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Improvement in Solution Search Performance of Deterministic PSO Using a Golden Angle

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Abstract

A particle swarm optimization (PSO) is one of the powerful systems for solving global optimization problems. The searching ability of such PSO depends on the inertia weight coefficient, and the acceleration coefficients. Since the acceleration coefficients are multiplied by a random vector, the system can be regarded as a stochastic system. In order to analyze the dynamics rigorously, we pay attention to a deterministic PSO, which does not contain any stochastic factors. On the other hand, the standard PSO may diverge depending on the random parameter. Because of this divergence property, the standard PSO has high performance compared with the deterministic PSO. Since the deterministic PSO does not have stochastic factors, the diversity of the particles of deterministic PSO is lost. Therefore, its searching ability is worse. In order to give diversity to the deterministic PSO, the golden angle is applied to the rotation angle parameter of the deterministic PSO. We confirm the performance of the searching ability of the proposed PSO.

1. Introduction

Searching for an optimal value of a given evaluation function is very important. In order to solve such optimization problems speedily, various meta-heuristic optimization algorithms have been proposed. Particle swarm optimization (PSO), which was originally proposed by Kennedy and Eberhart [1], [2], is one such metaheuristic algorithm. The PSO algorithm is a useful tool for optimization problems [3]-[6].

The original PSO is described as

\[ v_j^{t+1} = w v_j^t + c_1 r_1 (pbest_j^t - x_j^t) + c_2 r_2 (gbest^t - x_j^t) \]

\[ x_j^{t+1} = x_j^t + v_j^{t+1} \]

(1)

(2)

where \( w \geq 0 \) is an inertia weight coefficient, \( c_1 \geq 0 \) and \( c_2 \geq 0 \) are acceleration coefficients, and \( r_1 \in [0, 1]^N \) and \( r_2 \in [0, 1]^N \) are two separately generated uniformly distributed random number vectors. \( x_j^t \in \mathbb{R}^N \) denotes the location vector of the \( j \)-th particle on the \( t \)-th iteration in the \( N \)-dimensional parameter space, and \( v_j^t \in \mathbb{R}^N \) denotes the velocity vector of the \( j \)-th particle on the \( t \)-th iteration. \( pbest_j^t \in \mathbb{R}^N \) represents the location that gives the best value of the evaluation function of the \( j \)-th particle on the \( t \)-th iteration. \( gbest^t \in \mathbb{R}^N \) is the location that gives the best value of the evaluation function on the \( t \)-th iteration in the swarm.

The particles in the swarm fly through the \( N \)-dimensional space according with Eqs. (1) and (2). Each particle shares information of a current optimal value of the evaluation function and its corresponding location of the best particle. Also, each particle memorizes its record of the best evaluation value and its best location. On the basis of such information, the moving direction and velocity are calculated using Eq. (1). Namely, all particles will move toward a coordinate that gives the current best value of the evaluation function.

The dynamics of the PSO systems is very complicated. In order to analyze the dynamics of such PSO, Clerc and Kennedy proposed a simple deterministic PSO system, and analyzed its dynamics theoretically [4]. The simple deterministic PSO system does not contain stochastic factors; namely, the random coefficients have been omitted from the original PSO system. The analysis of such a deterministic PSO is very important for determining the effective parameters of the standard PSO [4], [7].

Moreover, we are trying to implement the proposed system in an electronic circuit [8]. Considering the implementation, it is desirable that the system does not contain any stochastic factors. Therefore, we pay attention to a deterministic system. The simple acceleration coefficients of the deterministic PSO system can be described as

\[
\begin{align*}
\psi &= \frac{c_1 pbest_j^t + c_2 gbest^t}{\psi} \\
p_j^t &= \psi
\end{align*}
\]

(3)

where \( p_j^t \) can be regarded as a desired fixed point.

In this case, Eqs. (1) and (2) can be transformed into the following matrix form:

\[
\begin{bmatrix}
v_j^{t+1} \\
y_j^{t+1}
\end{bmatrix} =
\begin{bmatrix}
w & -\psi \\
w & 1 - \psi
\end{bmatrix}
\begin{bmatrix}
v_j^t \\
y_j^t
\end{bmatrix}
\]

(4)
where \( y_j = x_j - p_j \).

The dynamics of the particles of the deterministic PSO is characterized by the eigenvalues \( \lambda \), as shown in Eq. (4).

\[
\lambda = \frac{(1 + w - c)}{2} \pm \frac{(1 + w - c)^2 - 4w}{2} \tag{5}
\]

The damping factor \( \Delta \) and the rotation angle \( \theta \) are given by the eigenvalues.

\[
\Delta = \sqrt{w} \tag{6}
\]

\[
\theta = \arctan \left( \frac{4w - (1 + w - c)^2}{1 + w - c} \right) \tag{7}
\]

The damping factor and the rotation angle are regarded as the parameters of the deterministic PSO. Note that this system does not contain stochastic factors, therefore, this system can be regarded as a deterministic system.

2. Influence of a Random Number

Since the deterministic PSO does not contain stochastic factors, the searching ability is deteriorated comparing with the conventional PSO.

In deterministic PSO, random numbers are assumed to be \( r_1 = r_2 = 1 \). On the other hand, since the values of \( r_1 \) and \( r_2 \) are uniform random numbers in the conventional PSO, an average is set to 0.5. Figure 1 shows the relationship of parameters \( w \) and \( c = c_1 r_1 + c_2 r_2 \). Note that the parameters \( r_1 \) and \( r_2 \) of the deterministic PSO are set as \( r_1 = r_2 = 0.5 \) to compare the performance with the conventional stochastic PSO. The parameters are set within the triangular area in Figure 1 to guarantee the stability of the system. In the case of the deterministic PSO, the parameters are given as a certain point. The parameters of the conventional stochastic PSO vary with the number of iterations. However, the parameters of the deterministic PSO are time-invariant. As an example, the results of the Rosenbrock function and Rastrigin function are shown in Figure 2. We apply \( w = 0.729 \) and \( c_1 = c_2 = 1.49445 \) as parameters of PSO. These results show the influence of the random number on the solution search performance.

Figure 2: Standard PSO vs deterministic PSO

3. Deterministic PSO Using a Golden Angle

We have proposed a parameter-setting procedure in which the golden angle is applied to the rotation angle to improve the performance of the searching ability. The golden angle is the smaller of the two angles created by sectioning the circumference of a circle according to the golden ratio.

\[
\phi = 2\pi \cdot \frac{1}{1 + \sqrt{5}} \approx 2\pi \cdot 0.3819 \text{ [rad]} \tag{8}
\]

In our proposed procedure, the rotation angle of each dimension of the particle is determined by the golden angle.

When the golden angle is applied, the rotation angle does not have overlap; namely, the system has diversity, as shown in Figure 3(a). The radius corresponds to the damping factor. Considering the parameter range of the conventional stochastic PSO, the range of the rotation angle is normalized, as illustrated in Figure 3(b). Similar to the conventional PSO, the rotation angle parameter is set for each iteration \( t \). The conventional PSO can obtain good solutions when the particles
exhibit damped oscillations. For this reason, we set the parameters of the deterministic PSO to generate such damped oscillations. Also, we discuss a method for the deterministic PSO to search the solution whose ability is equivalent to the conventional PSO. The rotation angle $\theta$ for every dimension $d$ of each particle $i$ is calculated as follows. $D$ is the dimension of an evaluation function.

$$\theta_{nt} = \{jD + (d + 1) + (t + 1)D\} \phi$$  \hspace{1cm} (9)

$$\theta = \frac{\theta_{nt}}{2\pi} \phi \quad \text{mod} \quad 2\pi$$  \hspace{1cm} (10)

In this article, using such a time-variant rotation angle, the parameter-setting procedure of the deterministic PSO is proposed. $w$ and $c$ are derived from the rotation angle using Eqs. (6) and (7).

![Diagram](a) Diversity of the parameter using a golden angle  \hspace{1cm} (b) Range of golden angle

Figure 3: Rotation angle $\theta$ acquired from a golden angle

4. Numerical Simulations

In order to confirm the performance of the procedure of PSO that uses the golden angle, we compare it with the conventional PSO. We carry out the numerical simulation for two cases. One is when the golden angle is used only at the initialization. The other one is when the golden angle is used for every iteration.

The numerical simulations are carried out using four standard benchmark functions, as shown in Table 1. Sphere and Rosenbrock functions are unimodal functions. Rastrigin and Griewank functions are multimodal functions. Except for the Rosenbrock function, the optimum value of each function is 0 and the optimum solution is $x_d = 0$. The optimum value of the Rosenbrock function is 0 and the corresponding optimum solution is $x_d = 1$.

The parameters searching range, initializing range, and $v_{\text{max}}$ are set as shown in Table 2 for each benchmark function. $v_{\text{max}}$ is a divergent control parameter. The upper bound is given at each particle velocity calculated in Eq. (1). However, this parameter is not needed in the deterministic PSO if the parameters satisfy the stable condition. Namely, we can set $v_{\text{max}} = \infty$. The initializing range is determined as an asymmetric range in the searching range. This operation provides biased initial values.

<table>
<thead>
<tr>
<th>Name</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>$f_1(x) = \sum_{d=1}^{N} x_d^2$</td>
</tr>
<tr>
<td>Rosenbrock</td>
<td>$f_2(x) = \sum_{d=1}^{N} (100(x_{d+1} - x_d^2)^2 + (x_d - 1)^2)$</td>
</tr>
<tr>
<td>Rastrigin</td>
<td>$f_3(x) = 10N + \sum_{d=1}^{N} ((x_d)^2 - 10 \cos(2\pi x_d))$</td>
</tr>
<tr>
<td>Griewank</td>
<td>$f_4(x) = 1 + \frac{1}{4000} \sum_{d=1}^{N} x_d^2 - \prod_{d=1}^{N} \cos \left( \frac{x_d}{\sqrt{d}} \right)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Function</th>
<th>Search Range</th>
<th>$v_{\text{max}}$</th>
<th>Initial Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1(x)$</td>
<td>$(-100, 100)$</td>
<td>100</td>
<td>$(50, 100)^N$</td>
</tr>
<tr>
<td>$f_2(x)$</td>
<td>$(-100, 100)$</td>
<td>100</td>
<td>$(50, 100)^N$</td>
</tr>
<tr>
<td>$f_3(x)$</td>
<td>$(-10, 10)$</td>
<td>10</td>
<td>$(2.56, 5.12)^N$</td>
</tr>
<tr>
<td>$f_4(x)$</td>
<td>$(-600, 600)$</td>
<td>600</td>
<td>$(300, 600)^N$</td>
</tr>
</tbody>
</table>

5. Results

The simulation results are illustrated in Figure 4. The horizontal axis denotes the number of iterations, and the vertical axis denotes the average evaluation value. We carry out 50 trials. The parameters of PSO are set to $w = 0.729$ and $c_1 = c_2 = 1.494$.

In this figure, GD-PSO is the case where the rotation angle is set with an unnormalized golden angle. G_{D}-PSO is the case where the parameters are set using the proposed procedure. The parameters of G_{D}-PSO have diversity.

For Sphere, Rosenbrock, and Griewank functions, G_{D}-PSO exhibits better performance than GD-PSO. G_{D}-PSO shows similar performance to the conventional PSO in spite of being a deterministic system. For the Rastrigin function, which is one of the multimodal functions, GD-PSO gives a similar result to the conventional PSO. We consider that the wide range of parameters of GD-PSO are effective for escaping from the local minimum.

6. Conclusions

In this article, we proposed a novel parameter-setting procedure that deterministically reproduces the diversity of
the conventional stochastic PSO. The proposed procedure is based on the effect of the stochastic factor of the conventional PSO. We confirmed that the deterministic PSO using the proposed procedure exhibits a similar performance to the conventional PSO when using the golden angle.

Acknowledgment

This work was supported by JSPS KAKENHI Grant-in-Aid for Scientific Research (C) 22560389

References


Figure 4: Standard PSO vs deterministic PSO with golden angle