General paper

The Numerical Simulation of the Mechanical Behavior of CFCC with Matrix Anisotropic Damage by Homogenization Method

Dongmei LUO* and Shigeo TAKEZONO*

** Department of Mechanical Engineering, Toyohashi University of Technology,
Tempaku-cho, Toyohashi, 441-8580, Japan*

Abstract: The purpose of the present paper is to establish a damage constitutive model for predicting the elastic-brittle mechanical behavior of continuous fiber reinforced ceramic matrix composites (CFCC). In this paper, the anisotropic damage is applied to describe the matrix phase damage which reflects all types of damage that the matrix material undergoes such as matrix cracking and transverse cracking. The asymptotic expansion homogenization method is used to obtain the effective mechanical properties of the composites, and to derive homogenized damage elastic concentration factor of unidirectional and cross-ply laminate composite materials. Internal variables are introduced to describe the evolution of the damage state under uniaxial loading and as a subsequence the degradation of the material stiffness. Results obtained from the numerical simulations include damage evolution prediction and non-linear stress-strain analyses of macro-microstructure, and they are compared well with existing experimental data.

Key words: *Anisotropic damage, Homogenization method, Continuous fiber reinforced ceramic matrix composites (CFCC), Mechanical behavior*

1. INTRODUCTION

The mechanical response of ceramic matrix composites (CMCs) has received considerable attention from both the experimental and theoretical viewpoint [1,2]. Continuous fiber reinforced ceramic matrix composites (CFCC) offer several advantages over conventional materials such as higher specific stiffness and strength, higher toughness and non-brittle failure as compared to monolithic ceramics for both room and elevated temperature applications. This type of material results from a combination of brittle materials (fibers and matrix), and ends up in a material which is no longer brittle, but instead is damage tolerant. In brittle-matrix composites, the matrix is less resistant than the fiber. The fibers strengthen the material so that its integrity can be kept when cracks propagate through the matrix, and matrix cracking and transverse cracking are the main damages in brittle-matrix composites. Micro-mechanics analyses have evolved in recent years to describe the physical behavior of constituent materials, the interaction of these materials and their resulting behavior of the composite [3-4]. These methods need not account for specific crack geometry but assume that many micro-cracks are distributed over a large area, and only a representative volume element or unit cell is introduced to model the initiation and growth of microscopic damages and their effects on the material behavior. The representative unit volume element is the smallest region over which the stresses and strains are assumed to be macroscopically uniform. However, within the unit cell the stresses and strains are non-uniform due to the heterogeneity of the material. Due to the high degree of material heterogeneity, it is extremely difficult to analyze each individual micro-

structure for composites. A natural way to overcome this difficulty is to find some kind of equivalent material model with no necessity to represent each individual microstructure. This model should characterize the average mechanical behavior and represent the heterogeneous effect of the composites. Clearly, the homogenization method is an effective method to make the macro/micro analysis simultaneously. The main attraction of the homogenization method is its systematic capability of development of the homogenized macroscopic constitutive relation of composite materials and its straightforward methodology to compute the stress even in the microstructure of composites [5]. The homogenization method has been widely applied to study mechanical behavior of metal-matrix composites and heterogeneous lamina [6-7]. It is noted that there are few literatures considering the macroscopic damage-induced strain into the method, and little work on the application of the method to brittle matrix laminate composites consisting of many lamina even with matrix anisotropic damage.

The main objective of this paper is to construct a two-scale (macro-micro) constitutive model to account for evolution of damage and stress analysis of CFCC structures. An anisotropic damage theory is used to describe the matrix damage in the irreversible thermodynamic frameworks. The homogenization method is developed by considering the effect of macroscopic damage-induced strain driven by the macroscopic strain, and it is used to analyze the mechanical behaviors of composites with damaged constituent phase.

This methodology describes material non-linearity and progressive stress redistribution effects, and micro-macro mechanics equations are programmed into a finite element program. To demonstrate the methodology and to illustrate the various capabilities of the computer code,

the unidirectional CFCC subjected to the state of constant macro-strain field in the axial (parallel to the fibers) and transverse (normal to the fibers) directions are discussed. The laminate composites with /0₃/90/0₃/, /0₃/90₂/0₃/ and /0₃/90₃/0₃/ stacking sequences are also analyzed, and the results are compared well with the experimental data.

2. THE ANISOTROPIC DAMAGE OF MATRIX IN CFCC

2.1. Damage in Ceramic Matrix Composites

For ceramic matrix composites, several micro-cracking mechanisms can occur in matrix: cracks can be parallel to or orthogonal to the fiber direction. These mechanisms are strongly anisotropic. The elastic stiffness for un-damaged constituent phases are given as $E_{ijkl}^{(m)}$ and $E_{ijkl}^{(f)}$, and the elastic stiffness of damaged matrix phase is assumed as $L_{ijkl}^{(m)}(D)$. Here, D is the second rank symmetric damage tensor which has been often employed in the development of anisotropic damage theory. The volume fractions for matrix and fiber are denoted as $v^{(m)}$ and $v^{(f)}$, respectively, such that $v^{(m)} + v^{(f)} = 1$.

The overall elastic properties are given as in [8]:

$$\bar{L}_{ijkl} = v^{(m)} L_{ijmn}^{(m)}(D) B_{mnkl}^{(m)} + v^{(f)} E_{ijmn}^{(f)} B_{mnkl}^{(f)}, \quad (1)$$

where, $B_{mnkl}^{(\eta)}$ ($\eta = m, f$) are the elastic strain concentration factors.

In a general state of deformation and damage, the effective stress tensor $\bar{\sigma}$ is related to the Cauchy stress tensor σ by the following linear transformation:

$$\bar{\sigma} = M : \sigma, \quad (2)$$

where M is the fourth-order damage effect tensor reflecting all kinds of damage which matrix phase undergoes. It was obtained by Voyiadjis and Kattan [9]:

$$M = \begin{bmatrix} \frac{1-D_{22}}{\Delta} & 0 & \frac{D_{12}}{\Delta} \\ 0 & \frac{1-D_{11}}{\Delta} & \frac{D_{12}}{\Delta} \\ \frac{D_{12}}{2\Delta} & \frac{D_{12}}{2\Delta} & \frac{(1-D_{11}) + (1-D_{22})}{2\Delta} \end{bmatrix} \quad (3)$$

Here, $\Delta = (1-D_{11})(1-D_{22}) - D_{12}^2$, where D_{11} , D_{22} and D_{12} represent the damage variables parallel to the stress directions of σ_{11} , σ_{22} and σ_{12} , respectively. The damage is assumed to be constant through the thickness of the ply.

The damaged matrix stiffness is given by

$$[L(D)]^{(m)} = [M] : [E]^{(m)}. \quad (4)$$

2.2. The Damage Constitutive Equation and Damage Evolution Law

For an isothermal process, the constitutive equation

is:

$$\sigma_{ij} = M^{-1} L_{ijkl} M^{-1} \varepsilon_{kl} = \bar{L}_{ijkl} \varepsilon_{kl}. \quad (5)$$

The generalized thermodynamic force conjugate to the kinematic damage variable [10] is:

$$Y = -\frac{1}{2} \sigma : \frac{\partial \bar{L}^{-1}}{\partial D} : \sigma = -\sigma : (\bar{L}^{-1} : M^{-1} : \frac{\partial M}{\partial D})^s : \sigma, \quad (6)$$

Y is often given the name of damage energy release rate, and the dissipation inequality thus becomes:

$$(-Y) \cdot \dot{D} \geq 0. \quad (7)$$

The evolution laws of anisotropic damages are characterized below [11]:

$$\dot{D}_{11} = \alpha \frac{Y_{11}(\dot{Y}_{11} + Y_{22}\dot{Y}_{22} + Y_{12}\dot{Y}_{12})}{2k_d(Y_{11}^2 + Y_{22}^2 + Y_{12}^2)}, \quad (8a)$$

$$\dot{D}_{22} = \alpha \frac{Y_{22}(\dot{Y}_{11} + \dot{Y}_{22} + Y_{12}\dot{Y}_{12})}{2k_d(Y_{11}^2 + Y_{22}^2 + Y_{12}^2)}, \quad (8b)$$

$$\dot{D}_{12} = \alpha \frac{Y_{12}(\dot{Y}_{11} + \dot{Y}_{22} + \dot{Y}_{12})}{2k_d(Y_{11}^2 + Y_{22}^2 + Y_{12}^2)}. \quad (8c)$$

Here α is a damage governing parameter: $\alpha=1$ for damage state, $\alpha=0$ for undamaged state. k_d is a damage parameter.

2.3. Anisotropic Damage Criterion

In this paper, the matrix damage equivalent strain, $\theta^{(m)}$, is defined as the square root of the matrix damage energy release rate [12]:

$$\theta^{(m)} = \sqrt{(-Y)^{(m)}}. \quad (9)$$

Supposing the damage evolution at time t can be expressed as

$$D(x, t) = f(k^{(m)}(x, t)), \quad (10)$$

here, $k^{(m)}(x, t)$ is the deformation history parameter, and it is determined by the evolution of damage equivalent strain $\theta^{(m)}$ as follows:

$$k^{(m)}(x, t) = \max\{\theta^{(m)}(x, \tau) | (\tau \leq t), k_0\}, \quad (11)$$

where the threshold value for damage initiation, k_0 , represents the extreme value of the equivalent strain prior to the initiation of damage which is also a damage material parameter. Here, we assume that the initiated

damages are the same in three damage directions. The evolution of damage equivalent strain can be also expressed by the Kuhn-Tucker relations:

$$\dot{k}^{(m)} \geq 0, \theta^{(m)} - k^{(m)} \leq 0, \dot{k}^{(m)}(\theta^{(m)} - k^{(m)}) = 0. \quad (12)$$

Together the definition of damage evolution with the energy dissipation inequality, yield:

$$\dot{D} = \dot{k}^{(m)} \frac{\partial f(k^{(m)}(x, t))}{\partial k^{(m)}} \geq 0. \quad (13)$$

Combining this inequality with Kuhn-Tucker relations, the damage evolution conditions become:

$$\begin{aligned} \text{If } \theta^{(m)} - k^{(m)} = 0, \dot{k}^{(m)} > 0 &\Rightarrow \text{damage process: } \dot{D} > 0, \\ \text{If } \theta^{(m)} - k^{(m)} \leq 0, \dot{k}^{(m)} = 0 &\Rightarrow \text{elastic process: } \dot{D} = 0. \end{aligned} \quad (14)$$

3. HOMOGENIZATION METHOD FOR DAMAGED COMPOSITES

3.1. Homogenization Method for Unit Cell

The homogenization procedure has already been fully derived in Pellegrino et al.[13], therefore we only briefly recall it here. We take into consideration a periodic composite material, which is a material made of a large number of regularly distributed and equal unit cells with linear boundary conditions on them. An asymptotic expansion of the displacement field is:

$$u^\xi(x) = u^0(x, y) + \xi u^1(x, y) + \xi^2 u^2(x, y) + \dots, \quad y = \frac{x}{\xi}, \quad (16)$$

here, x is a macroscopic coordinate vector, and $y=x/\xi$ is a microscopic position vector, and ξ denotes a very small positive number compared with the macroscopic dimension, as shown in Fig.1.

With consideration of the indirect differentiation rule, the strain field can be expanded as:

$$\begin{aligned} \varepsilon(u^\xi) &= \frac{1}{\xi} \varepsilon_y(u^0) + \varepsilon_x(u^0) + \varepsilon_y(u^1) \\ &+ \xi [\varepsilon_x(u^1) + \varepsilon_y(u^2)] + \dots \end{aligned} \quad (17)$$

The stress field:

$$\sigma^\xi(x) = \frac{1}{\xi} \sigma^0(x, y) + \sigma^1(x, y) + \xi \sigma^2(x, y) + \dots \quad (18)$$

The stresses and displacements fields satisfy the following equations on macroscopic domain:

$$\frac{\partial \sigma_{ij}^\xi}{\partial x_j} = -f_i, \quad (19a)$$

$$\sigma_{ij}^\xi = E_{ijmn} \varepsilon_{mn}^\xi(u^\xi), \quad \varepsilon_{ij}^\xi(u) = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad (19b)$$

$$\sigma_{ij}^\xi(x) \cdot n_j = F_i, \quad \text{on } G_F, \quad (19c)$$

$$u^\xi(x) = 0, \quad \text{on } G_0. \quad (19d)$$

Here, σ_{ij}^ξ and ε_{ij}^ξ are components of stress and strain tensors, F_i is a prescribed stress on the portion Γ_F of the boundary of Ω , and Γ_0 is the prescribed displacement boundary portion. (see Fig.1)

Substituting Eqs. (17), (18) into Eq. (19), we have:

$$\sigma^0 = 0, \quad u^0 = u^0(x), \quad (20a)$$

$$\text{div}_y \sigma^1 = 0, \quad \sigma^1 = E_{ijmn} [\varepsilon_{mny}(u^1) + \varepsilon_{mnx}(u^0)], \quad (20b)$$

$$\sigma_{ij,xj}^1 + \sigma_{ij,yj}^2 + f_i = 0. \quad (20c)$$

Equation (20b) is the micro-equation which relates the micro perturbation term u^1 to the zero-th order macro term u^0 . It is difficult to solve Eq. (20b) numerically because the changes in u^0 require repeated solutions of u^1 . However, by superposition, this can be circumvented. Due to the linearity of Eq. (20b), the solution of the perturbation displacement takes the form:

$$u^1(x, y) = H_{ikl}(y) [\varepsilon_{xkl}(u^0) + d_{kl}(x)], \quad (21)$$

where H_{ikl} is often called the elastic corrector or characteristic function, and it is a y -periodic third-order tensor. It is clear that H_{ikl} independent of u^0 , and it is obtained by solving an auxiliary equation in Θ by FEM. The $d_{kl}(x)$ is considered as macroscopic damage-induced strain driven by the macroscopic strain, which describes the couple effect between micro- to macroscopic analysis [14], and $\varepsilon_{xkl}(u^0) = \bar{\varepsilon}_{kl}$ is the macroscopic strain.

Based on the decomposition given in Eq. (21), Eq. (20b) takes the following form:

$$\{E_{ijkl} [(I_{klmn} + G_{klmn}) e_{xmn}(u^0) + G_{klmn} d_{mn}(x)]\}_{,y_j} = 0 \quad \text{on } \Theta, \quad (22)$$

where

$$I_{klmn} = \frac{1}{2} (\delta_{mk} \delta_{nl} + \delta_{nk} \delta_{ml}), \quad G_{klmn}(y) = H_{(k,y_l)mn}(y), \quad (23)$$

and δ_{mk} is the Kronecker delta, while G_{klmn} is known as a polarization function. Since Eq. (22) should be valid for arbitrary macroscopic strain fields $\bar{\varepsilon}_{kl}$, we may first consider the case of $d_{kl}(x)=0$, which yields the following equation within a unit cell,

$$\frac{\partial}{\partial y_j} [E_{ijkl} (I_{klmn} + H_{(k,y_l)mn})] = 0. \quad (24)$$

The weak form boundary value problem defined by Eq.(24) can be solved by using a finite element method. The detailed process can be referenced in the literature [15]. This process has been implemented into the ANSYS program through a user-supplied material model subroutine [16].

The elastic homogenized stiffness becomes:

$$E_{ijkl}^{(H)} = \frac{1}{\Theta} \int_{\Theta} E_{ijmn} A_{mnkl} d\Theta, \quad (25)$$

where Θ is the area (volume for three-direction) of a homogeneous unit cell, H denotes the homogenization method, and A_{ijkl} are the elastic strain concentration functions which are defined as:

$$A_{ijkl} = I_{ijkl} + G_{ijkl}. \quad (26)$$

In an absence of macro-damage induced strain, the asymptotic expansion of strain can be expressed in terms of the macroscopic strain $\bar{\varepsilon}_{kl}$ as follows:

$$\varepsilon_{ij} = A_{ijkl} \bar{\varepsilon}_{kl} + O(\xi). \quad (27)$$

By integrating Eq. (22) by parts with consideration of y -periodic boundary conditions yields:

$$\begin{aligned} d_{kl}(x) &= - \left\{ \int_{\Theta} G_{klst} E_{stij} G_{ijmn} d\Theta \right\}^{-1} \left\{ \int_{\Theta} G_{klst} E_{stij} A_{ijmn} d\Theta \right\} \bar{\varepsilon}_{mn} \\ &= D_{klmn}(x) \varepsilon_{xmn}(u^0), \end{aligned} \quad (28)$$

where

$$D_{klmn}(x) = - \left\{ \int_{\Theta} G_{klst} E_{stij} G_{ijmn} d\Theta \right\}^{-1} \left\{ \int_{\Theta} G_{klst} E_{stij} A_{ijmn} d\Theta \right\}.$$

In conjunction with Eq. (21) and Eq. (28), The asymptotic expansion of strain field (17) is written as:

$$\begin{aligned} \varepsilon_{ij}(x, y) &= A_{ijmn}(y) \varepsilon_{xmn}(x) + G_{ijkl}(y) D_{klmn}(x) \varepsilon_{xmn}(x) \\ &+ O(\xi), \end{aligned} \quad (29)$$

where $G_{ijkl}(y)$ is a damage strain influence function. Hence, the asymptotic expansion of the strain field is given as a sum of mechanical fields induced by the macroscopic strain via elastic strain concentration function and thermo-dynamical fields governed by

damage induced strain.

In the homogenization procedures it is necessary to define macroscopic quantities as functions of microscopic quantities. We integrate equilibrium Eq. (22) over Θ , the term $\int_{\Theta} \sigma_{ij,y_j}^2 d\Theta$ vanishes due to periodicity and we can obtain:

$$\left(\frac{1}{\Theta} \int_{\Theta} \sigma_{ij}^1 d\Theta \right)_{,x_j} + f_i = 0, \text{ in } O. \quad (30)$$

Substituting the asymptotic expansion of the strain field Eq.(29) into Eqs. (20b) and (30) yields the macroscopic equilibrium equation,

$$\left\{ \frac{1}{\Theta} \int_{\Theta} E_{ijkl} (A_{klmn} \bar{\varepsilon}_{mn} + G_{klmn} d_{mn}) d\Theta \right\}_{,x_j} + f_i = 0. \quad (31)$$

We define the macroscopic stress,

$$\bar{\sigma}_{ij} = \frac{1}{\Theta} \int_{\Theta} \sigma_{ij}^0 d\Theta = \bar{L}_{ijkl} \bar{\varepsilon}_{kl}, \quad (32)$$

here
$$\bar{L}_{ijkl} = \frac{1}{\Theta} \int_{\Theta} E_{ijst} (A_{stkl} + G_{stmn} D_{mnkl}) d\Theta.$$

The average strains in each sub-domain in representative volume element are obtained by integrating Eq. (29) over Θ ,

$$\varepsilon_{ij}^{(\eta)} = \frac{1}{\Theta} \int_{\Theta} \varepsilon_{ij} d\Theta = A_{ijkl}^{(\eta)} \bar{\varepsilon}_{kl} + G_{ijkl}^{(\eta)} D_{klmn} \bar{\varepsilon}_{mn} + O(\xi), \quad (33)$$

where

$$A_{ijkl}^{(\eta)} = \frac{1}{\Theta} \int_{\Theta} A_{ijkl} d\Theta, \quad (34)$$

$$G_{ijkl}^{(\eta)} = \frac{1}{\Theta} \int_{\Theta} G_{ijkl} d\Theta, \quad (\eta = m, f). \quad (35)$$

The corresponding phase stresses are given as:

$$\sigma_{ij}^{(\eta)} = L_{ijkl}^{(\eta)} \varepsilon_{kl}^{(\eta)} + O(\xi), \quad (\eta = m, f), \quad (36)$$

and the overall stresses becomes

$$\bar{\sigma}_{ij} = \nu^{(m)} \sigma_{ij}^{(m)} + \nu^{(f)} \sigma_{ij}^{(f)}. \quad (37)$$

Substituting Eqs. (33), (36) into Eq. (37), we obtain:

$$\begin{aligned} \bar{L}_{ijkl} &= \nu^{(m)} L_{ijmn}^{(m)} (D)(A_{mnkl}^{(m)} + G_{mnst}^{(m)} D_{stkl}) \\ &+ \nu^{(f)} L_{ijkl}^{(f)} (A_{mnkl}^{(f)} + G_{mnst}^{(f)} D_{stkl}), \end{aligned} \quad (38)$$

Comparing Eq. (38) with Eqs. (1), (5) and (32), the homogeneous damage elastic concentration factor $B_{mnkl}^{(\eta)} (\eta=m,f)$ become:

$$B_{mnkl}^{(m)} = A_{mnkl}^{(m)} + G_{mnst}^{(m)} D_{stkl}, \quad (39)$$

and

$$B_{mnkl}^{(f)} = A_{mnkl}^{(f)} + G_{mnst}^{(f)} D_{stkl}. \quad (40)$$

3.2. The Stiffness Matrix of the Laminate Composite

The heterogeneous materials involve several scales: a macroscopic scale corresponding to the structure, a mesoscopic scale relating to the ply and a microscopic scale up to the level of the fibers and matrix. A symmetric stacking of plies is considered here such that t is the thickness of the laminate consisting of n plies and t_k is the thickness of the k -th lamina.

Then the homogenized stiffness matrix for material direction in each ply can be expressed as in matrix form, and the average stress is expressed as:

$$\{\sigma\}_{ave} = [(1/t) \sum_{k=1}^n [L]_k t_k] \{\varepsilon\}. \quad (41)$$

Making use of Eq. (41), the gross damage homogenized elastic stiffness for the laminated composite as follows in matrix form:

$$[L_g] = \left[\frac{1}{t} \sum_{k=1}^n [L]_k t_k \right]. \quad (42)$$

Constant strain is assumed through the laminate thickness, and the stresses in each lamina are calculated as follows:

$$\{\sigma\}_k = [L]_k \{\varepsilon\}. \quad (43)$$

The typical microscopic unit cell and macroscopic laminate elements are shown in Fig. 2.

3.3. The Implementation of Macro/Microscopic Approach

The proposed model uses a macro/micro-mechanical approach through the mathematical homogenization method. In this approach the material behavior of the constituents is established individually. Due to the non-linear character of the problem an incremental analysis is employed. Prior to non-linear analysis elastic strain concentration factors, $A_{ijkl}(\gamma)$ are computed using Eqs. (24)

and (26) by finite element method for unit cell. Then, the phase elastic strain concentration factors are pre-computed using Eqs. (34) and (35). Using the conditions for damage evolution in Eqs. (14) and (15), each constituent is considered individually to investigate if the current applied stress increment induces further damage. The next step requires to update the appropriate variables affected by a change in the damage variables. The above microscopic analysis provides effective composite material properties for macro-analysis. The effective properties degradation as damage progresses in the matrix material. Thus, they must be recalculated in the micro-analysis along with progressive damage. Then the displacement vector u , overall strain $\bar{\varepsilon}_{mn}$, strain history parameter $k^{(m)}$, damage parameter D_{11} , D_{22} , D_{12} and displacement increment Δu can be calculated from the finite element analysis of the macroscopic problem. After updating the newly calculated values for the damage variables, they are used to verify whether the new stress and damage state satisfies the damage criterion. In the case that the criterion is satisfied, the procedure is continued for the next constituent following the same procedure as outlined in order to obtain the overall composite material behavior. Then the damage effect tensor is calculated together with the composite material behavior. The complete procedure is repeated for each load increment. It must be emphasized that in the event of damage, the quantities need to be calculated, and the new effective stress and strain concentration tensors are determined after each increment.

4. THE NUMERICAL RESULTS AND DISCUSSION

4.1. The Mechanical Behaviors of Unidirectional Laminate

A rectangular composites plate 300×100× t mm (t from 1.4 to 1.8mm, the average thickness of the ply is 0.2mm) with a centered hole (the radius of the hole is 25mm), is used to test the validity of the present model. The micro-level unit-cells are taken from the position of the maximum stress. Since the convergence of the finite element approximation is important, the materials of the composite are artificially chosen and the microstructure is set uniform throughout the macroscopic domain Ω . Then, the homogenized elastic constants are computed by using different meshes. Similarly, the macroscopic rectangular structure Ω is discretized by 57 and 176 meshes, respectively. The typical macroscopic and microscopic finite element meshes are shown in Fig.3 (a), and Fig.3 (b), respectively. The material system used here is Nicalon SiC fiber-reinforced calcium aluminosilicate composites (as shown in Table.1). The unidirectional CFCC plate is subject to the state of constant macro-strain field in the axial (parallel to the fibers) and transverse (normal to the fibers) directions. For cross-ply laminates, the applied load is aligned with the fiber axis in the outer 0° plies. The results are shown from Fig.4 to Fig.6, respectively. As shown in Fig.4, the rates of convergence of the homogenized elastic constants are very well for microscopic problem. The average stress-strain results for two

meshes are shown in Fig.5. It can be seen that the solutions converge with mesh refinement. Figure 5 also shows that the numerical results are in good agreement with the experimental data in [17]. It shows that when the damages reach their limits in the matrix phase, the matrix material is completely damaged, and the axial loading capacity of composite is provided by the fiber only. Figure 6 shows that when the matrix is totally damaged, it fails to transfer the load into the fiber, and the entire load carrying capacity rapidly decreases to some constant, and the strengthening effect of the fibers is nearly negligible. It shows a strain-softening behavior in transverse tension for unidirectional laminate too.

(a) Macroscopic mesh (b) Microscopic mesh
Fig.3. FEM mesh for macro-microscopic structure.

Table 1. The characteristics of matrix and fibers in the SiC/CAS composites.

Properties	CAS matrix	SiC Fibers
Elastic modulus (GPa)	98	193.2
Tensile strength (MPa)	124	2760
Volume fraction	65%	35%
Poisson's ratio	0.255	0.31
Fracture toughness (MPam ^{1/2})	1350	1300

Fig.4. The examination of convergence of homogenized elastic constants.

The damage parameters and load increments will influence the mechanical behavior. These parameters are chosen as $k_d=5.0$, $k_0=0.028$, $P=4.0\text{MPa}$ for longitudinal loading, $\Delta P=0.5\text{MPa}$ for transverse loading, respectively.

4.2. The Analysis of Cross-ply CFCC Plates with Different Stacking Sequences

Following the same method used for the unidirectional laminate, we simulate a series of laminated plates with different stacking sequences: $/0_3/90/0_3/$, $/0_3/90_2/0_3/$ and $/0_3/90_3/0_3/$. The choice of this particular system is due to the vast amount of experimental data that is readily available.

Fig.5. Longitudinal tensile stress-strain curves for SiC/CAS unidirectional composite.

Fig.6. Stress-strain curves for unidirectional SiC/CAS with different transverse load increments.

Fig.7. The stress-strain curves for unidirectional and cross-ply laminates.

Fig.8. Stiffness reduction as a function of the composite strain for $/0_3/90_0_3/$ and $/0_3/90_3_0_3/$.

Figure 7 shows the tensile stress-strain curves for $/0_8/$, $/0_3/90_0_3/$, $/0_3/90_2_0_3/$ and $/0_3/90_3_0_3/$ cross-ply laminates under tensile loading. The results are in good agreement with those of experimental data in [17]. The stiffness reductions for $/0_3/90_0_3/$ and $/0_3/90_3_0_3/$ are shown in Fig.8. It is seen that the numerical simulation results are more close to the experimental results than that of theoretical prediction by the shear lag analysis of Garrett and Bailey in [18]. It proves that the damage is actually the degradation of the material stiffness, and the overall stiffness reductions are large, which reflect the significant contribution made by the matrix to the laminate modulus. The model not only predicts the experimentally observed response, it also provides the proof for the determination of the material parameters.

4.3. The Analysis of Microscopic Mechanical Behavior

Since the above types of layups are symmetric, the strains in each lamina of the layups are the same. However, the stress and damage distributions are different for each lamina of the layups since each lamina has a different stiffness. When the mechanical behavior of the elementary ply is determined, the mechanical behavior of any laminate stacking sequence can be found. The evolutions of the stresses with strain in 0° and 90° plies are analyzed in Fig.9. After the development of da-

Fig.9. Numerical results of stresses in the 0° plies and 90° plies with strain for different cross-ply laminates.

Fig.10. The macro-microscopic stress-strain curves for $/0_3/90_3_0_3/$ laminate.

mage in the 90° plies, its stress is very small, and nearly keeps constant for each laminate. The stress in 0° plies still increases as the intact fibers in the 0° plies can take extra load. It shows that the stresses in the ply do depend on the ply orientations. The thicker the 90° plies are, the smaller the carrying stress in the 0° plies will be. The microscopic stress-strain relationship in each constituent phase can also be obtained in Fig.10. In this section the behavior of the $/0_3/90_3_0_3/$ composite is explained, and the response of the other composites can be explained in a similar manner. It is clear that the occurrence of anisotropic damage in matrix will result in the strain-softening of brittle matrix, and this is the reason of the non-linear mechanical behaviors of composites.

4.4. Damage Prediction

The damage evolutions are plotted in Fig.11 for unidirectional composites. This figure shows that the anisotropy of the damage in transverse loading greatly influence its load-carrying ability, and the first damages obviously initiate at very low strains, but most damages develop in the axial strains between 0.06-0.3%, then they remain almost the same till failure. The damage for longitudinal loading is only along fiber direction, and it shows that specimen failure occurs parallel to one of fiber directions in this case. Figure 12 is the damage evolution for $/0_3/90_3_0_3/$ laminate, which shows the anisotropy of the damage in the plane of each ply. Damage D_{11} in the 0° fiber orientation experiences larger than that in the 90° fiber orientation as expected, and the matrix micro-cracking at first occurs in the 0° ply. Damage D_{22} in the 90° ply is larger than that in the 0° ply, and the transverse cracking always first appears in 90° ply. With the increase of the thickness of 90° ply, the strain for the onset of transverse cracking decreases. In fact, the matrix cracking can decompose into longitudinal, transverse and shear direction, respectively, and the anisotropic damage describes the damage mechanism along different direction. It is obvious that the anisotropic damage theory is more advantageous than the concept of crack densities in explaining the failure mechanism of CMCs. The damage

Fig.11. Damage-strain curves for unidirectional composites with longitudinal and transverse load.

Fig.12. Damage evolutions in the 0° plies and 90° plies with the average strain for /0₃/90₃/0₃/ laminate.

Fig.13. Damage in interfacial and non-interfacial area.

D_{12} is along the shear direction, which reflects the fiber-matrix interfacial debonding, and it shows that the interfacial debonding is mainly governed by mode II shear.

The damage in interfacial area will progress more early than that in the other non-interfacial area, as shown in Fig.13.

5. CONCLUSIONS

The irreversible thermodynamic theory and homogenization method are coupled to establish the constitutive equation with matrix anisotropic damage in CFCC. The proposed model has been used to analyze the

mechanical behavior of unidirectional plate with different loading direction and cross-ply laminate with different stacking sequence. The finite element procedure makes the macro-microscopic analysis possible through the homogenization method. The following conclusions can be drawn:

(1) The macro- and microscopic stress-strain behaviors of the CFCC unidirectional composites and cross-ply laminates with anisotropic damage in matrix are simulated by the computer code, and the results are in good agreement well with those of experimental observation for the SiC/CAS composites system. This shows that the homogenized mesoscopic model can provide accurate behavior predictions for the symmetry laminates.

(2) The new expressions are derived for the stress and strain homogenized damage elastic concentration factors with macroscopic damage induced strain for the damaged composites.

(3) The second rank symmetric damage tensor is employed to describe the state of anisotropic damage, and completely identified model is able to predict the failure mechanism of the unidirectional and cross-ply laminate composites at room temperature.

Acknowledgment-We gratefully acknowledge the support of this research through the Scholarship from Ministry of Education, Science and Culture of Japan to the first author.

REFERENCES

1. V. Eddy, and O. V. D. Biest, *Acta. Mater.*, **45** (1997) 3349.
2. S. Baste, *Comp. Sci. and Tech.*, **61** (2001) 2285.
3. N. Lissart and J. Lamon, *Acta Mater.*, **45** (1997) 1025.
4. W. Curtin, *J. Am. Ceram. Soc.*, **74** (1991) 2837.
5. F. Devries, H. Dumontet, G. Duvaut and F. Lene, *Int. J. Numer. Meth. Engng.*, **27** (1989) 285.
6. J. M. Guedes and N. Kikuchi, *Comp. Meth. in Appl. Mech. and Engng.*, **83** (1990) 143.
7. F. R. Abadi, C. M. Chen and N. Kikuchi, *Computers & Structures*, **76** (2000) 319.
8. G. J Dvorak, Y. A Bahei-El-Din and A.W Wafa, *Comp. Mech.*, **14** (1994) 201.
9. G. Z. Voyiadjis and P. I. Kattan, *Int. J. Engrg. Sci.*, **30** (1992) 1089.
10. S. Murakami and K.Kamiya, *Int. J. Mech. Sci.*, **39** (1997) 473.
11. C.L.Chow and J. Wang, *Engng. Frac. Mech.*, **27** (1987) 547.
12. J. C. Simo and J. W. Ju, *Int. J.Solids and Stru.*, **20** (1987)821.
13. C. Pellegrino, U. Galvanetto and B. A. Schrefler, *Int. J. Numer. Meth. Engng.*, **46** (1999) 1609.
14. J. Fish, Q. Yu and K. Shek, *Int. J. Numer. Meth. Engng.*, **45** (1999) 1657.
15. J. Fish, P. Nayak and M. H. Holmes, *Comp. Mech.*, **14** (1994) 323.
16. ANSYS, User's Manual 1992, Houston.
17. S.W. Wang and A. P. Majidi, *J. Mater. Sci.*, **27** (1992) 5483.
18. K. W. Garrett and J. E. Bailey, *ibid.* **12** (1977)15.