PAPER

Construction of ZCZ Codes with Low Aperiodic Autocorrelation

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Abstract ZCZ-CDMA, which is a quasi-synchronous CDMA (code division multiple access) system using a set of sequences with a zero correlation zone called ZCZ code as a spreading code, is useful for short-range wireless communications because of its excellent properties such as co-channel interference-free performance and fast frame synchronization capability. In this paper, we propose a ZCZ code with good aperiodic autocorrelation to be used as a synchronization symbol and training signal for channel estimation. The ZCZ-code is derived from an even-shift orthogonal sequence called an E-sequence, whose out-of-phase aperiodic autocorrelation function takes a value of zero at any even shift. A ZCZ code of length $2^n$ is discussed expressed by both a logic function and matrices including the Sylvester type Hadamard matrix. The logic function gives a code generator consisting of $2^n$ counters, and feedforward logic, and the matrices provide a matched filter bank that can take the correlation between input signals and any sequence in, the ZCZ code simultaneously.

Keywords: signal design, spreading code, CDMA, code generator, matched filter bank

1. Introduction

A CDMA (code division multiple access) system is also useful in short-range wireless communications because of its inherent robustness against multipath fading as well as its multiple access capability and frequency utilization efficiency [1]. Generally, since co-channel interference, becomes large with the increase in the number of users, the CDMA system performance deteriorates. To decrease the influence of co-channel interference, CDMA requires complex functions such as precise transmittance power control, anti-interference processing and so on.

ZCZ-CDMA operating under quasi-synchronous timing control and using a set of sequences, whose periodic correlation function has a zero correlation zone, called ZCZ code, archives interference free performance and synchronization capability with fast frame acquisition [2]-[16]. Since ZCZ-CDMA is served similarly to single access communication, its hardware design can be simplified.

A ZCZ code with the minimum zero correlation zone, whose out-of-phase periodic autocorrelation function and cross-correlation functions take values of zero within one shift around each zero shift, is considered here because of its application in short-range and low speed communications. When high-speed transmission is considered, it will be useful for the block-coding technique using a ZCZ code with the minimum zero correlation zone since the system performance is almost maintained [17], [18]. If a ZCZ code with a wide zero correlation zone is used, ZCZ-CDMA has poor system performance since the user size becomes small as mentioned in a latter section [19].

Aiming at the further simplification of its hardware and system design, the authors examined the construction of a ZCZ code with the minimum zero correlation zone, which includes a sequence with a good aperiodic autocorrelation property to be used for a synchronization symbol [20]. Since all sequences except one in a ZCZ code may have comparatively high autocorrelation, it is desirable that all the sequences possess a low aperiodic autocorrelation property in order to provide ZCZ-CDMA with flexibility, such that two or more synchronous control symbols are needed, and spreading sequences are also used as pilot symbols for channel estimation [21].

In this paper, a ZCZ code for which any sequence possesses low aperiodic autocorrelation is proposed. It is constructed by using an even-shift orthogonal sequence called an E-sequence, whose aperiodic autocorrelation function takes a value of zero at any even shift [22]-[24]. E-sequences are included in complementary sequences, which are pairs of sequences for which the sum of the out-of-phase autocorrelation values of the same shifts takes a value of zero [25]-[27].

In Sect. 2 an E-sequence of length $2^n$ is introduced. It is expressed by a logic function and matrices including the
Sylvester type Hadamard matrix. In Sect. 3 it is shown that the E-sequence possesses a good aperiodic autocorrelation property by consideration of the logic function. Sections 2 and 3 include part of the contents of [24]. In Sect. 4 the construction of four types of ZCZ codes, which include a new ZCZ code of length $2^{m+1}$ derived from the E-sequence of length $2^m$, is discussed and the logic function of the new ZCZ code is formulated. In Sect. 5 it is shown that the new ZCZ code has a good autocorrelation property. In Sect. 6 its code generator and matched filter bank are presented. Sections 4-6 include part of the contents of [28].

2. E-sequence

2.1 Construction of E-sequence

Let $e$ be a binary sequence of length $N$ written as

$$e = (e_0, e_1, \ldots, e_s, \ldots, e_{N-1})$$

where $e_s$ takes a value of 1 or $-1$ for $0 \leq x \leq N - 1$, and 0 otherwise. The aperiodic correlation function between binary sequences $e$ and $\hat{e}$ at a shift $\tau$ is defined by

$$C_{ee}(\tau) = \sum_{t=0}^{N-1} e_t \hat{e}_{t-\tau} \quad 0 \leq \tau \leq N - 1$$

If the out-of-phase aperiodic autocorrelation function takes a value of zero at any even shift, i.e., $C_{ee}(2m) = 0$ for $m \neq 0$, $e$ is called the even-shift orthogonal sequence (E-sequence) [22]-[24] [29], [30].

A pair of binary sequences, $e$ and $\hat{e}$, of length $N$ are called complementary sequences if the sum of the aperiodic autocorrelation functions is zero everywhere except at a zero shift [25]-[27], i.e.,

$$C_{ee}(\tau) + C_{\hat{e}e}(\tau) = \begin{cases} 2N & \text{for } \tau = 0 \\ 0 & \text{otherwise} \end{cases}$$

It is known that the length of a complementary sequence is $K2^m$, where $K$ is 2, 10, 26 a product of these values.

Let $e$ and $\hat{e}$ be complementary sequences of length $N/2$. If and only if

$$e = (e_0, \hat{e}_0, e_1, \hat{e}_1, \ldots, e_{s}, \hat{e}_{s}, \ldots, e_{N/2-1}, \hat{e}_{N/2-1}) \quad (1)$$

$e$ is an E-sequence of length $N$ [23], [31], [32] since

$$C_{ee}(2m) = C_{\hat{e}e}(m) + C_{\hat{e}e}(m)$$

2.2 Logic functions of E-sequences of length $2^n$

Logic functions of E-sequences of length $N = 2^n$ are those of complementary sequences produced by the interleaving method as shown in Eq. (1) [31], [32].

Let $\mathbb{Z} = (x_0, x_1, \ldots, x_{n-1})$ be the space $V_n$ of binary $n$-tuples whose elements are coefficients expressed as the binary expansion of an integer $x$ ($0 \leq x < N$),

$$x = x_02^0 + x_12^1 + \ldots + x_{n-1}2^{n-1}$$

Let $E$ be a set of E-sequences of length $N = 2^n$, written as

$$E = \{ e^0, e^1, \ldots, e^{N-1} \}

$$

$$e^s = (e^s_0, e^s_1, \ldots, e^s_{N-1}) \quad e^s_r \in \{1, -1\}$$

Each element is expressed by

$$e^s_r = (-1)^{L^s_r}$$

with

$$f_s(\bar{x}) = g(\bar{x}) \oplus d \cdot \bar{x} \oplus d 

$$

$$g(\bar{x}) = y_0y_1 \oplus y_1y_2 \oplus \ldots \oplus y_{2^{N-2}}y_{2^{N-1}} \quad (2)$$

where $\oplus$ denotes addition over $GF(2)$, $y_j \in \{x_0, x_1, \ldots, x_{n-1}\}$ with $y_0 = x_0$ and $y_j \neq y_k$ for $j \neq k(\geq 1)$, $\bar{d} = (a_0, a_1, \ldots, a_{n-1})$ is a parameter to give different E-sequences, $d \cdot \bar{x} = a_0x_0 \oplus \ldots \oplus a_{n-1}x_{n-1}$, and $d \in \{0, 1\}$ is a parameter of the inversion of a sequence. To simplify the discussion, $d = 0$ in this paper.

If $y_0 \neq x_0$, the logic function produces complementary sequences excluding E-sequences.

2.3 Matrix expression of E-sequence

Let $E$ be a set of E-sequences of length $N$, which can be expressed as a matrix, rewritten by

$$E = (e^0, e^1, \ldots, e^{N-1})^T$$

$$e^s = (e^s_0, e^s_1, \ldots, e^s_{N-1}) \quad e^s_r \in \{1, -1\}$$

where $T$ denotes the transpose of a matrix and each row is equal to the E-sequence $e^s$. From Eq. (2), the set $E$ is written as

$$E = H_n \sigma_n$$

where $H_n$ denotes the Sylvester type Hadamard matrix of order $N(= 2^n)$ defined by

$$H_n = \begin{pmatrix} H_{n-1} & H_{n-1} \\ H_{n-1} & -H_{n-1} \end{pmatrix}, \quad H_1 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad (3)$$

and $\sigma_n$ is a diagonal matrix whose diagonal elements are equal to $(-1)^{a_i(\bar{x})}$.

Example 1: A set of E-sequences of length $N = 2^n$ generated by

$$f_s(\bar{x}) = x_0x_1 \oplus x_1x_2 \oplus a_0x_0 \oplus a_1x_1 \oplus a_2x_2$$
is written as

\[
E = H_3^n = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 
\end{pmatrix}
\]

The number of E-sequences of length \(2^n\) generated by the logic function is given as

\[2^n(n - 1)! \]

since the number of different \(\vec{d}'\) is \(2^n\) and the number of different expressions \(x_0 y_1 \oplus y_2 \oplus \cdots \oplus y_{n-1} = x_{n-1}\) is \((n - 1)! = (n - 1)(n - 2) \cdots 2\).

3. Aperiodic Autocorrelation Property of E-sequences

In this section, the autocorrelation function of an E-sequence is evaluated by consideration of its logic function. Two criteria are used for the evaluation. One is the merit factor \(F\) [27] defined by

\[F = \frac{|C_{\text{aut}}(0)|^2}{2 \sum_{\tau=1}^{N-1}|C_{\text{aut}}(\tau)|^2}\]  
(4)

and the other is the maximum absolute aperiodic correlation value \(C_{\text{max}}\) defined by

\[C_{\text{max}} = \max_{0\leq|\tau|\leq N-1} |C_{\text{aut}}(\tau)|\]

Note that, it is desirable for \(F\) to be as large as possible. On the other hand, it is desirable that \(C_{\text{max}}\) is small.

Table 1 and 2 show the aperiodic autocorrelation functions and merit factors for a set of E-sequences of length 16, which are produced by

\[f_0(x) = x_0 x_1 \oplus x_1 x_2 \oplus x_2 x_3 \oplus \vec{d} \cdot \vec{x}\]  
(5)

and

\[f_0(x) = x_0 x_1 \oplus x_1 x_3 \oplus x_3 x_2 \oplus \vec{d} \cdot \vec{x}\]  
(6)

respectively, with \(a = a_0 2^0 + a_1 2^1 + a_2 2^2 + a_3 2^3\). An E-sequence is expressed in hexadecimal. For example “121d” means

\[e = (+, +, +, +, - , +, +, +, +, +, +, -)
\]

where + and − denote 1 and −1, respectively.

Note that there exist sets of E-sequences with the same merit factor \(F\) generated by the logic function given by Eq. (2). It is presumed that if \(a_0 = x_0, a_1 = x_1, \cdots, a_{n-1} = x_{n-1}\), then the produced E-sequences of length \(2^n\) possess the same merit factor \(F\). This means that such an E-sequence is one of the complementary sequences made sequentially by only the interleaving method from sequences of length 2.

Table 3 shows values of \(C_{\text{max}}\) and \(F\) for all the E-sequences of length \(N = 2^n\) with \(3 \leq n \leq 7\), which can be generated by the logic function given by Eq. (2). Table 4 shows values of \(C_{\text{max}}\) and \(F\) for all the complementary sequences of length \(N = 2^n\) except for the E-sequences. Table 5 also shows the values of \(C_{\text{max}}\) and \(F\) for M-sequences of length \(N = 2^n - 1\) whose periodic autocorrelation functions take a value of −1 except for the zero-shift. These values are obtained for an optimal phase shift such that these merit factors become maximum. Here, it is well known that an M-sequence generated by a primitive polynomial has a good periodic correlation property [1], [2], [27].

Figure 1 shows that the merit factors of some E-sequences of period \(2^n\) are larger than those of some M-sequences with the optimal initial phase produced by certain primitive polynomials.

4. ZCZ Codes

4.1 Definition of ZCZ code

Let \(Z\) be a set of \(M\) biphase sequences of period \(L\),

\[Z = \{z'_0, z'_1, \ldots, z'_i, \ldots, z'_{M-1}\}\]

\[z'_i = (z'_0, z'_1, \ldots, z'_i, \ldots, z'_{L-1}), \quad z'_i \in \{1, -1\}\]

The periodic correlation function is defined by

\[R_{z'_i}(\tau) = \sum_{\tau=0}^{L-1} z'_i(\tau+k) \mod L \]

\[= C_{z'_i}(\tau) + C_{z'_i}(\tau - L)\]

The set \(Z\) is called a ZCZ code denoted by \(Z(L, M, Z_{cz})\) if

\[R_{z'_i}(\tau) = \left\{ \begin{array}{ll}
L & \text{for } \tau = 0, j = k \\
0 & \text{for } \tau = 0, j \neq k \\
0 & \text{for } 1 \leq |\tau| \leq Z_{cz}
\end{array} \right.\]

4.2 Construction of ZCZ code

As mentioned in Sect. 1, the ZCZ code \(Z(L, M = L/2, 1)\) is considered, which achieves the upper bound [19]

\[M \leq \frac{L}{Z_{cz} + 1}\]
Table 1 Autocorrelations of E-sequences derived from Eq. (5)

<table>
<thead>
<tr>
<th>(a)</th>
<th>(e)</th>
<th>(C_{ee}(r)) (0 \leq r \leq N - 1)</th>
<th>(F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1212</td>
<td>161010101010305010-1</td>
<td>3.20</td>
</tr>
<tr>
<td>1</td>
<td>4748</td>
<td>161010101010-3050101</td>
<td>3.20</td>
</tr>
<tr>
<td>2</td>
<td>2124</td>
<td>161010101010305010-1</td>
<td>3.20</td>
</tr>
<tr>
<td>3</td>
<td>7478</td>
<td>161010101010305010-1</td>
<td>3.20</td>
</tr>
<tr>
<td>4</td>
<td>1417</td>
<td>1610101010103050101</td>
<td>3.20</td>
</tr>
<tr>
<td>5</td>
<td>4847</td>
<td>161010101010305010-1</td>
<td>3.20</td>
</tr>
<tr>
<td>6</td>
<td>2228</td>
<td>161010101010305010-1</td>
<td>3.20</td>
</tr>
<tr>
<td>7</td>
<td>7746</td>
<td>161010101010-3050101</td>
<td>3.20</td>
</tr>
<tr>
<td>8</td>
<td>1472</td>
<td>1610101010101050101</td>
<td>3.20</td>
</tr>
<tr>
<td>9</td>
<td>4341</td>
<td>16101010101010305010-1</td>
<td>3.20</td>
</tr>
<tr>
<td>10</td>
<td>7746</td>
<td>16101010101010305010-1</td>
<td>3.20</td>
</tr>
<tr>
<td>11</td>
<td>4341</td>
<td>16101010101010305010-1</td>
<td>3.20</td>
</tr>
<tr>
<td>12</td>
<td>7746</td>
<td>1610101010105010101</td>
<td>3.20</td>
</tr>
<tr>
<td>13</td>
<td>4341</td>
<td>1610101010105010101</td>
<td>3.20</td>
</tr>
<tr>
<td>14</td>
<td>7746</td>
<td>1610101010105010101</td>
<td>3.20</td>
</tr>
<tr>
<td>15</td>
<td>4341</td>
<td>1610101010105010101</td>
<td>3.20</td>
</tr>
</tbody>
</table>

Table 2 Autocorrelations of E-sequences derived from Eq. (6)

<table>
<thead>
<tr>
<th>(a)</th>
<th>(e)</th>
<th>(C_{ee}(r)) (0 \leq r \leq N - 1)</th>
<th>(F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1323</td>
<td>165030305010103010-1</td>
<td>2.29</td>
</tr>
<tr>
<td>1</td>
<td>4258</td>
<td>1650303050101030101</td>
<td>2.29</td>
</tr>
<tr>
<td>2</td>
<td>2234</td>
<td>1650303050101030101</td>
<td>2.29</td>
</tr>
<tr>
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</tr>
<tr>
<td>4</td>
<td>1432</td>
<td>16503030501030101</td>
<td>2.29</td>
</tr>
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<td>5</td>
<td>4998</td>
<td>16503030501030101</td>
<td>2.29</td>
</tr>
<tr>
<td>6</td>
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<td>2.29</td>
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<tr>
<td>15</td>
<td>7648</td>
<td>16503030501030101</td>
<td>2.29</td>
</tr>
</tbody>
</table>

Table 3 Autocorrelations of E-sequences of length \(N = 2^n\)

<table>
<thead>
<tr>
<th>(N)</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_{ee})</td>
<td>3</td>
<td>3</td>
<td>7</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>(F)</td>
<td>2.67</td>
<td>2.29</td>
<td>3.33</td>
<td>2.13</td>
<td>4.57</td>
</tr>
</tbody>
</table>

Table 4 Autocorrelations of complementary sequences of length \(N = 2^n\)

<table>
<thead>
<tr>
<th>(N)</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_{ee})</td>
<td>3</td>
<td>3</td>
<td>7</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>(F)</td>
<td>2.67</td>
<td>2.29</td>
<td>3.33</td>
<td>2.13</td>
<td>4.57</td>
</tr>
</tbody>
</table>

Table 5 Autocorrelations of M-sequences

<table>
<thead>
<tr>
<th>(N)</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_{ee})</td>
<td>3</td>
<td>3</td>
<td>7</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>(F)</td>
<td>2.67</td>
<td>2.29</td>
<td>3.33</td>
<td>2.13</td>
<td>4.57</td>
</tr>
</tbody>
</table>

of order \(N\) defined by Eq. (3), \((I_n, D_n)\) is an \(N \times 2N\) concatenated matrix, \(I_n\) is the unit matrix of order \(N, D_n\) is a diagonal matrix whose elements take values of \((-1, 1, -1, 1, \ldots, -1, 1)\), \(\sigma_n\) is a column replacement matrix of order \(N\) whose elements can be \(-1\) in addition to \(1\) with \(N^2 - N\) zero elements, or a diagonal matrix whose elements take values of \(1\) and \(-1\), and \(\sigma_{n+1}\) is a circular permutation matrix of order \(2N\).

Four types of ZCZ code are listed in Table 6. ZCZ1 is a traditional ZCZ code [11], [14]; ZCZ2 includes a sequence with a good aperiodic autocorrelation property [20], ZCZ3 is a newly given ZCZ code using E-sequences, and ZCZ4 is a code given by one of the complementary sequences that do not include E-sequences, which is used for comparison with ZCZ3.

Example 2: \(Z(8,4,1)\) in ZCZ3, generated by the logic function

\[ f_d(x) = x_0 x_1 + d \cdot x^2 \]

is written as

\[ Z = (\varepsilon^0, \varepsilon^1, \varepsilon^2, \varepsilon^3)^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \]

A ZCZ code of length \(L = 2N = 2^{n+1}\) can be expressed by

\[ Z = \begin{bmatrix} \varepsilon^0 \\ \varepsilon^1 \\ \vdots \\ \varepsilon^{N-1} \end{bmatrix} = H_n \sigma_n (I_n, D_n) \rho_n \]

where \(H_n\) denotes the Sylvester type Hadamard matrix.

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where \( H_2 = (-1)^{\overrightarrow{\mathbf{d}} \cdot \overrightarrow{x}}, \overrightarrow{d}, \overrightarrow{x} \in V_2 \) and \( \rho_3 = I_3 \).

5. Evaluation of ZCZ Codes

In order to examine the aperiodic autocorrelation functions of the ZCZ codes in Table 6, the maximum (max), minimum (min) and average (avg) of the merit factors given by Eq. (4) are discussed. Tables 7 - 9 show the merit factors of ZCZ codes of lengths 16, 32 and 64, respectively. ZCZ1 has the lowest merit factors. ZCZ2 has the highest maximum merit factors but the minimum and average merit factors are low. This means that almost all sequences in ZCZ2 have high correlation values. The minimum and average merit factors of ZCZ3 are highest among the codes. Therefore ZCZ3 has low aperiodic autocorrelation. Since for ZCZ4, there are many merit factors, only some of them are listed in table 9.

It is shown that ZCZ3, given by a set of E-sequences with the same merit factors, also has the same sequences as shown in Tables 10 and 11.

Table 12 shows the values of \( C_{\text{max}} \) and \( F \) of all the ZCZ codes. Not all the merit factors of ZCZ2 of length 64 or 128 could be given, since there exist a very large number of ZCZ codes. ZCZ4 can not construct ZCZ codes of length 8, since all the complementary sequences of length 4 are E-sequences.

Figure 2 shows the maximum value of the average merit factors of the ZCZ codes except ZCZ2. ZCZ3 possesses a higher average merit factor than the other codes.

6. Code Generator and Matched Filter Bank

6.1 Logic function

Consideration of the construction of the ZCZ given by Eq. (7) and the logic function given by Eq. (2) gives the logic functions of \( Z(2^{n+1}, 2^n, 1) \) in ZCZ3 and ZCZ4 as

\[
z_d(\overrightarrow{x}) = (x_0 \oplus 1) f_d(\overrightarrow{x}) \oplus x_n f_d(\overrightarrow{x}) \oplus (x_0 \oplus 1)
\]

\[
e = y_1 y_2 \oplus \cdots \oplus y_{n-2} y_{n-1}
\]

\[
\delta \cdot \overrightarrow{d} \oplus x_0 x_n \oplus x_n
\]

where

\[
\overrightarrow{x} = (x_0, x_1, \cdots, x_{n-1}, x_n)
\]

A code generator is easily given as shown in the following example.

Example 3: A code generator of ZCZ3, \( Z(8, 4, 1) \), produced by

\[
z_d(\overrightarrow{x}) = x_0 x_1 \oplus x_0 y_0 \oplus x_1 y_1 \oplus x_0 x_2 \oplus x_2
\]

is shown in Figure 3, where \( \oplus \) and \( \oplus \) denote an XOR gate and AND gate, respectively.

6.2 Matched filter bank

Let us design a matched filter bank that simultaneously takes the correlation between an input signal and any se-
Table 9  Merit factors of Z(64, 32, 1)

<table>
<thead>
<tr>
<th>Type</th>
<th>max</th>
<th>min</th>
<th>avg</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZCZ1</td>
<td>0.42</td>
<td>0.10</td>
<td>0.23</td>
</tr>
<tr>
<td>ZCZ2</td>
<td>5.22</td>
<td>0.12</td>
<td>0.96</td>
</tr>
<tr>
<td>ZCZ3</td>
<td>3.05</td>
<td>3.05</td>
<td>3.05</td>
</tr>
<tr>
<td></td>
<td>3.76</td>
<td>2.56</td>
<td>3.11</td>
</tr>
<tr>
<td>ZCZ4</td>
<td>4.00</td>
<td>0.51</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>2.78</td>
<td>2.37</td>
<td>2.58</td>
</tr>
<tr>
<td></td>
<td>1.56</td>
<td>1.31</td>
<td>1.43</td>
</tr>
<tr>
<td></td>
<td>2.56</td>
<td>1.94</td>
<td>2.25</td>
</tr>
</tbody>
</table>

Table 10  Aperiodic autocorrelation functions of ZCZ3 Z(8, 4, 1)

<table>
<thead>
<tr>
<th>j</th>
<th>z`</th>
<th>C_{j,k}(\tau) (0 \leq \tau \leq 7)</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>e4</td>
<td>8 1 -2 1 0 1 -2 -1</td>
<td>2.67</td>
</tr>
<tr>
<td>1</td>
<td>b1</td>
<td>8 -1 2 0 1 -1 -2 -1</td>
<td>2.67</td>
</tr>
<tr>
<td>2</td>
<td>d1</td>
<td>8 -1 1 0 1 2 1 -1</td>
<td>2.67</td>
</tr>
<tr>
<td>3</td>
<td>d2</td>
<td>8 1 0 1 2 1 -1 -2</td>
<td>2.67</td>
</tr>
</tbody>
</table>

The sequence in a ZCZ code, which can reduce the number of circuit elements as much as possible.

Let \( x = (x_1, x_2, \ldots, x_M)^T \) and \( w = (w_1, w_2, \ldots, w_N)^T \) be column vectors with \( N = 2^m \), related to the numbers of inputs and outputs, respectively.

The matched filter can be expressed by

\[
    w = Zx
\]

(9)

where \( x \) denotes the input of a sequence, \( w \) the output of the correlation between \( x \) and any sequence, and \( Z \) the matrix given by Eq. (7).

The Sylvester type Hadamard matrix of order \( N \) can be factorized into \( n \) matrices with \( 2N \) nonzero elements, expressed by

\[
    H_n = \left( H_1 \otimes I_1 \otimes I_1 \otimes \cdots \otimes I_1 \right) \times \left( I_1 \otimes H_1 \otimes I_1 \otimes \cdots \otimes I_1 \right) \times \cdots \times \left( I_1 \otimes I_1 \otimes I_1 \otimes \cdots \otimes H_1 \right)
\]

where \( \otimes \) is the Kronecker product defined by

\[
    F \otimes G = \begin{bmatrix} f_1 G & f_2 G \\ f_3 G & f_4 G \end{bmatrix}, \quad F = \begin{bmatrix} f_1 & f_2 \\ f_3 & f_4 \end{bmatrix}
\]

The factorization of the Sylvester type Hadamard matrix can reduce the number of operations or circuit elements from \( O(N^2) \) to \( O(N \log N) \) [33, 34].

Example 4: Let us consider the matched filter bank of ZCZ3 of length \( 2^3 \) given by Eq. (8), generated by

\[
    z_{k}(\tau) = x_0 x_1 \otimes x_0 a_0 \otimes x_1 a_1 \otimes x_0 x_2 \otimes x_2
\]

Table 11  Aperiodic autocorrelation functions of ZCZ3 Z(16, 8, 1)

<table>
<thead>
<tr>
<th>j</th>
<th>z`</th>
<th>C_{j,k}(\tau) (0 \leq \tau \leq 15)</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>ed4</td>
<td>16 -1 2 -3 0 1 -2 -1 0 2 -1 -0 3 2 1 3.20</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>ed12</td>
<td>16 2 3 0 -1 -2 1 0 -1 -2 1 0 3 2 1 3.20</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>de7</td>
<td>16 1 -2 -0 3 2 1 0 -1 -0 3 2 1 -2 1 -1 3.20</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>b72</td>
<td>16 -1 -2 -1 0 3 2 -1 0 1 2 3 0 -1 -2 1 3.20</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>b74</td>
<td>16 -1 -2 -0 3 2 1 0 -1 -0 3 2 1 -2 1 -1 3.20</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>c7b</td>
<td>16 -1 -2 -3 0 1 2 1 0 -1 -2 1 0 3 2 1 3.20</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>d17b</td>
<td>16 -1 -2 -3 -2 0 -1 2 1 -2 0 3 0 1 2 1 3.20</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>b12</td>
<td>16 1 -2 -0 3 2 1 0 -1 -0 3 2 1 -2 1 -1 3.20</td>
<td></td>
</tr>
</tbody>
</table>

The ZCZ code is expressed by

\[
    Z = \left( z_0, z_1, z_2, z_3 \right)^T
\]

\[
    = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}
\]

\[
    \otimes_2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rho_3
\]

where \( \rho_3 = I_3 \) can be omitted. From Eq. (9),

\[
    w = H_2 \sigma_3 (I_2, D_2) \rho_3 x
\]

\[
    = H_2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
    \otimes_2 \begin{bmatrix} x_1 - x_5 \\ x_2 + x_6 \\ x_3 - x_7 \\ x_4 + x_8 \end{bmatrix}
\]

\[
    = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix}
\]

\[
    \otimes_2 \begin{bmatrix} x_1 - x_5 \\ x_2 + x_6 \\ x_3 - x_7 \\ -x_4 + x_8 \end{bmatrix}
\]

\[
    = \begin{bmatrix} x_1 - x_5 \\ x_2 + x_6 \\ x_3 - x_7 \\ -x_4 + x_8 \end{bmatrix}
\]
The matched filter derived from Eq. (10) is shown in Fig. 4, where $z^{-1}$ denotes a delay, $\oplus$ addition and $\Delta$ multiplication by -1.

7. Conclusion

In this paper, a ZCZ code constructed from E-sequences has been proposed, for which any sequence possesses good aperiodic autocorrelation. The ZCZ code has been discussed in terms of its formulated logic function and matrix expression. Furthermore, its generator and matched filter bank were given. The proposed ZCZ code can be applied to CDMA systems.

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