Circuit Theory Based on New Concepts and Its Application to Quantum Theory

7. Wave Digital Filters for Commensurate Transmission Line Circuits

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Abstract: Because commensurate transmission line circuits consisting of unit elements (UEs) are expressed using a z-transform, wave digital filters (WDFs) and a transient phenomenon can be obtained. There are two types of WDF, i.e., voltage and current WDFs, and hence the reactive power in the steady state can also be obtained. Moreover, normalized WDFs can be obtained using ideal transformers. By applying the construction method for WDFs to resonant tunneling diodes (RTDs), RTDs can be expressed similarly to WDFs. We show that physical phenomena defined as the wave function of quantum wells and the existence probability of electrons correspond to the voltage standing wave generated along the UEs of WDFs. Note that RTDs can be resonant even if the standing wave exists in the steady state.

Keywords: wave digital filter (WDF), resonant tunneling diode (RTD), digital lattice filter (DLF), reflection and transmission, resonant, reformulation of Maxwell's theory, unit element (UE), backward wave, normalized WDF, voltage standing wave, wave packet

1. Introduction

As mentioned in Session 1, the purpose of this lecture series is to apply circuit theory to quantum mechanics. In the previous sessions, we have mainly discussed cascade matrices, rather than scattering matrices, so that we can treat commensurate transmission line circuits [1] and reactive power, which are considered to be the key issues in the application of circuit theory.

There are two types of circuit response: transient and steady-state. Lumped constant circuits, such as usual LC circuits, are expressed using a Laplace transform but not a z-transform. Therefore, lumped constant circuits cannot be used to express transient phenomena. In contrast, commensurate transmission line circuits can be expressed using a z-transform, as described in Session 3, and can be used to obtain a transient response.

In this session, commensurate transmission line circuits are expressed as wave digital filters (WDFs) [2]. WDFs are considered to be suitable for use in circuit theory because there are voltage and current WDFs. Reactive power can be treated using these WDFs. Here, filters consisting of cascade-connected unit elements (UEs) alone are called bar-type filters [1] and are expressed as digital lattice filters (DLFs) [3]. We demonstrate that conventional DLFs are obtained by normalizing the voltage WDFs. However, current WDFs are also required to obtain reactive power, and DLFs alone would be insufficient for use in circuit theory. As an example of an application, we also show that resonant tunneling diodes (RTDs) [4] are expressed in the same way as DLFs.

2. Application of WDFs to Quantum Electrodynamics

Commensurate transmission line circuits [1] are considered to be important for discussing the development of physics theory from Maxwell’s equations to quantum mechanics. Therefore, such circuits are explained as follows.

In Ref. 5, the strange behavior of light particles and strange interactions between photons and electrons are described to explain quantum electrodynamics. We showed that the reflection and transmission of photons regarded as strange behavior in Ref. 5 are very similar to those used in circuit theory, demonstrating that circuit theory may be applicable to quantum electrodynamics [6]. Moreover, we consider that the behavior of light includes phenomena that can be explained by resonance. By showing that the
resonant phenomena of photons are similar to the resonance of RTDs [4], we attempt to apply circuit theory to discuss the existence probability and wave function of RTDs.

To explain the operation of RTDs, we need to apply circuit theory to the Schrödinger equations. This application is explained in Ref. 6, which is referred to in this session. Details will be explained in later sessions of this lecture series. Here, we discuss the existence probability and wave function, as defined in quantum mechanics, in quantum wells of RTDs on the basis of circuit theory.

Nahin explained Maxwell’s equations as follows [7]. For example, Feynman wrote: “In the general theory of quantum electrodynamics, one takes the vector and scalar potentials (A) as the fundamental quantities. E (electric field) and B (magnetic field) are slowly disappearing from the modern expression of physical laws....” Heaviside strongly rejected A, writing in an 1888 letter to Oliver Lodge of his desire to “murder” Maxwell’s “monster” (A).

The behavior of light discussed by Feynman [5] as an example of quantum electrodynamics is the reflection and transmission of photons. Reflection and transmission coefficients are physical quantities obtained when voltage and current, as well as the ratio of voltage to current, i.e., impedance, are defined. These quantities are not determined by potential (A). Voltage and current defined by the telegrapher’s equations correspond to an electric field (E) and a magnetic field (B) defined by Maxwell’s equations, respectively. This means that in Ref. 5, Feynman used the reformulation of Maxwell’s theory, which was proposed by Heaviside [7] and expresses the functions of electric and magnetic fields.

Because the reformulation of Maxwell’s theory proposed by Heaviside assumes three-dimensional space, applying circuit theory to the reformulated Maxwell’s theory is difficult. In contrast, the telegrapher’s equations are equivalent to the equations used in the reformulated Maxwell’s theory and assume one-dimensional space; hence, the application of circuit theory to the telegrapher’s equations is easy. Because of this, the application of circuit theory to quantum electrodynamics was made possible by using commensurate transmission line circuits [1] derived from the telegrapher’s equations. This indicates that quantum electrodynamics can be explained by applying the reformulated Maxwell’s theory that expresses the functions of electric and magnetic fields. We verify the validity of the reformulation of Maxwell’s theory proposed by Heaviside using an RTD as an example.

3. DLF for Circuit Consisting of UEs

Although Feynman [5] clearly stated that the reflection of photons is a probabilistic phenomenon, the reflection coefficient of light is a constant that depends on the dielectric constants of the two media and is a physical constant rather than a probability. In addition, the reflection coefficient is defined on the basis of the impedances of the two media and can thereby be explained by circuit theory in which voltage and current are defined. In short, the reflection and transmission coefficients are obtained by defining two functions, voltage and current.

In relation to the reflection and transmission of light, refraction is a well-known phenomenon. In Ref. 5, Feynman considered the case in which light entered glass perpendicularly and no refraction was observed. Therefore, the reflection and transmission caused by the difference in the dielectric constant were discussed in Ref. 5. The reflection and transmission coefficients of light are determined by the dielectric constant, which represents the conductance as the inverse of resistance. Therefore, the reflection and transmission of light are caused by the difference in impedance of two media.

Similarly to circuit responses, there are two types of reflection and transmission: transient and steady-state. Complex impedance represents the impedance in the steady state and can be used to express reflection and transmission in the steady state. When two different resistors or conductors are connected, instantaneous reflection and transmission coefficients that can be applied to a transient response are obtained. WDFs [2] and DLFs [3] are the techniques for representing the circuit state in terms of reflection and transmission.

Commensurate transmission line circuits consisting of cascade-connected UEs alone are called bar-type circuits [1] and can be expressed as not only the voltage and current WDFs but also the DLFs [3]. Although DLFs can be considered as a type of WDF, conventional DLFs [3] can actually be considered as normalized voltage WDFs. Note that Feynman’s method [5] corresponds to the DLF of normalized waves.

In this session, we attempt to construct two types of WDF, i.e., voltage and current WDFs, using the construction method of DLFs. To this end, we determine the reflection and transmission coefficients for voltage and current waves and construct voltage and current WDFs. Subsequently, normalized waves for the two WDFs are obtained to construct normalized voltage and current WDFs. By this method, we demonstrate that conventional DLFs are normalized voltage WDFs and also show that the current wave is not considered in the conventional method.

3.1 Reflection and transmission along UEs

In Sessions 5 and 6, we showed that resonance occurs in one-dimensional crystals. Resonance also occurs in circuits consisting of UEs, as shown in Sessions 2 and 3; however, the behavior of voltage and current along UEs was not discussed. Here, we discuss the phenomena that occur in UEs while resonance is taking place.
Numerical examples of resonance and a transient phenomenon that occurs until resonance is achieved were given in Session 3. Figure 7.1 shows a circuit obtained by generalizing the examples. In this figure, the input/output resistances and the characteristic resistance of the UE in the circuit shown in Fig. 3.1 (see Session 3) are denoted as $R_a$ and $R_0$, respectively.

The voltage and current along the UE were expressed by Eqs. (3.4a) and (3.4b), respectively, in Session 3 and are again given here.

$$V(x) = K_a(s)\exp(-su^{-1}x) + K_b(s)\exp(su^{-1}x) \quad (7.1a)$$

$$I(x) = \frac{K_a(s)}{R_a}\exp(-su^{-1}x) - \frac{K_b(s)}{R_0}\exp(su^{-1}x) \quad (7.1b)$$

By substituting the boundary conditions of the circuit shown in Fig. 7.1 into the above equations, two constants of integration, $K_a(s)$ and $K_b(s)$, can be obtained. As expressed by the second terms of the right-hand sides of Eqs. (7.1a) and (7.1b), the reflected voltage and current waves, in addition to the incident voltage and current waves, travel along the UE. That is, multiple reflected voltage and current waves travel in a loop. Therefore, the voltage and current waves are not expressed by the simple reflection and transmission coefficients. In the following, we examine the method of constructing a WDF by determining the reflection and transmission coefficients of a circuit in which a load resistor is connected to a voltage source (with nonzero internal resistance).

3.2 Reflection and transmission at voltage source

Complex impedance represents the steady state of a circuit. To examine the transient response, we should assume a circuit consisting of a resistor and a voltage source alone. Using such a circuit, we examine the maximum available power of the voltage source and impedance matching at the resistor and determine the reflection and transmission coefficients of voltage and current waves.

Assume the circuit shown in Fig. 7.2. The voltage source is the source of the circuit and is located at the left end. The internal resistance and voltage of the voltage source are denoted as $R_G$ and $E_G$, respectively. Note that resistance is generally non-negative. A load resistor (resistance, $R_R$) is connected to the voltage source.

The current that flows through the circuit, $I$, and the voltage across the load resistor, $V$, are respectively given by

$$I = \frac{E_G}{R_R + R_G} \quad (7.2a)$$

$$V = \frac{E_GR_R}{R_R + R_G} \quad (7.2b)$$

Therefore, the power at the load resistor $P$ is given by

$$P = \frac{E_G^2R_R}{(R_R + R_G)^2} \quad (7.2c)$$

To maximize $P$, $R_R$ should be changed to equal $R_G$, that is,

$$R_R = R_G \quad (7.3)$$

This equation represents impedance matching, and the maximum active power $P_i$ is given by

$$P_i = \frac{E_G^2}{4R_G} \quad (7.4)$$

This is called the maximum available power [8], which is the maximum power supplied from the voltage source to the load only when impedance matching is achieved. In other cases, the power supplied from the voltage source to the load is always less than the maximum available power.

3.3 Reflection and transmission coefficients

Considering that the power transmitted from a source to a load depends on the load resistance $R_R$, we assume that voltage, current, and power are reflected and transmitted. Namely, we consider the incident, reflected, and transmitted waves for voltage, current, and power.

Because it is ideal to deliver as much incident power to the load as possible, the maximum available power given by Eq. (7.4) is considered to be the incident power. The voltage and current at which the maximum available power is delivered to the load should be defined as the incident voltage $V_i$ and incident current $I_i$, respectively. $V_i$ and $I_i$ for the circuit shown in Fig. 7.2 are given by
Both the voltage and current reflection coefficients of the circuit are denoted as $r_b$ and given by [8]

$$ r_b = \frac{R_g - R_o}{R_g + R_o} \quad (7.6) $$

The transmission coefficient is given as the ratio of the amplitudes of the transmitted wave to the incident wave. The voltage and current transmission coefficients are considered to differ because the voltage and current are given by different equations [Eqs. (7.2b) and (7.2a), respectively]. Using $V_i$ given by Eq. (7.5b) and $I_i$ given by Eq. (7.5a), the voltage and current transmission coefficients, $t_v$ and $t_i$, respectively, are determined by

$$ t_v = \frac{V}{V_i} = \frac{2R_g}{R_g + R_o} = 1 + r_b \quad (7.7a) $$

$$ t_i = \frac{I}{I_i} = \frac{2R_o}{R_g + R_o} = 1 - r_b \quad (7.7b) $$

### 3.4 Analysis of circuit consisting of UEs

Here, we construct a WDF for the circuit shown in Fig. 7.1. In Fig. 7.1, the voltage and current waves are reflected and transmitted at port-1 and port-2. Referring to Eq. (7.6) and Fig. 7.2, the reflection coefficients at port-1 and port-2, $r_{1b}$ and $r_{2b}$, respectively, are given by

$$ r_{1b} = \frac{R_o - R_{in}}{R_o + R_{in}} \quad (7.8a) $$

$$ r_{2b} = \frac{R_o - R_{in}}{R_o + R_{in}} = -r_{1b} \quad (7.8b) $$

An important phenomenon in a circuit consisting of UEs is that voltage and current waves are repeatedly reflected, i.e., multiple reflections occur. The sign of the reflected voltage wave is different from that of the reflected current wave, as shown in Session 1. Concretely, the reflected voltage wave is positive, whereas the reflected current wave is negative. The reflected waves travel to the left and are again reflected and transmitted. The reflection coefficients of the leftward and rightward waves are of the same magnitude but of opposite sign, as shown in Ref. 5.

One more thing to note is that the sign of the reflection coefficient of the rightward wave at port-1 is opposite to that at port-2. Therefore, all the reflection and transmission coefficients can be expressed using $r_{1b}$. Considering the above, the voltage WDF for the circuit shown in Fig. 7.1 is represented by Fig. 7.3.

A current WDF can be obtained by using the voltage reflection coefficient of the opposite sign. The current WDF for the circuit shown in Fig. 7.1 is represented by Fig. 7.4.

### 4. Normalized WDF

There are WDFs for normalized waves, which are expressed using ideal transformers. In circuit diagrams, the normalized resistance is set to 1 Ω. However, the square root of 1 is also 1, which is inconvenient when handling equations. Here, the normalized resistance is denoted as $R_a$. Figure 7.5 shows the circuit obtained by rewriting the circuit shown in Fig. 7.1 for normalized waves using $R_a$ and ideal transformers.

The reflection coefficients of the WDF in Fig. 7.5 are the same as those of the WDFs shown in Figs. 7.3 and 7.4. However, the transmission coefficients depend on the ideal
transformers. Referring to Eq. (7.7a) obtained from the circuit shown in Fig. 7.2, the voltage transmission coefficient at port-1 of the circuit shown in Fig. 7.1 is expressed by

\[ t_v = \frac{2R_o}{R_o + R_o} = 1 + r_{ib} \]  

(7.9)

The voltage transmission coefficient depends on the ideal transformers as follows.

\[ \sqrt{\frac{R_o}{R_o + R_o}} = \sqrt{\frac{2R_o \sqrt{2R_o}}{R_o + R_o}} = \sqrt{(1 + r_{ib})(1 - r_{ib})} \]

(7.10)

The voltage transmission coefficient at port-2 can be similarly calculated to be \( \sqrt{1 - r_{ib}^2} \), as given by Eq. (7.10).

Therefore, the voltage WDF for the circuit shown in Fig. 7.5 is represented by Fig. 7.6. Note that Fig. 7.6 shows a conventional filter called the DLF [3], which is the filter for the normalized voltage wave.

![Diagram of Normalized Voltage Wave Digital Filter](image1)

Fig.7.6 Normalized voltage wave digital filter (normalized voltage WDF) for the circuit shown in Fig.7.5

Let us stop to examine the phenomenon in more detail. As we mentioned before, circuit responses are of two types, transient and steady-state, and the transient response can be obtained for circuits that can be expressed using a z-transform. In practice, the reflection and transmission coefficients of the WDFs shown in Figs. 7.3 and 7.4 are instantaneous coefficients, meaning that the transient response is obtained. You may think that the transient response can also be obtained for the filter shown in Fig. 7.6, which is expressed using a z-transform, similarly to the filter shown in Fig. 7.3. Here, we should consider the following.

The reason why we should reconsider is described as follows. The circuit shown in Fig. 7.5 is the base for the filter shown in Fig. 7.6 and has ideal transformers. As explained in Session 2, when ideal transformers are used, voltage and current do not continuously change but change discontinuously. This change is an ultimate operation using infinity \( \propto \) and cannot be considered to be instantaneous. Therefore, the WDFs of the circuit having ideal transformers are represented using digital elements that are neither instantaneous nor in the steady state. We refer to such digital elements of the ideal transformers as semi-instantaneous elements [9]. The normalized WDF shown in Fig. 7.6 uses semi-instantaneous elements and may need more time until the circuit reaches the steady state than the filter shown in Fig. 7.3.

In the WDFs, backward waves are generated upon reflection, making the simulation of the transient response difficult. You can now understand how difficult it is to obtain the transient response of usual circuits such as lumped constant circuits.

We return to the original topic. There are two types of WDF, voltage and current WDFs. Because the normalized voltage WDF was obtained as above, the normalized current WDF should also be obtained. Namely, the normalized current WDF shown in Fig. 7.4 can be obtained. Here, attention should still be paid to the transmission coefficient. Referring to Eq. (7.7b) obtained from the circuit shown in Fig. 7.2, the current transmission coefficient at port-1 of the circuit shown in Fig. 7.1 is expressed by

\[ t_i = \frac{2R_o}{R_o + R_o} = 1 - r_{ib} \]  

(7.11)

The current transmission coefficient depends on the ideal transformers as follows.

\[ \sqrt{\frac{R_o}{R_o + R_o}} = \sqrt{\frac{2R_o \sqrt{2R_o}}{R_o + R_o}} = \sqrt{(1 + r_{ib})(1 - r_{ib})} \]

(7.12)

Thus, the current transmission coefficient is equal to the voltage transmission coefficient. Moreover, the current transmission coefficient at port-2 can be similarly calculated to be \( \sqrt{1 - r_{ib}^2} \), as given by Eq. (7.12). In summary, for the normalized waves, the voltage transmission coefficients are the same as the current transmission coefficients. However, the current WDF for the circuit shown in Fig. 7.5 has reflection coefficients of the opposite sign from those of the voltage WDF, as shown in Fig. 7.7.

![Diagram of Normalized Current Wave Digital Filter](image2)

Fig.7.7 Normalized current wave digital filter (normalized current WDF) for the circuit shown in Fig.7.5
Fig. 7.8 (a) Potential barrier, (b) absolute value of transfer wave function (probability amplitude) for energy

5. An Example from Resonant Tunneling Diodes

RTDs [4] are the quantum elements that should be examined after discussing the Schrödinger equation used in quantum mechanics. The tunneling effect involves the reflection and transmission of waves [10] and can be examined by applying circuit theory [6]. The method described in Ref. 6 is explained in the following.

Because quantum mechanics treats the wave nature of electrons, there are phenomena characteristic to quantum mechanics, one of which is the tunneling effect. Electrons have characteristic energy and are generally not considered to pass through a potential barrier higher than the energy. However, electrons with wave nature can pass through such a barrier, which is known as the tunneling effect.

Fig. 7.9 Potential structure and characteristic for a resonant tunneling diode (RTD): (a) Potential structure, (b) Absolute value of transfer wave function for energy, (c) Absolute value of wave function at resonant tunneling energy level $E_1$. 

The tunneling effect occurs for a quantum barrier width of up to 20–30 Å. Figures 7.8(a) and 7.8(b) show a potential barrier with a width of 20 Å and the absolute value of the transmission probability amplitude for electrons with respect to the quantum barrier energy $E$, respectively.

When two potential barriers are used, a resonant phenomenon occurs, which is known as the resonant tunneling effect. Assume an RTD with a double-barrier quantum well consisting of two 20-Å-wide quantum barriers that are 50 Å apart from each other. Figures 7.9(a)–7.9(c) show the potential structure, the absolute value of the transmission probability amplitude $T_{br}$, $|T_{br}|$, and the absolute value of the wave function of the energy $E_1$ of the electrons that undergo the resonant tunneling effect, $|\Psi|$, for the RTD, respectively.

The phenomenon shown in the figures seems very strange. For $E_1$, at which the resonant tunneling effect occurs, the absolute value of the transmission probability amplitude is approximately 0.182 $\approx 2/11$ when only one quantum barrier is used. When two quantum barriers are used, the transmission probability amplitude does not decrease but is surprisingly 1, implying the occurrence of resonance. This quantum phenomenon is clarified using an equivalent circuit as follows.

In Ref. [6], the equivalent circuit of an RTD is obtained. The transmission probability amplitude corresponds to the semi-instantaneous transmission coefficient $T_{br}$ of the normalized voltage WDF shown in Fig. 7.6. Therefore, we obtain

$$\sqrt{1 - r_{lb}^2} = \frac{2}{11} \tag{7.13}$$

$$\therefore r_{lb} = \sqrt{1 - \left(\frac{2}{11}\right)^2} = \sqrt{1 - \frac{4}{121}} \tag{7.14a}$$

The reflection coefficient is given by a fraction [Eq. (7.6)]. When Eq. (7.14a) is expressed by an approximation, we obtain

$$r_{lb} \approx \sqrt{\frac{1 - \frac{2}{121}}{1 + \frac{2}{121}}} \approx \sqrt{\left(\frac{1 - \frac{1}{121}}{1 + \frac{1}{121}}\right)^2} \tag{7.14b}$$

$$\therefore r_{lb} = \frac{121 - 1}{121 + 1} \tag{7.14c}$$

Here, $\approx$ denotes ‘approximately equal to’.

By comparing the reflection coefficients given by Eqs. (7.14c) and (7.8a), a circuit with input/output resistances of 1 Ω and a UE characteristic resistance of 121 Ω can be obtained. Figure 7.10 shows a circuit whose reflection coefficient is given by Eq. (7.14c) and which achieves resonance.

For the circuit of the RTD shown in Fig. 7.10, the UE characteristic resistance is 121 Ω at $E_1$ and not equal to the normalized resistance (1 Ω). To modify this, ideal transformers with a transformation ratio of $\sqrt{121} = 11$ are required, as shown in Fig. 7.11.

Which part of the equivalent circuit shown in Fig. 7.11 corresponds to the absolute value of the wave function [Fig. 7.9(c)] of the RTD shown in Fig. 7.9(a)? When focusing on the circuit shown in Fig. 7.10, for example, the load resistance and UE characteristic resistance are 1 and 121 Ω, respectively, and a standing wave exists along the UE because of impedance mismatching. Similarly, a standing wave exists in the circuit shown in Fig. 7.11 because of impedance mismatching. The voltage standing wave along the UE of the circuit shown in Fig. 7.11 represents the absolute value of the wave function shown in Fig. 7.9(c).

In quantum mechanics, the square of the wave function is defined as the existence probability of electrons. The voltage standing wave along the UE of the circuit shown in Fig. 7.10 represents the existence probability. Therefore, the voltage along the equivalent circuit of the RTD corresponds to the wave function defined in quantum mechanics.

Moreover, the wave function of resonance in the RTD shown in Fig. 7.9(c) can be considered as a wave packet. In
this case, the wave packet consists of resonant waves and, when moving, the wave packet can be considered as a bundle of energy, similar to a tsunami, because the resonant waves do not disperse. If a physical phenomenon that satisfies the natures of waves and particles is assumed to be resonant, a resonant wave packet is not a bundle of waves of different frequencies that can be expressed using a Fourier series but a bundle of waves of the same frequency. Therefore, the wave packet moves as a bundle of energy at a constant velocity without dispersion.

6. Summary

Thus far, we have shown that resonance can be obtained using UEs. The process that occurs until the resonance is achieved, i.e., a transient phenomenon, was described in Session 3. In the transient phenomenon, the wave is reflected at the load and returns to the source along the UE as a backward wave, which plays an important role in the analysis and construction of circuits. Analysis of a circuit to clarify the existence of the reflected wave is required and such a circuit is realized as a WDF, as explained in this session. For the WDFs consisting of cascade-connected UEs alone, the transmission coefficient is determined when the reflection coefficient is determined. The reactive power generated along the UE in the steady state can be determined using voltage and current WDFs. Therefore, by using both the voltage and current WDFs shown in Figs. 7.6 and 7.7, respectively, the power-of-a-point theorem shown in Fig. 2.3 (see Session 2) can be used and the active and reactive powers in Fig. 3.3 (see Session 3) can be determined. The waveform obtained for resonance can be considered as a wave packet, indicating that the meaning of the wave packet is different from the original meaning.

RTDs are quantum elements using electron waves and are known to undergo reflection and transmission [10]. By applying circuit theory to the reflection and transmission in an RTD, we showed that the wave function in the RTD corresponds to the voltage standing wave of the normalized WDF and that the square of the wave function, defined as the existence probability of electrons, corresponds to the voltage standing wave of the normalized WDF. Thus far, we have shown that resonance can be obtained using UEs. The process that occurs until the resonance is achieved, i.e., a transient phenomenon, was described in Session 3. In the transient phenomenon, the wave is reflected at the load and returns to the source along the UE as a backward wave, which plays an important role in the analysis and construction of circuits. Analysis of a circuit to clarify the existence of the reflected wave is required and such a circuit is realized as a WDF, as explained in this session. For the WDFs consisting of cascade-connected UEs alone, the transmission coefficient is determined when the reflection coefficient is determined. The reactive power generated along the UE in the steady state can be determined using voltage and current WDFs. Therefore, by using both the voltage and current WDFs shown in Figs. 7.6 and 7.7, respectively, the power-of-a-point theorem shown in Fig. 2.3 (see Session 2) can be used and the active and reactive powers in Fig. 3.3 (see Session 3) can be determined. The waveform obtained for resonance can be considered as a wave packet, indicating that the meaning of the wave packet is different from the original meaning.

RTDs are quantum elements using electron waves and are known to undergo reflection and transmission [10]. By applying circuit theory to the reflection and transmission in an RTD, we showed that the wave function in the RTD corresponds to the voltage standing wave of the normalized WDF and that the square of the wave function, defined as the existence probability of electrons, corresponds to the voltage standing wave of the RTD. Note that the RTD can be resonant even if the standing wave is present in the steady state.

When WDFs are used, we can obtain the transient response as well as the standing wave generated by the difference in impedance. The standing wave is a packet of resonant waves and is expressed as if the wave is stationary but, from the viewpoint of WDFs, the standing wave is expressed using a z-transform and is constantly moving. Therefore, the standing wave might move by some chance as if it were a tsunami and it is considered to be the wave packet itself.

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References


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