Circuit Theory Based on New Concepts and Its Application to Quantum Theory

6. Transmission Circuit Theory Based on Eigen-Values of Cascade Matrix

Nobuo Nagai¹ (Hokkaido University) and Takashi Yahagi² (Signal Processing Technology Laboratory)

E-mail: ¹nagai@es.hokudai.ac.jp, ²yahagi@risp.jp

Abstract  First, we demonstrate that the eigen-values and corresponding eigen-vectors of the cascade matrix of a lossless reciprocal circuit are the iterative parameters known in classical circuit theory. When these iterative parameters are used, conjugate impedance matching is demonstrated to be a valid matching method of circuit theory. In addition, the iterative impedances at the left and right of an asymmetric circuit are complex conjugates; hence, an impedance-matched network can be obtained by iteratively connecting the same circuits. In other words, the roles of the regularly spaced knots, which were proposed by O. Heaviside and are the bases of the lumped loading coils, are shown to be valid from the theory of iterative parameters. Networks with a periodic structure obtained by iteratively connecting the same circuits are effective for obtaining resonances and eigen-oscillations.

Keywords: lossless and reciprocal circuit, eigen-value of cascade matrix, iterative parameters, passband, stop-band, conjugate impedance matching, iterative connection, periodic structure, evanescent mode, tunneling effect, bandgap

1. Introduction

In Session 5, we focused on a one-dimensional crystal consisting of many molecules and atoms and demonstrated that its electrically equivalent circuit is given by an LC ladder circuit. LC circuits are reciprocal and lossless and can be expressed using a cascade matrix. In Session 5, we also calculated the eigen-values of the cascade matrix and demonstrated that calculating the eigen-values is equal to determining iterative parameters [1]. We also showed that the complex impedance and phase of the LC ladder circuit in the passband can be obtained.

Transmission circuits, such as filters [2], are designed using scattering matrices [2],[3], which are used in circuit theory. Unlike scattering theory [4] used in physics, the scattering matrices are considered to be the best design method in science and technology. However, they also have faults, which may be compensated for by cascade matrices. In this session, we discuss the features of the cascade matrix.

2. Features of Cascade Matrix

In electronics, voltage and current are treated as a pair and satisfy linear equations. In circuit theory, they are expressed using a $2 \times 2$ matrix, for example, a cascade matrix, an impedance matrix, an admittance matrix, or a scattering matrix (S-matrix).

In network (circuit) synthesis and filter design theory, a scattering matrix is used as a general method [2],[3]. When the scattering matrix is used, the reflection and transmission of active power at the input/output terminals are considered, and hence, resistors are used for the input/output terminals. Therefore, only active power is focused on. If the input/output terminals are short-circuited or opened, i.e., the active power at the input/output terminals is 0, the scattering matrix cannot be used.

When an oscillation remains in the circuit even if the active power at the input/output terminals is 0, it is the eigen-oscillation and is considered to be a phenomenon induced by the reactive power alone. That is, the eigen-oscillation cannot be obtained when the scattering matrix is used. When a cascade matrix is used, however, the input/output terminals can be short-circuited or open, and
the complex power can be treated, enabling the determination of the reactive power.

In quantum theory, the stationary state, quantum level, and resonance are treated. They are considered to be related to the oscillation and thereby related to the reactive power. In this session, we use the cascade matrix, instead of the scattering matrix, to examine the application of circuit theory to quantum theory.

In conventional circuit synthesis, both scattering and cascade matrices are used [2],[3]. The cascade matrix used in this case is a positive real matrix expressed as a function of the real coefficient related to the Laplace variable s. Therefore, the relationship between this cascade matrix and the complex factors, such as complex power and reactive power, is not explicitly expressed. Moreover, you may think that the transient response of the positive real matrix can be obtained because the matrix is expressed using s. However, the eigen-values of even the basic section of an LC ladder circuit are expressed using an irrational expression of s; hence, the inverse Laplace transform of the eigen-values cannot be obtained, as shown in Section 5. Therefore, we examine only the steady-state response, instead of the transient response, of the positive real matrix when a cascade matrix is given.

In this lecture series, we discuss the application of circuit theory to quantum theory. Cascade matrices can be obtained from the Schrödinger and Dirac equations used in quantum theory [5]. However, these cascade matrices are not positive real matrices but complex function matrices, requiring the re-examination of the cascade matrices. Note that the unit elements, used as the components of the commensurate transmission line circuits discussed in the previous sessions, are not positive real matrices but complex function matrices.

As discussed in Session 4, the laws of conservation of energy and momentum are both important in mechanics, and heat generation is related to the regulation of circuits in which the two laws hold simultaneously. In contrast, for circuits with unitarity, i.e., lossless lines and circuits in which no heat is generated, the question may arise of how the circuits are regurated. We consider that this question may be answered by considering the reactive power and that the cascade matrix should be used to examine the wave nature of the circuits.

3. Cascade Matrix Representing Lossless Reciprocal Circuit

Resonance has a wave nature, and may also have a particle nature because the complex power concentrated on a resonant circuit has the potential to serve as energy. Through this lecture series, it has been shown that the resonance can be obtained for lossless transmission lines or circuits, which are expressed using a cascade matrix. In the following, we first give a cascade matrix and then demonstrate how a lossless reciprocal circuit is expressed using the cascade matrix.

Two physical quantities, voltage and current, are defined in circuit theory. To describe the relationship between the two quantities, a $2 \times 2$ matrix is required, and a transmission circuit is expressed using the following cascade matrix with complex numbers as its elements.

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix}$$

(6.1)

Here, the positive direction of $I_2$ is from left to right.

A condition required for achieving resonance is that the circuit is lossless. A two-terminal pair circuit having reactances [2] is lossless, and this condition is generalized as follows [6].

[Condition under which cascade matrix represents lossless circuit]

The condition under which a cascade matrix represents a lossless two-terminal pair circuit is given by [6]

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

(6.2)

Under this condition, the cascade matrix is considered to be a J-unitary matrix [6]. By calculating the determinants of the two sides, we obtain

$$|AD-BC|^2 = 1$$

(6.3a)

This can be generally rewritten as

$$AD - BC = \exp(i\beta)$$

(6.3b)

The Tellegen medium and the bi-isotropic medium are known to satisfy Eq. (6.3b) [5],[7] and to serve as lossless lines. When these media are used as a transmission line, the propagation velocity of the forward wave is different from that of the backward wave in a transmission line that satisfies Eq. (6.3b). An example is the case in which a time lag in the propagation of sound is generated that depends on the direction when wind blows at a certain velocity in one direction. For circuits, a difference between the phase of the waves traveling from the source to the load and that from the load to the source is generated in the steady-state.

For a general circuit without a gyrator, a phase difference between the waves traveling in left and right directions is not generated. That is, the circuit is reciprocal and we obtain

$$AD - BC = 1$$

(6.4)

If the circuit is reciprocal, the condition given by Eq. (6.2) is simplified as follows.

[Conditions under which cascade matrix represents lossless reciprocal circuit] [8]

The conditions under which a cascade matrix represents a lossless reciprocal two-terminal pair circuit are that Eq. (6.4) is satisfied and that

$$A \text{ and } D \text{ are real numbers, whereas } B \text{ and } C \text{ are purely imaginary numbers.}$$

(6.5)
4. Eigen-Value Problem and Iterative Parameters of Cascade Circuit

In Ref. [1], iterative parameters are derived from the eigen-value problem of a cascade matrix, but their properties are not accurately explained because of insufficient handling of iterative impedances. In this section, the eigen-values and eigen-vectors of a cascade matrix that represent a lossless reciprocal circuit are determined so that impedance matching can be understood using the iterative parameters. The cascade matrix is expressed using four functions, \( A, B, C, \) and \( D, \) and the four functions obtained from the eigen-values and eigen-vectors of the cascade matrix are iterative parameters. The conversion of the matrix elements into functions that are meaningful in circuit theory is the essence of the theory of iterative parameters.

4.1 Eigen-values and eigen-vectors of cascade matrix representing lossless reciprocal circuit

Here, we determine the eigen-values and eigen-vectors of a cascade matrix representing a lossless reciprocal circuit. The eigen-values are the solutions of the quadratic equation of \( A \) derived from the following determinant.

\[
\begin{vmatrix}
A - \Lambda & B \\
C & D - \Lambda
\end{vmatrix} = 0
\]  \( (6.6a) \)

\[
\therefore \Lambda^2 - (A + D)\Lambda + AD - BC = 0
\]  \( (6.6b) \)

Because Eqs. (6.6a) and (6.6b) do not include the complex conjugates of matrix elements (e.g., \( A^* \)), the properties of the circuit are not completely represented by the condition for lossless circuits alone. Therefore, we assume that the circuit is reciprocal, i.e., Eq. (6.4) holds. Namely, the constant term of the quadratic equation given by Eq. (6.6b) is 1, and the effectiveness of this assumption will be shown later. The eigen-values of the cascade matrix obtained as the solutions of Eq. (6.6b) are given by

\[
\Lambda = \frac{(A + D)}{2} \pm \sqrt{\frac{(A + D)^2 - 4}{4}} \]  \( (6.7a) \)

This is rewritten as

\[
\Lambda_1 = \frac{A + D}{2} + \sqrt{\frac{(A + D)^2 - 4}{2}}
\]  \( (6.7b) \)

\[
\Lambda_2 = \frac{A + D}{2} - \sqrt{\frac{(A + D)^2 - 4}{2}}
\]  \( (6.7c) \)

The eigenvector corresponding to \( \Lambda_1 \) given by Eq. (6.7b) is expressed by

\[
\begin{bmatrix}
Z_{j1} \\
1
\end{bmatrix}
\]  \( (6.8a) \)

Here,

\[
Z_{j1} = \frac{\sqrt{(A + D)^2 - 4}}{2C} + \frac{A - D}{2C}
\]  \( (6.8b) \)

\( Z_{j1} \) is called the iterative impedance of the rightward electromagnetic wave.

The eigen-vector corresponding to \( \Lambda_2 \) given by Eq. (6.7c) is expressed by

\[
\begin{bmatrix}
Z_{j2} \\
-1
\end{bmatrix}
\]  \( (6.9a) \)

Here,

\[
Z_{j2} = \frac{\sqrt{(A + D)^2 - 4}}{2C} - \frac{A - D}{2C}
\]  \( (6.9b) \)

\( Z_{j2} \) is called the iterative impedance of the leftward electromagnetic wave.

The four functions obtained from the eigen-values and eigen-vectors of the cascade matrix, i.e., \( \Lambda_1, \Lambda_2, Z_{j1}, \) and \( Z_{j2}, \) are collectively called the iterative parameters.

Here, we express the cascade matrix given by Eq. (6.1) using the iterative parameters. Because the eigen-values and eigen-vectors of the cascade matrix are given by Eqs. (6.7b), (6.7c), (6.8b), and (6.9b), the cascade matrix is expressed using the iterative parameters as

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = \begin{bmatrix}
Z_{j1} & Z_{j2} \\
1 & -1
\end{bmatrix} \begin{bmatrix}
\Lambda_1 & 0 \\
0 & \Lambda_2
\end{bmatrix} \begin{bmatrix}
Z_{j1} & Z_{j2}
\end{bmatrix}^{-1}
\]

\[
= \frac{1}{Z_{j1} + Z_{j2}} \begin{bmatrix}
\Lambda_1 - \Lambda_2 & Z_{j1}Z_{j2}(\Lambda_1 - \Lambda_2)
\end{bmatrix}
\]  \( (6.10) \)

4.2 Iterative parameters in the passband

Considering the fact that circuits can be used as filters, the waves of some frequencies easily pass through the filters but those of other frequencies pass with difficulty, that is, a passband and a stopband exist. The iterative parameters are useful for separating these bands and thus representing the properties of circuits. In this section, we discuss how the passband is expressed using the iterative parameters.

Here, we assume that a given circuit is lossless and reciprocal. Namely, \( A \) and \( D \) in Eqs. (6.7b) and (6.7c) are real numbers, and therefore, \( (A + D)/2 \) is a real number. If

\[
-1 < \frac{A + D}{2} < 1
\]  \( (6.11a) \)

holds,

\[
\frac{\sqrt{(A + D)^2 - 4}}{2}
\]  \( (6.11b) \)

is purely imaginary and satisfies

\[
\frac{(A + D)^2}{2} + \frac{\sqrt{(A + D)^2 - 4}^2}{2} = 1
\]  \( (6.11c) \)

Therefore, we can assume

\[
\frac{A + D}{2} = \cos \beta_j
\]  \( (6.12a) \)
\[
\sqrt{(A + D)^2 - 4} = \frac{A - D}{2C}
\]

Therefore, \( A_1 \) and \( A_2 \) in Eqs. (6.7b) and (6.7c), respectively, can be expressed by

\[
\Lambda_1 = \exp(j\beta_1)
\]

(6.13a)

\[
\Lambda_2 = \exp(-j\beta_1)
\]

(6.13b)

The vectors in Eqs. (6.13a) and (6.13b) are similar to the exponential functions using the phase constant obtained from lossless telegrapher’s equations [5]. These vectors enable the transmission of waves (in the steady-state), as explained with the unit element; that is, the circuit has a passband. The vectors in Eqs. (6.13a) and (6.13b) are called the pseudorotation vectors [5].

Assuming that the eigen-values are given by Eqs. (6.13a) and (6.13b), we focus on the iterative impedances \( Z_{11} \) and \( Z_{22} \) given by Eqs. (6.8b) and (6.9b), respectively. The second-row first-column element of the cascade matrix, \( C \), is purely imaginary because the circuit is lossless and reciprocal. Hence, the first and second terms of Eqs. (6.8b) and (6.9b) are expressed using the real number \( R \) and the purely imaginary number \( jX \) as

\[
\sqrt{(A + D)^2 - 4} = R_J
\]

(6.14a)

\[
\frac{A - D}{2C} = jX_J
\]

(6.14b)

Using these equations, \( Z_{11} \) and \( Z_{22} \) can be rewritten as

\[
Z_{11} = R_J + jX_J
\]

(6.15a)

\[
Z_{22} = R_J - jX_J = Z_{11}^*
\]

(6.15b)

Thus, \( Z_{11} \) and \( Z_{22} \) are complex conjugates. The features of the passband of the lossless reciprocal circuit are the pseudorotation vectors given by Eqs. (6.13a) and (6.13b) and the iterative impedances that are complex conjugates containing a real part. Note that the circuits treated by the theory of iterative parameters are not symmetric and that their asymmetry is represented by the iterative impedances of the left and right of the circuit, which are not the same but are complex conjugates.

Here, we rewrite the cascade matrix given by Eq. (6.1) using the iterative parameters given by Eqs. (6.13a), (6.13b), (6.14a), and (6.14b). To do this, we assume

\[
\frac{A + D}{2} = \cos \beta_J = c
\]

(6.16a)

\[
\sqrt{(A + D)^2 - 4} = j\sin \beta_J = js
\]

(6.16b)

\[
\sqrt{(A + D)^2 - 4} = R_J
\]

(6.16c)

\[
\frac{A - D}{2C} = jX_J
\]

(6.16d)

The cascade matrix of the circuit (filter) having the passband that satisfies Eq. (6.11a) is expressed by

\[
\frac{1}{R_J} \begin{pmatrix} R_Jc - XJs & js(R_J^2 + X_J^2) \\ js & R_Jc + XJs \end{pmatrix}
\]

(6.17)

\section{4.3 Iterative parameters in the stopband}

The stopband is also considered in the theory of iterative parameters, as the passband and stopband were considered in the theory of image parameters. To examine the properties of the stopband, we assume

\[
\frac{A + D}{2} < -1
\]

(6.18a)

In this case, the following is obtained as the relationship regarding the eigen-values given by Eqs. (6.7b) and (6.7c).

\[
\frac{(A + D)^2}{2} - \sqrt{\left(\frac{(A + D)^2}{2} - 4\right)^2} = 1
\]

(6.18b)

Therefore, we obtain

\[
cosh \gamma_J = \frac{A + D}{2}
\]

(6.19a)

\[
sinh \gamma_J = \sqrt{\left(\frac{(A + D)^2}{2} - 4\right)^2}
\]

(6.19b)

\[
\Lambda_1 = \exp(\gamma_J)
\]

(6.19c)

\[
\Lambda_2 = \exp(-\gamma_J)
\]

(6.19d)

From Eq. (6.18a), \( A_1 \) and \( A_2 \) are negative real numbers. Hence, \( \gamma_J \) in the above equations has the imaginary term \( j(2\pi n + \pi) \).

Here, \( n \) is an integer.

Therefore, \( \gamma_J \) is given by

\[
\gamma_J = j(2\pi n + \pi) + \alpha_J
\]

(6.20a)

When Eq. (6.18) is satisfied, the iterative impedances \( Z_{11} \) and \( Z_{22} \), given by Eqs. (6.8b) and (6.9b), respectively, are purely imaginary numbers and expressed by

\[
Z_{11} = jX_{11}
\]

(6.21a)

\[
Z_{22} = jX_{22}
\]

(6.21b)

Next, we assume

\[
1 < \frac{A + D}{2}
\]

(6.22a)

In this case, the following is obtained as the relationship regarding the eigen-values given by Eqs. (6.7b) and (6.7c).

\[
\frac{(A + D)^2}{2} - \sqrt{\left(\frac{(A + D)^2}{2} - 4\right)^2} = 1
\]

(6.22b)

Therefore, we obtain

\[
cosh \gamma_J = \frac{A + D}{2}
\]

(6.23a)

\[
sinh \gamma_J = \sqrt{\left(\frac{(A + D)^2}{2} - 4\right)^2}
\]

(6.23b)

\[
\Lambda_1 = \exp(\gamma_J)
\]

(6.23c)

\[
\Lambda_2 = \exp(-\gamma_J)
\]

(6.23d)
From Eq. (6.22a), $A_1$ and $A_2$ are positive real numbers. Hence, the imaginary part of $\gamma_J$ can be 0. Because Eqs. (6.13) and (6.20) have an imaginary term, however, $\gamma_J$ is assumed to have the imaginary term

$$j(2\pi n)$$

Here, $n$ is an integer.

Therefore, $\gamma_J$ is given by

$$\gamma_J = j(2\pi n) + \alpha_J$$

When Eq. (6.22a) is satisfied, the iterative impedances $Z_{1n}$ and $Z_{2n}$, given by Eqs. (6.8b) and (6.9b), respectively, are purely imaginary numbers and expressed by

$$Z_{1n} = jX_{1n}$$

$$Z_{2n} = jX_{2n}$$

Here, we discuss the stopband when Eqs. (6.18a) and (6.22a) are satisfied. The circuit treated here is lossless but has the stopband, where the attenuation constant is $\alpha_J$ as given in Eqs. (6.20b) and (6.24b).

The attenuation constant of transmission lines represents the attenuation of electromagnetic waves caused by the generation of heat at resistors. However, a lossless circuit is considered here, and no heat is generated in the circuit. What then does the attenuation constant of the lossless circuit represent? The answer is related to the impedance of the periodic structure obtained by the theory of resonances and eigen-oscillations in the passband. Thus, achieve conjugate matching or increase the number of resonances and eigen-oscillations. Namely, the role of the regularly spaced knots proposed by Heaviside [10] is to construct a circuit with a periodic structure, that is, to achieve conjugate matching or increase the number of resonances and eigen-oscillations in the passband. Thus, the periodic structure can be understood by the theory of iterative parameters.

6. A Feature of Cascade Matrix of Lossless Reciprocal Circuit

For crystals consisting of molecules or atoms (for example, two types of atom with different masses) [11], the passband is divided into the first and second Brillouin zones, or the bandgap exists in terms of quantum mechanics [12]. These phenomena are considered to be caused by the presence of the stopband discussed in Section 4.3 from the viewpoint of circuit theory. In this section, we discuss when the stopband of the cascade matrix appears.

Here, the conditions for the cascade matrix of a lossless reciprocal circuit, as given by Eqs. (6.4) and (6.5), are provided again.

$$AD - BC = 1$$

(6.28)

$A$ and $D$ are real numbers, whereas $B$ and $C$ are purely imaginary numbers.

(6.29)

The condition for the passband was given by Eq. (6.11a), which is also provided again.

$$-1 < \frac{A + D}{2} < 1$$

(6.30)

As a point to be noted, when the signs of the purely imaginary numbers $B$ and $C$ are different, we obtain

$$BC > 0$$

(6.31)

From Eqs. (6.28) and (6.31), we obtain

$$AD = 1 + BC > 1$$

(6.32)

When the signs of $A$ and $D$ are different, Eq. (6.32) does not hold. When the signs of $A$ and $D$ are the same, the following two equations hold.
In this case, Eq. (6.30) does not hold and the passband does not exist. Namely, when the signs of $B$ and $C$ are different, the stopband appears instead of the passband.

For general LC circuits without attenuation poles, all elements of the cascade matrix are expressed by polynomials of $\omega$. Therefore, the difference in the signs of the polynomials causes the zero-point problem of the polynomials. The purely imaginary numbers $B$ and $C$ are expressed using the sinusoidal functions within the passband, as given by Eq. (6.17). Because $\sin(n\pi) = 0$, the zero point of the polynomial is equal to the phase $n\pi$. If $B$ and $C$ of the cascade matrix are expressed by different polynomials, the zero points of the polynomials differ. The point near the zero point represents the iterative phase near $n\pi$, increasing the possibility that the stopband may appear at an iterative phase of $n\pi$. For example, when crystals consisting of atoms of different masses are iteratively connected as shown in Session 5, the stopband appears for an iterative phase of $n\pi$, the passband appears separately, and a bandgap appears.

As discussed in Section 4.3, the evanescent mode and the tunnel effect correspond to the stopband. In these cases, Eqs. (6.18a) and (6.22a) are satisfied even when the signs of $B$ and $C$ are the same.

7. Non-Periodic Structures

The passband of a periodic structure obtained by iteratively connecting $n$ basic sections is the same as that of the basic section. Because the iterative phase increases $n$-fold, the number of points with an iterative phase of $n\pi$ increases, as explained in Session 5, and the number of resonances increases. In this section, we focus on the features of a network with a nonperiodic structure obtained by cascade-connecting many basic sections and adding a circuit different from the basic section.

The LC circuit shown in Fig. 6.1 is assumed to be the basic section.

![Fig.6.1 Basic section (Asymmetric LC circuit)](image)

Figure 6.2 shows an LC ladder network in which four circuits (the basic section shown in Fig. 6.1) are cascade-connected (iteratively connected) and a series component $L$ is connected to the right of the ladder network. The cascade matrix of this ladder network is determined as

$$
\begin{pmatrix}
A_s & B_s \\
C_s & D_s
\end{pmatrix} = \left(1 - \omega^2 LC \right) \begin{pmatrix}
1 & j\omega L \\
j\omega C & 1
\end{pmatrix} \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix} (6.34)
$$

The ladder network shown in Fig. 6.2 is not periodic but is symmetric. For symmetric circuits, the elements of the cascade matrix $A_s$ and $D_s$ are equal. Therefore, the functions that represent the cascade matrix are $A_s$, $B_s$, and $C_s$. From Eqs. (6.8b) and (6.9b), the resistance in the passband is given by

$$
Z_{j1} = Z_{j2} = \frac{\sqrt{A_s^2 - 1}}{C_s} = R_e (6.35)
$$

The three parameters obtained from the eigen-values and eigen-vectors of the cascade matrix of the symmetric circuit are denoted as $A_1$, $A_2$, and $R_e$ and are collectively called the semi-image parameters (although the circuit is not image-connected).

![Fig.6.3 Semi-image phase $\beta_e$ in the passband of the symmetric ladder network shown in Fig. 6.2](image)
The semi-image parameters of the ladder network shown in Fig. 6.2 are determined and illustrated as follows. Figure 6.3 shows the region of the passband in which the semi-image phase $\beta_e$ is determined. In contrast to the complex iterative impedance, the semi-image impedance is a real number because of the symmetric ladder network, and the semi-image resistance $R_e$ can be determined for the passband, as shown in Fig. 6.4. In this figure, the cutoff frequency $\left(2/\sqrt{LC}\right)$ of the basic section shown in Fig. 6.1 corresponds to 2 on the horizontal axis.

The passband shown in Fig. 6.3 is narrower than that shown in Fig. 5.6 of Session 5. The eigen-values are real numbers for the angular frequencies corresponding to $\beta_e$ of $\pi$, $2\pi$, and $3\pi$, which start at a value of $\omega$ of nearly 0.6, 1.2, and 1.6 (cutoff angular frequency, 2), respectively, indicating that the stopband appears at these angular frequencies. For the periodic structure, the passband appeared at these angular frequencies. By connecting a series component $L$ to the ladder network to give it a nonperiodic structure, multiple reflections occur during the transient response and the multiple-reflected waves become out of phase, changing the passband to the stopband, which corresponds to the bandgap.

As shown in Fig. 6.4, the semi-image resistance $R_e$ in the passband is discontinuous and changes from $\infty$ to 0. This is because the semi-image impedance of the stopband is generated because of the nonperiodic structure, and it is not a real number (resistance) but a purely imaginary number (reactance).

**8. Summary**

As explained in Ref. [2], the theory of the construction of filters started from the theory of image parameters proposed by O. J. Zobel. Because of this, classical circuit theory is considered to be the theory of image parameters, which is used as a method of designing high-order filters. Moreover, two image resistances can be obtained and impedance matching can be easily understood with this theory. Therefore, the theory of image parameters is considered to be an easy way to understand circuit theory.

In contrast, the theory of iterative parameters [1], one of the classical circuit theories, gives two different iterative impedances, the meanings of which are difficult to understand, and thus it has not been discussed deeply. As shown in this session, analyzing the theory of iterative parameters as the eigen-value problem of the cascade matrix yields two iterative impedances as the complex conjugates in the passband. Thus, conjugate impedance matching is proven to be a valid matching method of circuit theory. When the same circuits are iteratively connected to construct a network with a periodic structure, impedance matching occurs and no waves are reflected within the structure in the steady-state. This means that the numbers of resonances and eigen-oscillations increase. The roles of the regularly spaced knots, which were proposed by Heaviside [10] and are the bases of the lumped loading coils, can be explained using the theory of iterative parameters.

The theory of image parameters is useful for obtaining the resonance. We will further discuss the theory of image parameters by comparing it with the theories of iterative parameters and semi-image parameters.

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**References**

Nobuo Nagai received his B.S. and D.Eng. degrees from Hokkaido University in 1961 and 1971, respectively. In 1961, he joined Hokkaido University as an Assistant and in 1972 he became an Associate Professor, and from 1980 to 1992 he was a Professor in the Research Institute of Applied Electricity. From 1992 to 2001, he was a Professor in the Research Institute for Electronic Science, Hokkaido University. In 2001, he retired and became an Emeritus Professor. His research interests are circuit theory and digital signal processing. He is interested in the application of above theory to quantum theory. Dr. Nagai is a Life Fellow of the Institute of Electronics, Information and Communication Engineers, Japan.

Takashi Yahagi received his B.E., M.S. and Ph.D. degrees all from the Tokyo Institute of Technology in 1966, 1968 and 1971, respectively. In 1971, he joined Chiba University as a Lecturer and in 1974 he became an Associate Professor, and from 1984 to 2008 he was a Professor at the same university. Since 2008 he has been with the Signal Processing Research Laboratory. In 1997, he founded the Research Institute of Signal Processing, Japan (RISP). Since 1997 he has been President of RISP. From 1997 to 2013 he was Editor-in-Chief of the Journal of Signal Processing (JSP). Since 2013 he has been Honorary Editor-in-Chief of JSP. He was the author of “Theory of Digital Signal Processing (Vols. 1-3)”, (1985, 1985, 1986), Corona Pub.Co., Ltd. (Tokyo, Japan). He was also the editor and author of “Library of Digital Signal Processing (Vols. 1-10)”, (1996, 2001, 1996, 2000, 2005, 2008, 1997, 1999, 1998, 1997), Corona Pub.Co., Ltd. (Tokyo, Japan). He was the editor of “My Research History (Vols. 1 and 2)” (2003, 2003), RISP. Dr. Yahagi is a Life Fellow of the Institute of Electronics, Information and Communication Engineers, Japan.