Assessment of Damage Tolerance and Reliability for Ceramics

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Abstract: The risk of fracture does exist even in the carefully designed ceramic components under excess stress unexpected in design procedure such as high stresses caused by restraining the deformation at the contact region of the component. To assess the safety and the reliability of ceramics components under such circumstances, it is indispensable to take the damage tolerance of the material into consideration. Ceramics is thought to be perfectly brittle and have no damage tolerance. But, though very little, it has the damage tolerance. The important problems open to us now are to estimate the extent of the damage tolerance of ceramics quantitatively and to clear the relation between the damage tolerance of the material and the reliability of the component. In this paper, we demonstrate the existence of the damage tolerance in the ceramics by tests using specimens of a porous cordierite. The nonlinear stress-strain curve of the brittle material is thought to be a reflection of the damage tolerance of the material and it is shown that the nonlinear stress-strain curve obtained can be simulated by the distributed micro cracks model developed here. The relation between the damage tolerance and the reliability is also discussed through several simulation works. These simulations show that when the damage tolerance of the material is getting larger, the reliability of the component becomes higher. This conclusion is supported by the fracture test results using notched specimens of a porous cordierite where the notch sensitivity is shown to be very small in this material. This means that this material has the high reliability under the stress concentration.

Key words: Ceramics, Porous ceramics, Damage tolerance, Reliability, Nonlinear stress-strain curve, Simulation, Weibull modulus, R-curve, Material model

1. INTRODUCTION

Ceramics is the brittle material which has the scatters in the strength. The design guide for components of such material has already been developed[1] and widely utilized in the design procedure. When all of the stress distribution of the component can be known, it is proved that the design guide works well. But unfortunately it is very difficult to know the magnitude of the stress distribution exactly with localized peaks such as that caused by the deformation restraint. Examples of such stresses are thermal stress, contact stress and stress at joining part. Accordingly there may be the cases that the fracture will occur even in the component carefully designed according to the design guide. So it is necessary to test the component under actual loading conditions and to make sure of the safety of it.

This is not the case of the component of metallic material where the peak stresses are well reduced by the plastic deformation. This is the remarkable difference between ceramics and metals when they are used for mechanical use.

However, does the ceramics have not the damage tolerance at all? Yes, it has the damage tolerance in some extent. There are some experimental evidences such as the formation of the depression in the ceramics under indentation testing, R-curve behavior observed during crack extension test and so on, which show that, though very little, the ceramics does have the damage tolerance.

The purpose of this paper is to assess the damage tolerance of ceramics quantitatively and develop the reliability analysis method taking the effect of the damage tolerance into consideration.

A porous ceramics was used as the test material because it exhibits the lager inelastic behavior than the monolithic ceramics and is easier to handle in the experiment. Here, the damage tolerance is defined as the character that the local or microscopic damage does not directly cause the whole destruction.

2. NONLINEAR STRESS-STRAIN BEHAVIOR OF POROUS CORDIERITE

Notched plate specimens (thickness is 3mm) shown in Fig.1 were used in the test under tension loading. Test material was a porous cordierite (percentage of porosity is 30%) and was cut from a ceramic filter tube of an actual plant. The stabilizer was used to achieve the stable fracture. Figure 2 shows schematically the whole view of the stabilizer and the test specimen (shown in the hatch) installed in the stabilizer through the pins. When the compression load is applied at the top of the stabilizer by the testing machine, the tensile loading is applied to the specimen installed in the stabilizer. Since the rigidity of the columns of the stabilizer is much greater than that of the specimen, the influence of the rigidity change of the specimen caused by the fracture or damage of the specimen is small on the rigidity of the testing system. As a result, an almost complete displacement control was achieved and the stable fracture test was conducted.

The strain was measured by uniaxial strain gauges stamped in the center of both surfaces of the specimen. The load applied to the specimen was calculated from the measured elastic strains of the two centered columns of the stabilizer.
Figure 3 shows the nonlinear stress-strain curve obtained. The value of strain in the figure is the average of the strains of both surfaces and the value of stress in the figure represents the value of the load divided by the minimum cross-sectional area of the specimen. The portion of C-D-E of the stress-strain curve represents the unloading behavior of strain gage area by shielding effect caused by occurrence of large cracks in the area other than the gage area, while the specimen was still continued to be loaded under strain control.

The stress-strain curve obtained is not the true one because the stress distribution existed in the specimen. But it was experimentally shown that the porous cordierite exhibits the nonlinear stress-strain behavior and then fractures as shown in Fig. 3.

3. MODELING OF STRESS-STRAIN BEHAVIOR

The porous ceramics is assumed to be a brittle solid which contains a lot of micro cracks. The micro cracks grow up according to R-curve when this material is under loading, and, as a result, a macroscopic nonlinear strain is caused. In this procedure the number of micro cracks is assumed to remain unchanged.

Consider the behavior of a solid which has a penny shaped crack perpendicular to the stress axis under uniform tension as shown in Fig. 4 (a). The response of the solid can be written by Eqs. (1) and (2)[2].

\[ u = u_0 + \int_0^\sigma G dA. \]  
\[ \varepsilon = \frac{u}{L_1} = \frac{\sigma}{E} + 4\pi^2 F^2 \frac{a^3}{3E' L_1 L_2 L_3}, \]  

where \( u \) is displacement at loading point, \( u_0 \) is displacement at loading point of the solid without cracks, \( G \) is energy release rate, \( P \) is load, \( A \) is the area of a crack, \( a \) is the radius of penny shaped crack, \( \varepsilon \) is average strain, \( \sigma \) is stress, \( E \) is Young’s modulus, \( E' = E/(1-\nu^2) \), \( \nu \) is Poisson’s ratio and \( F \) is shape factor \((=2/\pi)\). \( L_1, L_2 \) and \( L_3 \) are the sizes of the body as shown in Fig. 4.

From Eq. (2), Eq. (3) is obtained for the case of solid body with \( N \) cracks of the same size as shown in Fig. 4(b).

\[ \varepsilon = \frac{\sigma}{E} + \frac{16}{3} \frac{N}{L_1 L_2 L_3} \frac{\sigma a^3}{E'}. \]  

When cracks are uniformly distributed in space, the density \( q \) of cracks is given as

\[ q = \frac{N}{L_1 L_2 L_3}, \]  

and strain \( \varepsilon \) is given from Eq. (3) as

\[ \varepsilon = \frac{\sigma}{E} + \frac{16}{3} q \frac{\sigma a^3}{E'}. \]  

It is considered that the averaged response of ceramics
is approximately represented by Eq. (5). Equation (5) represents a class of the material model and is called here the distributed micro cracks model, where a crack will develop in its size under loading according to the R-curve relation.

To examine the describing ability of the model, the simulation for stress-strain curve shown in Fig. 3 was conducted under the assumption that the curve shown in Fig. 3 was the one in the case of uniaxial tension. Values of parameters used in the simulation are as follows. Initial crack size $a_0$ is thought to be approximately same as that of grain size and given as

$$a_0 = 50 \mu m. \quad (6)$$

Referring to the data [3] for porous SiC in the range of $\delta a \leq 0.5 \text{mm}$, R-curve behavior is given as

$$K_R / K_0 = (\Delta a / a_0)^{0.362}, \quad (7)$$

where $K_R$ is the crack growth resistance represented in the stress intensity factor, and $K_0$ is a material constant which was given as

$$K_0 = 4.48 \times 10^{-2} \text{MPa}\sqrt{\text{m}}. \quad (8)$$

Young’s modulus $E$ and Poisson’s ratio $\nu$ are given respectively as

$$E = 72.8 \text{GPa}, \quad \nu = 0.27. \quad (9)$$

Calculated stress-strain curves with different crack density $q$ are shown in Fig. 5. When the value of crack density $q$ is chosen as

$$q = 1 \times 10^{12} (1/\text{m}^3), \quad (10)$$
calculated stress-strain curve is a good approximation of that shown in Fig. 3 except for the unloading portion of the curve.

4. RELIABILITY OF DAMAGE TOLERANT CERAMICS (Part 1, Apparent Weibull modulus)

Defects of various sizes included in the ceramics are modeled as cracks. Existing probability $P_r$ of cracks of size $a$ or more is assumed to be written as

$$P_r = 1 - \exp \left\{ - \left( a / a_0 \right)^m \right\}, \quad (11)$$

where $a_0$ and $m$ are material constants and $m$ will be shown later to be equal to the Weibull modulus in the strength distribution. According to the linear fracture mechanics, fracture condition of a crack is written as

$$K = K_c. \quad (12)$$

$K$ is the stress intensity factor and is written as

$$K = F \sigma \sqrt{a}, \quad (13)$$

where $\sigma$, $a$ and $F$ are stress, crack size and shape factor of a crack respectively. $K_c$ is the critical value of $K$. The failure probability of a ceramic component is equal to the existing probability of a crack whose size is equal to $a$ or more in the component if it is assumed that the failure of the ceramic component occurs when Eq. (12) is satisfied with one arbitrary crack according to the weakest link theory. From this consideration, the expression for the strength distribution of a component is obtained as[4]

$$P_r = 1 - \exp \left\{ - \left( \sigma / \sigma_0 \right)^m \right\}. \quad (14)$$

Here $\sigma_0$ is the material constant defined as

$$\sigma_0 = \frac{K_c}{F \sqrt{a_0}}. \quad (15)$$

![Fig. 4. Uniaxial tension of a component with micro cracks, (a) component with one micro crack, (b) component with many micro cracks of same size.](image)

![Fig. 5. Calculated stress-strain curve.](image)
The strength distribution expressed by Eq. (14) is called the Weibull distribution and \( m \) appeared in the equation is called Weibull modulus representing the extent of the scatter of strength.

Here the strength distribution is calculated for the case where the initial crack size distribution is represented by Eq. (11) and fracture of a crack is controlled by \( R \)-curve behavior shown in Eq. (7). Given the initial size of a crack, the strength of an element of a component corresponding to the crack can be calculated by the model shown in Chapter 3. If the failure of a component is described by the weakest link theory, the failure probability of an element obtained from Eq. (11) is the failure probability of a component as it is. When \( m=20 \) is assumed in Eq. (11) and values of other material constants are those shown in Chapter 3, the calculated strength is plotted on the Weibull chart with the corresponding failure probability as shown in Fig. 6. Here the initial crack size distribution is adjusted for 50% strength to be 41MPa. As shown in Fig. 6, the strength distribution thus obtained becomes a straight line on the Weibull chart. The apparent Weibull modulus \( m' \) obtained from the inclination of this straight line becomes 30 which is larger than that for the case of the perfectly brittle material where the strength distribution is written by Eq. (14) and Weibull modulus \( m \) equals to 20. This shows that the reliability of the material with \( R \)-curve behavior in crack extension is higher than that of the perfectly brittle material.

This conclusion has already obtained by Shetty et al. They showed that if \( R \)-curve behavior is represented by the type of Eq. (7), the relation

\[
m' = \frac{m}{1-2n}
\]

holds, where \( n \) represents the exponent of the power-law function at the right side of Eq. (7). While they handled only the case where \( R \)-curve is represented by the type of Eq. (7), the method mentioned here can handle any types of equation for \( R \)-curve behavior.

5. RELIABILITY OF DAMAGE TOLERANT CERAMICS (Part 2, Simulation of fracture process)

5.1. Network Model

In the preceding Chapter, the strength distribution was described for the case where the fracture of a microelement of a component is determined by the fracture of a crack following the \( R \)-curve behavior and the fracture of a component is determined by the fracture of the weakest microelement. Such model can only simulate the fracture behavior of a component in which the stress state is statically determinate. The actual material is statically indeterminate in the microscopic level and can not be described by the model in the preceding Chapter. The stress is expected to be eased at the damaged microelement in the statically indeterminate material and the evolution rate of damage is also expected to be lessened at that microelement. As a result, it is expected that the reliability of such material improves further if it has the damage tolerance in microscopic level.

To understand such an effect on the brittle material, a simulation study was conducted on the brittle network shown in Fig. 7. Each element of the network is a brittle rod with the same cross sectional area, length and elastic constants. It has the critical strain energy of \( E_c \) at which the fracture occurs. Elements are connected each other through a pin, and only the axial stress is acted on the element. The pin at the connecting point can move in the vertical direction freely but cannot move in the horizontal direction. The critical strain energy is calculated by Eq. (18) using the strength \( \sigma_c \). The distribution of \( \sigma_c \) is described by the Weibull distribution of Eq. (17). \( \sigma_c \) is randomly distributed to each element.

\[
P_r = 1 - \exp \left\{ - \left( \frac{\sigma_c}{\sigma_0} \right)^m \right\}
\]

\[
E_c = \frac{\sigma_c^2}{2E} S L.
\]

Here, \( P_r \) is the cumulative failure probability, \( m \) and \( \sigma_0 \) are Weibull parameters, \( E \) is Young's modulus, \( S \) is the cross sectional area of an element and \( L \) is the length of an element.

The fracture process of this network under uniform tension in the vertical direction may be described as follows. Initially equal amount of strain energy is accumulated in each element under uniform tension and it grows up according to the increase of applied loading. When the strain energy accumulated in the weakest element reach the critical value, the fracture of that element occurs. The energy of fractured element is distributed to other elements. The element which receives
this energy increases own energy and there is the case whether the fracture condition is satisfied or not. If the fracture condition is satisfied, the subsequent fracture may occur and if not, by increasing the applied load, the subsequent fracture of the weakest element among the remainders can be caused to occur. Thus, fracture process can be simulated. The energy redistribution procedure through the occurrence of fractures can be calculated by detailed structural analyses. But it may require a great deal of calculation time to conduct such a simulation rigorously. In order to roughly understand the nature of the fracture process of the damage tolerant material, a local rule for energy distribution is employed which may be a sort of approach employed in the study of complexity science [8].

5.2. Local Rule for Energy Distribution [7]

5.2.1. Energy to be released

Figure 8 shows a part of the network. When the element AB is broken, it is assumed that the energy to be released is the sum of the energy accumulated in the element AB and a part of the energy accumulated in the elements A3 and B4 which are located in the upper and lower region of the element AB. The amount of stress relief at the fracture of the element AB is considered to be greater in elements of A3 and B4 than that of elements of A1, A2, B5 and B6 under the loading condition shown in the Fig. 7. So only energy release of A3 and B4 is considered here for simplicity. Accordingly energy $E'$ to be released is written as

$$E' = E_{AB} + (1 - \alpha)E_{A3} + (1 - \alpha)E_{B4}.$$  (19)

Here, $E_{AB}$ etc. show the energy of element AB etc. and $1 - \alpha$ represents the rate of the energy released along with the stress relief. Each half of this energy will be distributed from node A and B respectively. It is assumed that the energy dissipation is caused by the inelastic work according to the fracture. The rate of the dissipation of energy is written as $\beta$ and the energy $E''$ to be distributed from node A or B to neighboring elements becomes as

$$E'' = \frac{1}{2} (1 - \beta)E'.$$  (20)

5.2.2. Local rules for energy distribution

In the case where three non-fractured elements exist around node A, the energy received by each element around node A is settled as

$$E_{A1} = \frac{1}{2} E'', \quad E_{A2} = E_{A3} = \frac{1}{4} E''.$$  (21)

The case where two non-fractured elements exist around node A is separated into two cases. The one is that where one non-fractured element is in the upper position of node A and another is in the lower position and the energy distribution rule is settled as

$$E_{A1} = E_{A2} = \frac{1}{2} E''.$$  (22)

Another case is that where two non-fractured elements are in the upper position of node A. In this case these two elements do not keep any stress which is obvious from the figure. So these elements release the all energy at another node other than node A, which is the sum of the accumulated energy in itself and the received energy from node A. In the case where the only one element exists around node A, this element receives all the energy released from node A. But because this element does not keep any stress, the sum of the accumulated energy in itself and the received energy from node A is released from another node. This element does not contain any energy over the future and is registered as an element unbroken but without energy. In addition to above it is considered that the energy distribution is not done over the boundary and all energy is lost at the boundary node at fracture.

The local rule described above is mere an example. Also, other rules can be adopted and better results may be expected. But the investigation for the better selection of the local rule is out of scope of this paper.

5.3. Calculated Results

Though Fig. 7 shows the network of $12 \times 12$, calculations are conducted with the network of $72 \times 72$. 

![Fig. 7. Brittle network.](image)

![Fig. 8. Network element.](image)
Assessment of Damage Tolerance for Ceramics

Calculations are conducted under the condition which is one case for \(\alpha = 0\) and two cases for \(\beta\) representing the rate of energy dissipation as \(\beta = 0\) and \(\beta = 0.2\). The distribution of critical energy is obtained by distributing the uniform random numbers to \(P_r\) of Eq. (17).

The evolution of damaged zone calculated for uniform tension is shown in Fig. 9 where notion 2 indicates the non-fractured element, notion 0 fractured element and notion 1 non-fractured element without energy, which shows the resembling nature with fracture pattern[6] of granite experimentally observed by moire interferometry shown in Fig. 10.

Figure 11 shows the relation between the energy level of the whole structure and the number of the fractured elements (number of the fracture events). This figure can be interpreted to show the relation between load and (inelastic) deformation. Also Fig. 12 shows the same relation for the network with a center hole where four elements at the center of the network were removed beforehand.

5.4. Discussions

It is revealed from Fig. 11 that the deformation up to the maximum load is greater and the maximum load itself is greater and consequently the system is more reliable in the case where the energy dissipation occurs at fracture of the element \((\beta = 0.2)\) than the case where there is no energy dissipation at fracture of the element \((\beta = 0)\). And it is found by the comparison of calculated results shown in Figs. 11 and 12 that this effect is more remarkable for the case of stress concentrations. The material with energy dissipation can be understood to be the damage tolerant material.

6. RELIABILITY OF DAMAGE TOLERANT CERAMICS (Part 3, Notch Sensitivity)

Notch sensitivity of a porous cordierite was investigated under tension using specimens with semi-circular notches in both sides. Specimens were prepared from a porous ceramics filter tube of cordierite (percentage of porosity is 35.6%). Though the material used here is basically the same described in Chapter 2, the material was cut from another filter tube than that for the material in Chapter 2. So, there are some discrepancies in the percentage of the porosity.

Fig. 9. Propagation of damaged zone.

Fig. 10. Crack propagation pattern in granite[6].

Fig. 11. Load-deflection curve (uniform loading).

Fig. 12. Load-deflection curve (loading with stress concentration).
Configurations of specimens are shown in Fig. 13 and their thickness is 3mm. Four kinds of specimens with different notch radius were tested. Stress concentration factors of specimens are $K_t = 2.31$ (R=0.5mm), 1.79 ($R=1\text{mm}$), 1.53 ($R=1.5\text{mm}$) and 1.37 ($R=2\text{mm}$) respectively, where the nominal stress is the average stress at minimum cross section. The effective volumes $V_E$ of specimens are 0.190mm$^3$, 0.416mm$^3$, 0.825mm$^3$ and 1.437mm$^3$ respectively. They are calculated from elastic stress distributions of specimens obtained by FEM.

Test results are shown in Fig. 14 with notation of “EXP”. Estimated results by two methods without considering the damage tolerance of the material are also shown in Fig. 14. Estimations noted by “Pred by Smax” are those predicted by one method in which the fracture occurs when the maximum stress in the specimen reaches the limit. Estimations noted by “Pred by $V_E$” are those predicted by another method[4] in which the fracture occurs when the stress arranged by $V_E$ reaches the limit.

This figure reveals that the strength of this material in the nominal stress is not affected by stress concentrations extensively and the notch sensitivity of this material is very small. Also it shows that predictions by two methods without considering the damage tolerance give too small values of strength and excessive fracture possibility. This fact shows that the reliability of the brittle material is improved by the damage tolerance. And this is also the proof of the conclusions obtained in the previous Chapters.

7. CONCLUSIONS

The relation between damage tolerance of ceramics and structural reliability is investigated by tests for porous ceramics and simulations. Conclusions obtained are as follows:

1. Porous cordierite is shown to have the nonlinear stress-strain curve and damage tolerance by tensile tests.
2. This nonlinear stress-strain curve is simulated well by the distributed micro cracks model developed here considering R-curve behavior for cracks.
3. Several simulations proved that reliability of the material having damage tolerance or structures made of such material is greater than that of the material having no damage tolerance or structures made of such material. In addition, this was confirmed by test using notched specimens of porous cordierite.

REFERENCES


Fig. 13. Notched specimen.

Fig. 14. Comparison of fracture strength in the nominal stress.