on the penalty parameter.

algorithm could miss the reliability-based design. However, the proposed tunneling method successfully obtains the global design point and hence the reliability-based design.

5.2.2. Effect of penalty parameter

The effect of the penalty parameter on the reliability analysis is investigated. The changes of the obtained reliability index on the penalty parameters for the near-optimal laminates, $[+42^\circ/-42^\circ]_s$, $[+61^\circ/-61^\circ/0^\circ]_s$ and $[+47^\circ/-47^\circ/0^\circ/90^\circ]_s$ are shown in Fig. 9. The horizontal axis indicates the penalty parameter k in Eq. (19). The numbers of calling the tunneling function are also compared in Fig. 10. The number is very large, because the finite difference method is used in the sensitivity analysis.

Fig. 10. Effect of penalty parameter on the number of calling tunneling function.

Fig. 11. Tunneling method in other load cases on tetra-axial laminate, $[+\theta/-\theta/0^\circ/90^\circ]_s$.

From these figures, a large value of the penalty parameter ($k > 10^8$) is recommended as an adequate selection. Even for cases of the smaller penalty parameter ($k < 10^4$), the correct reliability index is sometimes obtained. In these cases, the algorithm only happens to find the solution as a local design point. The algorithm does not evaluate that the obtained point is a global one and then proceeds to the next minimization phase. The algorithm is stopped only after exceeding the limit number of the repetition of the two phases (20 in this study). Therefore, the algorithm requires a large number of the tunneling function evaluations.

On the other hand, for larger penalty parameters ($k > 10^8$), the algorithm evaluates that the obtained point is a global one. Hence, the number of the tunneling function calling is smaller.

5.2.3. Efficiency of proposed method in other load cases

Finally, the efficiency of the proposed method is investigated under the other two load cases. The mean applied

load conditions are $(\sigma_1, \sigma_2, \sigma_6) = (0.1, -0.1, 0.01)$ (GPa) and $(0.1, 0.5, 0.01)$ (GPa). Both load conditions are also assumed to be normally distributed with coefficient of variations of 0.2. The changes of the reliability under both load cases in terms of ply orientation angle of the tetra-axial laminate are illustrated in Fig. 11. It is found that the reliability curves seems to consist of two curves which correspond to each failure mode. The dominant failure mode will shift at the intersection point. The tunneling method successfully track the dominant failure modes, though the SQP method converges to the local design point which does not belong to a dominant mode.

6. CONCLUSION

In this study, the tunneling algorithm is applied to the FORM that is formulated as a nonlinear optimization problem with an equality constraint. A new tunneling function suitable to the FORM is proposed, where the equality constraint is treated as a penalty function with introducing a pole at the local point. The penalty parameter is determined as a multiplier of $h(\mathbf{U} = \mathbf{0})$, the value of the limit state function at the origin in $\mathbf{U}$ -space. Because the reliability index generally takes larger value, as the limit state function value at the origin is larger.

The efficiency of the proposed method is demonstrated through the reliability analysis of the laminated composite plate subject to in-plane loads. Also, the selection of the penalty parameter is suggested.

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