Adaptive Fuzzy Controller for a Class of Nonlinear Systems Using Tunable Triangular Membership Functions

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In order to design an Adaptive Fuzzy Control System based on the Lyapunov synthesis approach, the fuzzy systems can be viewed as some fuzzy approximators to approximate the unknown functions in the control system. To enhance the quality of the control system, obviously, it needs to tune all parameters in the fuzzy approximators so that the approximation can be improved. On the other hand, fuzzy rules with triangular membership functions in the precedent of the fuzzy rules are used in many industrial control systems. However, how to stably tune the parameters involved in the triangular membership function is still an open problem. The goal of this paper is to design a controller for a class of nonlinear systems using triangular fuzzy functions. The adaptive laws to tune all the parameters in the system are developed. It is shown that the proposed adaptive controller guarantees tracking error, between outputs of the considered system and desired values, to be asymptotically in decay.

Keywords: Fuzzy approximator; adaptive control; global stability; nonlinear systems

1 Introduction

For the last decade, various approaches for adaptive fuzzy control systems have been proposed to deal with the nonlinear systems with poorly understood dynamics. Amongst the approaches, one important way is to consider the fuzzy system as fuzzy approximator to approximate the unknown functions in the plant to be controlled [1]-[3]. In this case, the control performance is closely connected with the precisions of the fuzzy approximators. In order to enhance the ability of the precision, it clearly needs to tune up all parameters involved in the fuzzy approximators. In our previous paper [2], we have successfully tuned the parameters up not only in the consequent, but also in the precedent of the fuzzy rules. In the proposed way, we used the Gaussian membership function in the precedent of the fuzzy rules:

\[ \mu(x) = \exp\left(-\frac{(x-\xi)^2}{\sigma}\right) \]  

where \( x \) is an input variable; \( \xi \) indicates the position of the fuzzy membership function; and \( \sigma \) indicates the variance of the bell-shaped curve.

On the other hand, fuzzy system with triangular membership functions in the precedent of the fuzzy rules is widely used in many industrial control systems. However, compared with the Gaussian one above, which has an unique expression, the triangular membership function (Fig.1):

\[ \mu_t(x) = \begin{cases} \frac{1}{m-l}(x-m)+1 & l \leq x < m \\ \frac{1}{r-m}(x-m)+1 & m \leq x \leq r \\ 0 & \text{otherwise} \end{cases} \]  

(2)

can not be expressed uniquely. Though we can, of course, tune the parameters such as \( l, m, \) and \( r \) using the steepest descent method, genetic algorithm and so on, they are difficult to fuse them into a control system, especially when we pay more attention on the stability of a control system.

In this paper, after we give a brief review of the general fuzzy system, a theorem regarding the fuzzy approximator is proposed. Then, we exploit the fuzzy system to develop an adaptive fuzzy controller for a class of nonlinear systems in which the all parameters involved in the systems are stably tuned. Finally, the performance of the proposed adaptive
fuzzy controller is verified through computer simulations.

2 Fuzzy System

We consider a fuzzy system in which there are three principal elements: fuzzy rule base, fuzzy inference engine, and defuzzifier. Let input space \( X = R^n \) be a compact product space. Assume that there are \( N \) fuzzy rules in the rule base and each of which has the following form:

\[
R_j: \text{IF } x_i \text{ is } A_i^j, \ldots, x_n \text{ is } A^n_j \text{ THEN } z \text{ is } w_j
\]

where \( j = 1, 2, \ldots, N \) is rule's number; \( w_j \) are some real (crisp) values; \( A_i^j (i=1, 2, \ldots, n) \) are the fuzzy sets, which are characterized by the following triangular membership functions:

\[
\mu_{A_i}(x_i) = \begin{cases} 
\frac{x_i - m_i}{l_i} & l_i < x_i < m_i \\
\frac{m_i - x_i}{l_i} & m_i < x_i < m_i + r_i \\
0 & \text{otherwise}
\end{cases}
\]

where \( l_i, m_i, \) and \( r_i \) are the left end, medium point, and the right end, respectively. The output of such a fuzzy system is given by,

\[
\mathcal{F}(X) = \sum_{j=1}^{N} w_j \left( \bigwedge_{i=1}^{n} \mu_{A_i}(x_i) \right)
\]

where,

\[
\bigwedge_{i=1}^{n} \mu_{A_i}(x_i) = \mu_{A_1}(x_1) \land \cdots \land \mu_{A_n}(x_n)
\]

In order to ease the procedure of design, the location of the neighboring fuzzy membership functions are chosen so that they always overlap at the membership value \( \mu = 0.5 \) in lots of fuzzy control systems. It means, as shown in Fig.2, one point, whose membership function value is 1 in a fuzzy set, is the point, whose membership function value is 0 in the neighboring fuzzy sets. In this way, it limits the flexibility of the fuzzy control systems to either approxi-
Substituting (8) into (5), follows,
\[ \prod_{j=1}^{n} \mu_{j}(x_{i}) = (k_{j}x_{i} + b_{j}) \wedge \cdots \wedge (k_{j}^{*}x_{i} + b_{j}^{*}) = k_{j}x_{i} + b_{j} \] (9)

where \( k_{j} \in \{ k_{1}, k_{2}, \ldots, k_{n}^{*} \} \), \( b_{j} \in \{ b_{1}, b_{2}, \ldots, b_{n}^{*} \} \), \( x_{i} \in \{ x_{1}, x_{2}, \ldots, x_{n} \} \). And substituting (9) into (4), one can be rewritten as,
\[ \mathcal{F}(X) = W^{T}G(X)K + W^{T}B \] (10)

where \( W = [w_{1}, w_{2}, \ldots, w_{n}] \), \( K = [k_{1}, k_{2}, \ldots, k_{n}] \), \( G(X) = \text{diag}(x^{1}, x^{2}, \ldots, x^{n}) \in R^{n \times n} \), which is a function of matrix with respect to input vector \( X = [x_{1}, x_{2}, \ldots, x_{n}]^{T} \), and \( B' = [b_{1}, b_{2}, \ldots, b_{n}] \).

Note that, at first glance, the fuzzy system (10) appears linearly, i.e., it can be rewritten as,
\[ \mathcal{F}(X) = \mathcal{K}^{T}\mathcal{G}(X) + C \] (11)

where \( C = W^{T}B \), \( \mathcal{K}^{T}\mathcal{G}(X) = W^{T}G(X)K \) and \( \mathcal{G}(X) \) is available as well as \( G(X) \). Surely, when we put such a form into a control system and take some adaptive control techniques, it is easy to tune the parameters \( \mathcal{K} \) and \( \mathcal{C} \) in (11). However, \( \mathcal{K} \) and \( \mathcal{C} \) are related with the parameters \( W, K, \) and \( B \) in (10). Clearly, it is impossible to get the three values of the parameters \( W, K, \) and \( B \) from the two parameters \( \mathcal{K} \) and \( \mathcal{C} \). In the following section, we will show a theorem that can give a linear expression regarding the fuzzy system (10).

3 Adaptive Fuzzy Controller

3.1 Problem description

We are interested in the single-input/single-output nonlinear system:
\[ x^{(n)} + f(\cdot) = b(\cdot)u(t) \] (12)

where \( f \) and \( b \) are smooth (i.e., infinitely differentiable) functions of
\[ X^{T}(t) = [x(t), \dot{x}(t), \ldots, \dot{x}^{(n-1)}(t)] \]

\( u(t) \) is the control input; \( x \) is the state of system; and \( n \) is the system order. It is assumed that the order \( n \) is known but the nonlinear functions \( f(\cdot) \) and \( b(\cdot) \) are unknown. It should be noted that more general classes of nonlinear systems could be transformed into the structure like (12) [4].

The control objective is to design an adaptive controller so that the overall system is stabilized and the output \( x(t) \) is forced to follow a desired value \( x_{d}(t) \). Defining the tracking error,
\[ \tilde{x} = x - x_{d} \] (13)

the problem is thus to design a control input \( u(t) \) which ensures that \( \tilde{x} \to 0 \) as \( t \to \infty \). In this paper, we take \( b=1 \) to lay stress on clearing approach for the control system.

3.2 Linearizing fuzzy system

To proceed with the development of our system, we have to approximate function \( f \) in (12). Here the fuzzy system described in the previous section is used. If we denote \( \mathcal{F}^{*} = W^{*T}G(X)K + W^{*T}B \) to be the optimal fuzzy approximator of the unknown function \( f \), it is reasonable to assume that the error between \( f \) and \( \mathcal{F}^{*} \), \( |\varepsilon_{f}| = f - \mathcal{F}^{*} \), has an upper boundary, i.e.,
\[ |\varepsilon_{f}| \leq \varepsilon_{f}^{*} \] (14)

However, the optimal parameters \( W^{*}, K^{*}, \) and \( B^{*} \) in \( \mathcal{F}^{*} \) cannot be used because we do not know how much they are. Thus, the estimator, denoted \( \hat{\mathcal{F}} = \hat{W}^{T}G(X)\hat{K} + \hat{W}^{T}\hat{B} \), is adopted instead of the optimal fuzzy approximator \( \mathcal{F}^{*} \). Clearly, there are errors between the optimal fuzzy approximator and fuzzy estimator, i.e.,
\[ \hat{W} = W^{*} - \hat{W} \] (15)
\[ \hat{K} = K^{*} - \hat{K} \] (16)
\[ \hat{B} = B^{*} - \hat{B} \] (17)

Regarding the fuzzy approximator and estimator, we give the following theorem.

Theorem 1 Estimating error,
\[ \hat{f} = f - \hat{\mathcal{F}} \] (18)

satisfies the following relations:
\[ \hat{f} = \frac{1}{2} W^{T}G(X)\hat{K} + \frac{1}{2} W^{T}\hat{B} \]
\[ + \frac{1}{2} \hat{W}^{T}G(X)\hat{K} + \frac{1}{2} \hat{W}^{T}\hat{B} + d_{f} \] (19)
where $\theta^* \in \mathbb{R}^{n \times 1}$ is an unknown constant vector; and $Y_f$ is a known function of vector.

**Proof:**

Substituting (15) ~ (17) into (18), one follows,

$$
\dot{\gamma} = \mathcal{F}^* + \epsilon_f - \dot{W}^T G(X) \dot{K} - \dot{W}^T B + \epsilon_f
$$

$$
= W^* G(X) K^* + W^* B^* - \dot{W}^T \dot{B} + \epsilon_f
$$

$$
= W^* G(X) K^* + W^* B^* + W^* G(X) \dot{K}^T
$$

$$
+ W^T \dot{B} - W^T G(X) \dot{K} - W^T \dot{B}
$$

$$
= W^* G(X) K + W^* \dot{B} + W^* G(X) \dot{K} + W^T \dot{B} + \epsilon_f
$$

$$
= W^* G(X) K^* + W^* \dot{B}^* + W^* G(X) \dot{K}
$$

$$
+ W^T \dot{B} + \epsilon_f
$$

$$
= \frac{1}{2} \left[ \dot{W}^T G(X) K^* + \dot{W}^T B^* + \dot{W}^T G(X) \dot{K}
$$

$$
+ \dot{W}^T \dot{B} + \epsilon_f + W^* G(X) \dot{K} + W^* \dot{B}
$$

$$
+ W^T \dot{B} + \epsilon_f \right]
$$

$$
= \frac{1}{2} \dot{W}^T G(X) \dot{K} + \frac{1}{2} \dot{W}^T \dot{B} + \frac{1}{2} \dot{W}^T G(X) \dot{K}
$$

$$
+ \frac{1}{2} \dot{W}^T \dot{B} + d_f
$$

(21)

where,

$$
d_f = \frac{1}{2} \dot{W} G(X) K^* + \frac{1}{2} \dot{W} B^* + \frac{1}{2} W^* G(X) \dot{K}
$$

$$
+ \frac{1}{2} W^* \dot{B} + \epsilon_f
$$

(22)

And, taking norm on the both sides of (15) ~ (17) yields the following inequalities:

$$
\| \dot{W} \| \leq \| W^* \| + \| \dot{W} \| = \| W \| + \| \dot{W} \|
$$

(23)

$$
\| \dot{K} \| \leq \| K^* \| + \| \dot{K} \| = \| K \| + \| \dot{K} \|
$$

(24)

$$
\| \dot{B} \| \leq \| B^* \| + \| \dot{B} \| = \| B \| + \| \dot{B} \|
$$

(25)

where $\dot{W}$, $\dot{K}$, and $\dot{B}$ are the boundaries of their corresponding vectors. From (22) ~ (25), we have,

$$
| d_f | \leq \theta^T Y_f
$$

(26)

where,

$$
\theta^* = \left[ \dot{W} B + \epsilon_f, \dot{W} K, \frac{1}{2} \dot{K}, \frac{1}{2} B, \frac{1}{2} \dot{B} \right]
$$

(27)

$$
Y_f = \left[ 1, \| G(X) \|, \| \dot{W} \| \| G(X) \|, \| \dot{W} \| \right]
$$

(28)

Note that, by using this theorem, the estimating error (18) can be expressed linearly with the estimator parameters $\dot{W}$, $\dot{K}$, and $\dot{B}$. Based on such a property, we develop our adaptive control system using variable structure theory.

### 3.3 Switching function

The sliding mode hyperplane is firstly defined as

$$
s(t) = \left( \frac{d}{dt} + \lambda \right) x(t)^{n-1} x(t)
$$

(29)

where $\lambda$ is a positive constant, which defines the bandwidth of the error dynamics of the system. The equation $s(t) = 0$ defines a time-varying hyperplane $R^n$ on which the tracking error $\dot{x}(t)$ decays to zero, so that perfect tracking can be asymptotically
obtained by maintaining this condition [5]. In this case the control objective becomes the design of a controller that ensures $s(t) = 0$. (29) can be written as:

$$s(t) = \Delta^T \dot{X}(t)$$

(30)

where $\Delta = [\lambda^{-1}, (n-1) \lambda, \cdots, (n-1) \lambda, 1]$, $\dot{X} = X - X_d$, $X_d = [x_d, \dot{x}_d, \cdots, x_d^{(n-1)}]$. Differentiating both sides of (30) by time, we have:

$$\dot{s}(t) = \Delta^T \ddot{X}(t) + \dddot{s}(t)$$

(31)

where $\Delta = [0, \lambda^{-1}, \cdots, (n-1) \lambda]$.

3.4 Design of controller

Looking at (31), it naturally suggests that when $f$ is known, a control input:

$$u(t) = -k_d s(t) + f + x_d^T \dot{X}(t) - \Delta^T \dot{X}(t) \quad k_d > 0$$

(32)

leads to a closed-loop system $s(t) \dot{s}(t) = -k_d s^2(t)$, and hence, $s(t) \rightarrow 0$ as $t \rightarrow \infty$. However, the problem is how $u(t)$ can be determined when $f$ is unknown. Inspired by the above control structure in (32), using fuzzy estimator $\hat{f}$, our adaptive fuzzy controller is now described below:

$$u(t) = -k_d s(t) + u_{ad}(t) + u_{ro}(t) \quad k_d > 0$$

(33)

where $u_{ad}(t)$, expressed by:

$$u_{ad}(t) = x_d^T \dot{X}(t) - \Delta^T \dot{X}(t)$$

(34)

is a linear combination with tracking error; $u_{ro}(t)$, expressed by:

$$u_{ro}(t) = \hat{V}^T G(X) \hat{K} + \hat{V}^T \hat{B} - \hat{\theta}_f Y_{sgn}(s)$$

(35)

is the adaptive component of the control law, which attempts to cover the unknown function $f$ in the plant. And the adaptive component is synthesized by:

$$\dot{\hat{V}} = -\frac{1}{2} \Gamma_w s(t) (G(X) \hat{K} + \hat{B})$$

(36)

$$\dot{\hat{K}} = -\frac{1}{2} \Gamma_w s(t) (G(X) \hat{W})$$

(37)

$$\dot{\hat{B}} = -\frac{1}{2} \Gamma_w s(t) \hat{W}$$

(38)

$$\dot{\hat{\theta}}_f = \Gamma_s Y_f |s(t)|$$

(39)

where $\Gamma_w \in R^{n \times n}$, $\Gamma_k \in R^{n \times n}$, $\Gamma_b \in R^{n \times n}$, and $\Gamma_s \in R^{n \times n}$ are some appropriately symmetric positive definite matrices, which define the adaptive rates.

3.5 Analysis of stability

The stability of the closed-loop system described by (12), (33) - (35), and (36) - (39) is established in the following theorem.

Theorem 2 If the plant (12) is controlled by (33) - (35), and the adaptive component is synthesized by (36) - (39), then all the signals involved in this control system are bounded, and tracking error decays to zero.

Proof:

Substituting (33) - (35) and (19) into (31), one has:

$$\dot{s}(t) = -k_d s(t) + \hat{W}^T G(X) \hat{K} + \hat{W}^T \hat{B} - \hat{\theta}_f Y_{sgn}(s) - f$$

leads to a closed-loop system $s(t) \dot{s}(t) = -k_d s^2(t)$, and hence, $s(t) \rightarrow 0$ as $t \rightarrow \infty$. However, the problem is how $u(t)$ can be determined when $f$ is unknown. Inspired by the above control structure in (32), using fuzzy estimator $\hat{f}$, our adaptive fuzzy controller is now described below:

$$u(t) = -k_d s(t) + u_{ad}(t) + u_{ro}(t) \quad k_d > 0$$

(33)

where $u_{ad}(t)$, expressed by:

$$u_{ad}(t) = x_d^T \dot{X}(t) - \Delta^T \dot{X}(t)$$

(34)

is a linear combination with tracking error; $u_{ro}(t)$, expressed by:

$$u_{ro}(t) = \hat{W}^T G(X) \hat{K} + \hat{W}^T \hat{B} - \hat{\theta}_f Y_{sgn}(s)$$

(35)

is the adaptive component of the control law, which attempts to cover the unknown function $f$ in the plant. And the adaptive component is synthesized by:

$$\dot{\hat{V}} = -\frac{1}{2} \Gamma_w s(t) (G(X) \hat{K} + \hat{B})$$

(36)

$$\dot{\hat{K}} = -\frac{1}{2} \Gamma_w s(t) (G(X) \hat{W})$$

(37)

$$\dot{\hat{B}} = -\frac{1}{2} \Gamma_w s(t) \hat{W}$$

(38)

$$\dot{\hat{\theta}}_f = \Gamma_s Y_f |s(t)|$$

(39)

Now, consider the following Lyapunov function candidate,

$$V(t) = \frac{1}{2} \hat{s}^2(t) + \hat{W}^T \Gamma \hat{W} + \hat{B}^T \Gamma \hat{B}$$

(41)

where $\hat{\theta}_f = \theta_f - \hat{\theta}_f$. The derivative of the both sides of (41) yields,

$$\dot{V}(t) = s(t) \dot{s}(t) - \hat{W}^T \Gamma \hat{W} - \hat{B}^T \Gamma \hat{B}$$

(42)
Substituting (40) into the expression above, one follows,

\[
\dot{V}(t) = - k_s s^2(t) - \hat{\theta}^T Y_s |s(t)| - \frac{1}{2} \hat{W}^T G(X) \hat{K}_s(t) - \frac{1}{2} \hat{W}^T \dot{B}_s(t) - \frac{1}{2} \hat{W}^T G(X) \hat{K}_s(t) - \frac{1}{2} \hat{W}^T \dot{B}_s(t) - d_s(t) - \hat{W}^T \Gamma^w \hat{W} - \hat{K}^T \Gamma^w \hat{K} - \hat{B}^T \Gamma^w \hat{B} - \hat{\delta}^T \Gamma^w \hat{\delta}.
\]

(43)

Combining (43) and (36) ~ (39), we have

\[
\dot{V}(t) = - k_s s^2(t) - \hat{\theta}^T Y_s |s(t)| - \frac{1}{2} \hat{W}^T G(X) \hat{K}_s(t) - \frac{1}{2} \hat{W}^T \dot{B}_s(t) - \frac{1}{2} \hat{W}^T G(X) \hat{K}_s(t) - \frac{1}{2} \hat{W}^T \dot{B}_s(t) - d_s(t) - \hat{W}^T \Gamma^w \hat{W} + \frac{1}{2} s(t) \left( G(X) \hat{K} + \hat{B} \right) \Gamma^w \hat{W} + \frac{1}{2} s(t) \hat{W}^T G(X) \Gamma^w \Gamma^w \hat{K} + \frac{1}{2} s(t) \hat{W}^T \Gamma^w \Gamma^w \hat{K} + \frac{1}{2} s(t) \hat{W}^T \Gamma^w \Gamma^w \hat{K} + \frac{1}{2} s(t) \hat{W}^T \Gamma^w \Gamma^w \hat{K} + \frac{1}{2} s(t) \hat{W}^T \Gamma^w \hat{W} - Y_s(t) \Gamma^w \hat{K} = - k_s s^2(t) - d_s(t) - \hat{\theta}^T Y_s |s(t)| = - k_s s^2(t) - d_s(t) - \theta^T Y_s |s(t)| = - k_s s^2(t) - d_s(t) - \theta^T Y_s |s(t)|. \tag{44}
\]

Besides, because of \(|d_s| < \theta^T Y_s|s(t)|\), the equation above becomes,

\[
\dot{V}(t) \leq - k_s s^2(t). \tag{45}
\]

It means that the all signals involved in this system are bounded, and \(s(t) \to 0\) as \(t \to \infty\).

4 Simulation Examples

To clarify the proposed design procedure, we apply the adaptive fuzzy controller developed in previous section to control an unstable nonlinear system (Example 1) and a chaotic system (Example 2).

**Example 1.** A nonlinear system is described as

\[
x(t) = \frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} + u(t) \tag{46}
\]

Without the control, i.e., \(u(t) = 0\), it can be easily seen that the system is unstable, because of \(f = \frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} > 0\) for \(x(t) > 0\), and \(f = \frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} < 0\) for \(x(t) < 0\) (Fig.4). The control objective is to force the system state \(x(t)\) to the origin, i.e., \(x_d(t) = 0\). The simulation is conducted with the following fuzzy rules:

\[
R_j: \text{IF } x \text{ is } A_j \text{ THEN } f = w_j
\]

where \(j\) is rule's number; \(A_j\) is a fuzzy set, and \(w_j\) is a singleton value. So, first of all, the problem is how to divide the domain of \(x\) into a few fuzzy sets. As shown in Fig.4, for such a curve of function \(f\), it is reasonable to consider five to seven fuzzy sets over the domain by our experience (or something like intuition). The following reason is strongly supporting our consideration. In most cases, such an unknown function is approximately described in the form of fuzzy rules, which is exactly the form of human-language-like description, by the expert's knowledge, experience and so on. This is the primary reason we use fuzzy control, otherwise we can use the other soft computing methods like neural-network, genetic algorithm, or maybe a purely mathematical method in the sense of the approximating function. So in the case of describing \(f\) by a (human) expert, how many cases should be considered? Five to seven seem reasonable. In this simulation, we adopt the number is five, in other words, there are five fuzzy sets over the domain of \(x\). Consequently, it yields five fuzzy rules at most, where each fuzzy rule \(R_j\) \((j=1, 2, \ldots, 5)\) has a consequent \(w_j\). Furthermore, five fuzzy are initially given as in Fig.5, and \(w_j\) is initially assigned to be value 1.

![Figure 4: Nonlinear function f in (46)](image-url)
Control law (33) was used where $k_d=3$. The adaptive component $u_m(t)$ is synthesized by (35) and (36)-(39) where $\Gamma_w=0.9I$, $\Gamma_x=0.2I$, and $\Gamma_y=0.1I$ with $I$ being some appropriate identity matrices. In addition, we take the values that $\lambda=1$ and initial state $x(0)=3$.

Simulation results are shown in Fig.6-7. Fig.6 shows the evolution of $x(t)$ where a good performance is observed. The amount of control effort required to achieve the above level of the performance is illustrated in Fig.7. The final tuned membership functions in precedent are shown in Fig.8. Comparing the initial membership functions in Fig. 5, it is obvious that a few membership functions have been changed. Since the system has an initial value $x(0)=3$ $(>0)$ and speedily converged to the origin $x_d=0$ without any overshoots, the membership functions in positive region were fired, then tuned, and others in negative region were not tuned.

Example 2. In this example, we consider the Duffing forced-oscillation system:

\[ \ddot{x}(t) = -ax(t) - bx^3(t) + c \cos(t) + u \]  

which is chaotic in unforced case, i.e., $u=0$. The unforced trajectory of the system is shown in Fig.9 in phase plane $(x, \dot{x})$ for $x(0)=\dot{x}(0)=2$, $a=0.11$, $b=1$, $c=12$, and time period $[0, 60]$. Now, we use
the control approach proposed in this paper to force the state \( x(t) \) to follow a desired trajectory \( x_d(t) = \sin(t) \). In the phase plane, the desired trajectory is an unit circle \( x^2(t) + \dot{x}^2(t) = 1 \). In this simulation, we choose the initial membership functions as shown in Fig.5 for both \( x(t) \) and \( \dot{x}(t) \). Clearly, the two input variables lead \( 5 \times 5 = 25 \) fuzzy rules as follows:

\[ R_1 : \text{IF } x \text{ is } A^1_1, \dot{x} \text{ is } A^2_1 \text{ THEN } f \text{ is } w_1 \]

To verify the control scheme, suppose that we have no knowledge regarding the function \( f = -a \dot{x}(t) - b x(t) + \cos(t) \), so the initial consequents \( w_i \) are selected randomly. In addition, the all parameters involved in the control law \( u(t) \) are chosen as same as Example 1.

Simulation result is shown in Fig.10 where the closed-loop trajectories are depicted with the initial conditions \( x(0) = \dot{x}(0) = 2 \). We see that our control scheme can control the system with some unknown time-variable facts such as \( \cos(t) \) to track a desired trajectory well.

5 Conclusion

In this paper, we gave a design for a class of adaptive fuzzy control system in which the fuzzy rules are composed of triangular membership functions. The main contribution in this paper is the approach to stably tune up the parameters in the triangular membership functions. Also, we showed that the all signals involved in this control system are bounded, and tracking error decays to zero. Simulation results for different nonlinear systems confirmed the theoretic analysis. As the future works, we want to further verify the approach proposed in this paper through a proper mechanical experiment, and extend the system to a general one such as MIMO without control gain being 1.

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