Enhanced OFDM Performance with Pilot-Aided Reduced Peak-to-Average Power Ratio

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Abstract  Orthogonal frequency division multiplexing (OFDM) plays a vital role as a physical layer technology. However, its inclusion in fifth-generation wireless systems is under threat due to its low power efficiency caused by a high peak-to-average power ratio (PAPR). Attempting to solve this problem, we propose a new spectrally efficient PAPR reduction scheme in this paper. We change pilot symbols’ positions among the data symbols iteratively, maintaining all the conditions required for accurate channel estimation. The particular combination of pilot and data symbols giving the lowest PAPR is transmitted. Although the pilots’ locations are varied from frame to frame, no side information (SI) is sent to the receiver to convey information about the pilot symbols’ locations. The pilot symbols, rather, are detected blindly at the receiver, utilizing only the equidistance and large-power properties of the pilot symbols, having detection accuracies of 95% and 85% over additive white Gaussian noise (AWGN) and fading channels at 0 dB signal-to-noise ratio, respectively. This facilitates bandwidth saving since the reservation of part of the available bandwidth to send SI to the receiver is no longer required. PAPR is found to be reduced up to 2.2 dB at a clipping probability of \(10^{-2}\) by the proposed scheme. The pilot detection accuracy as well as the corresponding bit error rate (BER) of the proposed system are investigated over both AWGN and fading channels, and a significant improvement in BER is observed in the presence of a high-power amplifier at the transmitter. The reduced PAPR of the proposed system culminates in power saving of up to 2.6 W. This makes the proposed OFDM system a spectrally and energy-efficient physical layer technology.

Keywords: OFDM, PAPR, pilot, capacity improvement, energy efficiency, 5G

1. Introduction

Green communication is set to be a feature of future fifth-generation (5G) cellular communication\(^1\) to reduce operating expenditure (OPEX) and CO\(_2\) emission, making communication systems more environmentally friendly. Cellular communication accounts for 0.5% of global CO\(_2\) emission\(^2\). A high-power amplifier (HPA) consumes 80% of the energy of a base station (BS)\(^3\), which in turn is responsible for 60% to 80% of the total energy consumption of a network\(^4\). The efficiency of a typical HPA ranges from 20% to 30%\(^5\). The input power that an HPA cannot convert to an output is dissipated as heat. For this reason, improving the efficiency of the HPA can significantly reduce energy consumption.
has low power efficiency in this region. This further degrades the efficiency of the HPA to less than 10% in the case of a linear amplifier such as a Class A power amplifier[6]. In addition to the efficiency degradation of the HPA, the high PAPR escalates in-band and out-of-band (OOB) distortions. To fulfill the energy efficiency requirement of the 5G, the HPA must be operated close to its saturation region, and one way to do this is to reduce the PAPR.

There have been a plethora of works carried out to address this high-PAPR problem. Details of each of these methods can be found in Ref.[7]. In all methods, some extra processing is carried out at the transmitter to reduce the PAPR level of the OFDM signal. All of these methods have some advantages and limitations. The common demerit of most of the PAPR reduction methods is the requirement of side information (SI), where extra bits for SI need to be sent to the receiver to inform it of the modification in the original data sequence before it is transmitted so that the receiver can extract the original sequence from the received modified sequence. For example, in selective mapping (SLM)[8], the data sequence is multiplied by a set of phase sequences and the resulting multiplied sequence with the lowest PAPR is selected for transmission. The phase sequence giving the lowest-PAPR OFDM signal is transmitted as SI. SI affects the system performance in two ways: firstly, a certain amount of bandwidth needs to be reserved for sending SI, which reduces the spectral efficiency and hence the data rate, and secondly, the bit error rate (BER), which ultimately defines the performance of a communication system, strongly depends on the accurate reception of the SI at the receiver. To ensure satisfactory SI reception at the receiver, a secure channel is used. This requires extra processing and logistic support. Nevertheless, the perfect reception of the SI cannot be ensured due to propagation channel contamination. This inaccurate SI causes erroneous data sequence recovery at the receiver, thereby degrading the BER performance. These setbacks have motivated researchers to find a ‘blind’ PAPR reduction technique that does not require SI transmission. The receiver itself recovers the original sequence from the received sequence without further information. A well-known technique of this kind is companding[9]. The companding-based technique markedly reduces PAPR. The average signal power, however, increases and adjacent channel interference (ACI) increases owing to the increased OOB distortion caused by the disruption of orthogonality of the OFDM signal due to the post-IFFT operation involved in it. Recently, the smart utilization of existing resources of OFDM to reduce PAPR has been reported. Some of the null subcarriers, which are subcarriers having no power, have been exploited for PAPR reduction[10]. Usually, null subcarriers are placed on both sides of data subcarriers. The positions of a predefined number of null subcarriers are switched with the data subcarriers iteratively, and the PAPR of the corresponding subcarriers’ arrangement is computed for each switching. The subcarriers’ arrangement providing the lowest PAPR is selected for transmission. The null subcarriers and data subcarriers are deswitched at the receiver on the basis of the lowest power of the null subcarriers. The computational complexity of this technique is prohibitively high. The method in Ref.[11], however, reduces the computational overhead significantly. Nonetheless, such techniques remain unattractive due to their poor BER performance caused by the inaccurate null detection, especially over a fading channel. There are several works on pilot-tone-assisted modulation (PTAM)[12],[13], in which pilot tones are utilized for PAPR reduction. In PTAM, a certain number of subcarriers, called pilot tones, are reserved for channel estimation and synchronization. In addition to these conventional functions, it has also been proposed that pilot tones can be used intelligently for PAPR reduction. In Ref.[14], the phases and amplitudes of clipped data symbols are sent to the receiver using pilot tones and null subcarriers, respectively. This approach reduces OOB distortion significantly compared with the conventional clipping. BER, however, experiences a similar degradation to that when clipping is employed. Since only the amplitudes of the pilot tones are utilized at the receiver, the phases of the pilot tones are exploited in Ref.[15], where an exhaustive search is performed to find the pilot subcarriers’ phases, which are later utilized to reduce the pilot power. This reduced pilot power results in PAPR reduction of an OFDM system. Such a technique does not require any SI transmission but suffers from a prohibitively high computational overhead caused by the exhaustive search. Positions and phases of the pilot symbols are used for PAPR reduction in Ref.[16]. Pilot symbols’ positions are ‘pseudorandomly’ changed among the data symbols, and PAPR is computed for each arrangement of pilot and data symbols. The arrangement providing the lowest PAPR is transmitted. According to Ref.[13], the channel estimation performance is markedly degraded if the distance between any two neighboring pilot symbols is not constant and not all pilot symbols have the same level of power. In Ref.[16], the pilot symbols are made ‘equiprobable’ among data subcarriers allocating various levels of power to different pilot symbols. This equiprobability, however, does not ensure an equal space between any two neighboring pilot symbols. Therefore, the channel estimation performance of this technique is adversely affected. In addition, a very large power is assigned to the pilots, which increases the mean power of the OFDM signal, thereby increasing OPEX[7]. Since the pilot symbols’ locations are randomly changed, the large pilot
power is the only clue[16] for pilot detection at the receiver. However, the pilot detection accuracy and BER of the proposed system have not been discussed. The BER will be unsatisfactory if the pilot detection is carried out using only a high pilot power. This technique is unfeasible in practice due to the breach of the equipower and equidistance properties, the extremely low pilot detection accuracy and the high average signal power.

In this paper, a new PAPR reduction technique is proposed utilizing pilot symbols in addition to their conventional roles in channel estimation and synchronization. To ensure the optimal channel estimation, all pilots are allocated equal power and a certain constant distance between any two neighboring pilots is maintained. We shift pilot symbols among the data symbols by one position at a time. In each a shift, the PAPR of the arrangement of the corresponding pilot and data symbols is computed. The arrangement providing the lowest PAPR is transmitted. The shifting of the pilot symbols among the data symbols avoids the necessity of the mathematical computation involved in random number generation required in Ref.[16]. Although the relative gap among the pilots is not changed, the positions of the pilots among the data symbols are varied over the OFDM symbols. This makes it necessary to know the pilot symbols’ positions at the receiver. We propose a robust pilot detection algorithm exploiting the constant distance between the pilots and the larger power allocated to each pilot. The performance of the proposed system is investigated in terms of PAPR reduction capability, BER, power savings and throughput improvement. Although this detection algorithm increases the computational complexity of the receiver, it paves the way for a bandwidth- and energy-efficient OFDM system.

2. Problem Formulation

OFDM is attractive to system designers for its robustness in combating multipath fading. It achieves this merit by dividing the whole bandwidth into a number of narrow orthogonal subcarriers in such a way that frequency-selective fading can no longer exist in the narrow subcarriers; hence, flat fading is the only effect of the fading channel on OFDM, and this effect can easily be mitigated using a one-tap equalizer. For a frequency-domain-mapped data vector \( X_k \) and \( N \) inverse discrete Fourier transform (IDFT) points, the time-domain OFDM symbol components can be obtained by the following equation:

\[
x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}
\]  

It is evident from this equation that the IDFT redistributes the energies of its input symbols among the components of the OFDM symbol. Each of the input symbols is multiplied by distinct sine waves and sampling is performed on the resulting signal obtained after adding all the sine-wave-multiplied symbols. The value of each sample depends on the amplitudes and phases of all the sine waves at a particular time. A particular sample can have a large power if all the sine waves are added coherently at that specific instant. Although a very small number of samples have a large power, this increases the peak-to-average power ratio, i.e., PAPR, which is defined by

\[
PAPR = \frac{\max(|x_n|^2)}{E\{|x_n|^2\}}
\]  

where \(|x_n|^2\) is the power of the \(n\)th sample and \(E\{|x_n|^2\}\) is the expected value of \(|x_n|^2\), where \(n\) is an integer satisfying \(0 \leq n \leq N-1\). If samples are taken at the Nyquest rate (that is, \(N\) samples), the computed peak is lower than its corresponding continuous-time waveform[17]. To improve the accuracy of determining the peak value, \(NL\) samples are taken, where \(L\) is an integer number called the oversampling factor. An oversampling factor having a value of at least 4 can provide a good approximation of the continuous-time waveform[18]. When oversampling is carried out, Eq.(1) becomes

\[
x_n = \frac{1}{NL} \sum_{k=0}^{NL-1} X_k e^{j2\pi kn/(NL)}
\]  

3. Proposed System

Our proposed PAPR reduction scheme causes changes in both the transmitter and receiver of the conventional OFDM system. On the transmitting side, it changes the pilot symbols’ positions iteratively in a predetermined manner among the frequency domain data symbols, which is followed by the computation of PAPR of the corresponding time-domain signal. The arrangement of the pilot and data symbols providing the lowest PAPR is transmitted. Since the pilots’ locations are not fixed in different OFDM symbols and the transmitter does not send any SI to inform the receiver about the pilots’ locations, the receiver must detect the pilot symbols’ locations blindly. Therefore, we propose a blind pilot detection algorithm based on the equidistance, equipower and relatively large power properties of the pilot symbols. The proposed transmitter and receiver are described in the following subsections.

3.1 Transmitter Design

A block diagram of our proposed system is shown in Fig.1. For simplicity, only the baseband modulation
Fig. 1  Block diagram of the proposed system

is considered. A mapped data vector of length $N_d$ is sent to a pilot insertion block after performing a serial-to-parallel (S/P) conversion operation on it. The pilot insertion block takes a pilot symbol vector of length $N_p$ in addition to the data vector and inserts these pilot symbols inside the sequence of data symbols to produce a vector of length $N = N_d + N_p$. The PAPR of this combined vector of length $N$ is computed after carrying out an inverse fast Fourier transform (IFFT) operation on it. This vector along with its PAPR is stored, and the control returns to the pilot insertion block (the dotted arrow in the transmitter indicates this iterative process), where the previously inserted pilots are shifted. The shifting procedure of the pilots is illustrated in Fig.2. In the first step (see Fig.2(a)), for a pilot symbol sequence of length $N_p$, the first pilot is inserted immediately before the first data symbol and the second pilot is maintained at a predetermined distance from the first pilot. The third pilot is placed in the data sequence in such a way that the same predetermined distance is maintained from the second pilot. The third pilot is placed in the data sequence in such a way that the same predetermined distance is maintained from the second pilot. In this way, each pilot symbol maintains a constant distance from the immediate pilot symbols located on both its sides. For example, for $N = 8$ and $N_p = 2$, the predetermined constant distance between any two neighboring pilots is 4 and the indexes of the first and second pilots are 1 and 5, respectively (see Fig.2(a)). For this arrangement, PAPR is computed after the IFFT operation. In the second step, each pilot symbol interchanges its location with the data symbol located immediately to the right of it, that is, while the data symbol immediately to the right occupies the location of the pilot symbol located to its left, the pilot symbol occupies the position of the data symbol. All pilot symbols are shifted in the same manner. As a result, it appears as if the pilot symbols are shifted by one position to the right among the data symbols. Figure 2(b) portrays this new scenario. Similarly to in the first step, the IFFT of this combination of pilot and data symbols is performed and the PAPR of the corresponding time-domain signal is computed. This process of shifting the pilots inside the data sequence, followed by a frequency-domain to time-domain conversion and PAPR computation of the corresponding time-domain signal, continues as long as the index of the right most pilot symbol remains less than or equal to $N$. All arrangements of pilot and data symbols, with their corresponding PAPR, are stored, and the sequence having the lowest PAPR is selected for transmission. Unlike PTAM, where pilots’ locations are known to both sides of the communication system, the pilots’ locations are not known to the receiver and no SI is sent to the receiver to inform it which symbols are pilots; the receiver detects pilot symbols blindly.

In general, for a system with an OFDM symbol length of $N$ and $N_p$ pilots, the constant distance between any two neighboring pilots is given by

$$R = \frac{N}{N_p} \quad (4)$$

If the difference in indexes between any two neighboring pilots is $R$, the index of any pilot in the combined
sequence, $\beta_i$, can be found for $1 \leq i \leq N_p$ by the following equation:
\[
\beta_i = K_i R + r_o
\]  
(5)
where $K_i$ is an integer taking any value from 0 to $N_p - 1$, and $r_o$ is the index of the first pilot in the combined sequence, ranging from 1 to $R$.

3.2 Receiver design

The receiver of the conventional PTAM OFDM knows the pilot locations in the received time-domain symbols. The receiver of our proposed system, however, does not have any information about the pilot symbols’ locations. Therefore, this blindness of the receiver regarding the pilot locations forces it to detect the pilot symbols from the received symbol sequence. The proposed receiver design is shown in the system model in Fig.1. Our proposed pilot detection algorithm is based on the relatively high power of pilot symbols compared with that of data symbols, the constant distance between any two neighboring pilots and the equipower property of the pilot symbols. The received serial time-domain symbol sequence, contaminated by thermal noise and the communication channel, is first converted to a parallel sequence before it is converted back to the frequency-domain symbol sequence. The pilot detection is carried out on the frequency-domain symbols. The step-by-step procedure of the detection process is shown in Fig.3. A set of frequency-domain symbols having amplitudes greater than a certain threshold are separated for further processing. The equidistance property of the pilot symbols is then utilized to find which of these separated symbols are pilot symbols. To show how this equidistance property is utilized, let us consider a vector $X_p = [x_{p1} x_{p2} x_{p3} x_{p4} \ldots]^T$, where each element is an integer and maintains a certain constant distance, $x_{q1}$ (that is, $x_{p2} - x_{p1} = x_{p3} - x_{p2} = x_{p4} - x_{p3} = \ldots = x_{q1}$), from its neighboring elements. Also let us consider another vector $X_q = [x_{q1} x_{q2} x_{q3} \ldots]^T$, where the elements of $X_q$ are integers satisfying $\frac{x_{q1}^2 + 1}{x_{q1}} = \kappa + 1$ and $\kappa = [1, 2, 3, \ldots]$. We use the addition operation of modular arithmetic[19] to limit the sum of two positive integers to less than or equal to a specified integer number. For example, if we perform the modular addition of 10 to 15 using modulus 16, the sum will be 9. In our example, the modular addition of $x_{p1}$ to the vector $X_q$ can reproduce the rest of the members of the vector $X_p$. For example, assume that $X_p = [1 17 33 49]^T$ and $X_q = [16 32 48]^T$. If we add $x_{p1} = 1$ to $X_q$ using modular arithmetic with modulus 64, the result will be $[17 33 49]^T$, the rest of the elements of $X_p$. However, if there is an element in $X_p$ that does not satisfy this property (that is, the distance from its neighboring elements is $x_{q1}$), the modular addition of this element to $X_q$ cannot reproduce any element of $X_p$. For example, let us consider $X_p = [1 3 17 33 49]^T$. If $x_{p2} = 3$ is added to $X_q$ using modular arithmetic, the result is $[19 35 51]^T$. Since $x_{p2} = 3$ does not maintain the property mentioned above, it cannot reproduce any element of $X_p$. Since the pilot symbols are placed equidistantly among the data symbols, this property can easily lead to the detection of the indexes of the pilot symbols among the segregated indexes. To improve the performance of the pilot detection, the whole set of candidate pilot symbols’ indexes is computed from each pilot symbol’s index found in the previous step. Then the set of indexes having the highest power is finally selected as the set of indexes of the pilots.

Suppose that $R_x$ is the received time-domain sequence. The corresponding frequency-domain sequence $Y$ is obtained as
\[
Y = FFT(R_x)
\]  
(6)
Each element of $Y$ is associated with an index, where the index of an element is its position among all elements of $Y$; for example, the index of the fifth element of $Y$ is 5. Suppose that the index vector is $A = [a_1 \ a_2 \ a_3 \ \ldots \ a_N]^T = [1 \ 2 \ 3 \ \ldots \ N]^T$. There is a one-to-one correspondence between the elements of $A$ and those of $Y$, that is, the index of the $n^{th}$ element of $Y$, $y_n$, is $a_n$, where $n$ is an integer ranging from 1 to $N$. The next step is to segregate the elements of $Y$ that exceed a certain threshold amplitude. If $B$ is the vector of length $N$ that contains the differences between the amplitude of each element of $Y$ and the threshold amplitude, each element of $B$, $b_i$, is obtained in the following way:
\[
b_i = |y_i| - \gamma \sqrt{P}
\]  
(7)
where \(|y_i|\) is the amplitude of the \(i^{th}\) element of \(Y\), \(\gamma\) is the design parameter, which defines how many elements of \(Y\) are considered for further processing as candidate pilot symbols, ranging for \(0 < \gamma < 1\), and \(P\) is the power assigned to the pilot symbols, which is known to the receiver. For all \(b_i > 0\), the index of the corresponding \(y_i\), \(a_i\), is segregated in the vector \(Q = [q_1 \ q_2 \ q_3 \ \ldots \ q_M]^T\) of length \(M\) such that \(0 < M < N\), that is, 
\[
q_j = a_i \quad \text{if} \quad b_i > 0
\]
where \(0 < j \leq M\). For a noise-free communication channel, the utilization of only the higher pilot power would be sufficient to detect pilots. In such a case, the vector \(Q\) would consist of the indexes of all the pilots. Since there are no noise-free channels in practice, \(Q\) consists of true pilot indexes and spurious pilot indexes due to the variation of the signal power over time caused by thermal noise. For this reason, a series of operations are carried out on these segregated indexes of the candidate pilot symbols exceeding the threshold amplitude to find the indexes of the true pilots. The cyclic distances of each pilot symbol from the other pilot symbols, which are known to the transmitter and receiver, are given by 
\[
S_i = T_i R
\]
where \(T = [T_1 \ T_2 \ T_3 \ \ldots \ T_{N_p-1}]^T = [1 \ 2 \ 3 \ \ldots \ N_p - 1]^T\) and \(S = [S_1 \ S_2 \ S_3 \ \ldots \ S_{N_p-1}]^T\), which will be called the distance vector hereafter. We now utilize the equidistance property of the pilot symbols to find which of the indexes of \(Q\) are the indexes of the true pilot symbols. A new matrix, \(D\), in which each row contains the result of the modular addition of each element of \(Q\) to the distance vector \(S\), is created. Each element of \(D\), \(D_{ij}\), is given by 
\[
D_{ij} = \begin{cases} v_j \text{mod}(N + 1) + 1 & \text{for } v_j \text{mod}(N + 1) < v_j \\ v_j & \text{otherwise} \end{cases}
\]
where \(i = 1, 2, 3, \ldots, M, \ j = 1, 2, 3, \ldots, N_p - 1, \ v_j = q_i + S_j\) and \(\text{mod}\) represents modulo operator.

A count vector \(C = [c_1 \ c_2 \ c_3 \ \ldots \ c_M]^T\) is defined in which each element, \(c_i\), represents the number of elements of \(Q\) produced in matrix \(D\) by each element of \(Q, q_i\). The desired pilot locations are determined from \(C\). The index of each pilot symbol corresponds to an index from \(Q\) that provides one of the \(N_p\) largest elements of \(C\). This results in a set of estimated pilot symbol’s indexes, \(U\), in which each element is defined as 
\[
u_i = q_j
\]
if \(c_j\) is among the \(N_p\) largest elements of \(C\), where \(i\) is an integer and ranges from \(0 < i \leq N_p\).

This pilot detection procedure can detect pilot symbols well. However, its performance degrades at a low SNR because \(Q\) cannot hold the indexes of most of the pilots and many segregated spurious indexes also satisfy the distance vector \(S\) at a low SNR. In this case, the number of elements reproduced by the spurious pilots outnumbers the number of elements produced by the true pilot indexes. As a result, the vector \(U\) consists of a small number of pilot symbols’ indexes and the detection accuracy degrades significantly. To make our detection technique robust in low-SNR environments, the initially estimated pilot index vector \(U\) is processed further. Since both the transmitter and receiver know the distance vector \(S\), the complete set of pilot indexes is created using \(S\) from each element of \(U\). The indexes of the \(N_p\) pilot symbols are calculated from \(U\) as 
\[
Z_{ij} = \begin{cases} w_j \text{mod}(N + 1) + 1 & \text{for } w_j \text{mod}(N + 1) < w_j \\ w_j & \text{otherwise} \end{cases}
\]
where \(w_j = (u_i + nR)\), and \(n, i\) and \(j\) are integers all ranging from 1 to \(N_p\). This gives \(N_p\) sets of candidate pilot indexes. For each set, the total power of the symbols having the corresponding indexes is computed. The set of indexes giving the highest total power is finally selected as the indexes of the desired pilot symbols. The power of each of the \(N_p\) sets is given by 
\[
\alpha_i = \sum_{j=1}^{N_p} |Z_{ij}|^2
\]
where \(\alpha_i\) is the total power of the symbols of the \(i^{th}\) set of indexes and \(i = 1, 2, 3, \ldots, N_p\). These detected pilots are then used in the usual manner for channel estimation and equalization. The equalized signal is demapped using the corresponding demodulator before P/S conversion. A decoded binary data stream is then obtained from the serial decimal symbols.

There is further scope to improve the performance of the proposed pilot detection scheme depending on the application concerned. For a delay-tolerant system, we can improve the detection accuracy in an extremely low SNR case by incorporating further processing. The idea is to use a soft \(\gamma\) parameter instead of a hard \(\gamma\). In the technique discussed above, we have used a specific value of \(\gamma\) (which we call a hard \(\gamma\)) for all SNR cases. Since the channel noise is completely random, this hard \(\gamma\) may end up capturing no or a very small number of elements in \(Q\), especially at an extremely low SNR. This will undoubtedly cause the erroneous detection of pilots, thereby deteriorating the BER performance. To prevent such a scenario, we can use a soft \(\gamma\) in the sense that if a specific \(\gamma\) results in fewer than \(N_p\) indexes in \(Q\), the parameter \(\gamma\) will be decreased by a certain amount to make the number of elements contained in \(Q\) at least equal to \(N_p\); the parameter \(\gamma\) is further decreased if this new value of \(\gamma\)
does not result in \( N_p \) or more elements in \( Q \). This process continues until \( Q \) has at least \( N_p \) elements or the allowable maximum number of iterations is reached.

3.3 Example of pilot detection

The pilot detection algorithm can be better understood by the following example. Suppose that \( R = 16 \), \( N = 64 \), \( \beta = [1 17 33 49]^{T} \) and \( S = [16 32 48]^{T} \). Also suppose that the indexes of the symbols whose amplitudes exceed a certain threshold \( \gamma \) are \( Q = [7 17 1 23 33 61 19] \). Then the matrix \( D \) is given by

\[
D = \begin{bmatrix}
23 & 39 & 55 \\
33 & 49 & 1 \\
17 & 33 & 49 \\
39 & 55 & 7 \\
49 & 1 & 17 \\
13 & 29 & 45 \\
35 & 51 & 3
\end{bmatrix}
\]

where the first row of \( D \) is the result of the modular addition of the first element of \( Q \) to \( S \), the second row is the result of the same operation between the second element of \( Q \) and \( S \), and so on. The second, third and fifth elements of \( Q \) maintain the distance vector \( S \), hence the modular addition of each of these elements produces several elements of \( Q \) in a specific row (see rows 2, 3 and 5) of \( D \). However, since the first and fourth elements of \( Q \) maintain the distance vector \( S \) with one element each, they each reproduce only one element of \( Q \). The sixth and last elements of \( Q \) are not located at the distance defined by \( S \), hence they cannot reproduce any other elements of \( Q \). This outcome is exploited to find the indexes of the true pilot symbols.

The first and second elements of \( Q \) reproduce one (23) and two (1 and 33) elements of \( Q \) in the first and second rows of \( D \), respectively. For this reason, the first two elements of the count vector \( C \) are 1 and 2, respectively. Thus, \( C \) is \([1 2 2 1 2 0 0])^{T} \). It can be seen that the second, third and fifth elements of \( Q \) reproduce the largest number of elements of \( Q \); the first and fourth elements reproduce one element each. Out of these two elements, one is selected randomly to obtain the indexes of \( N_p = 4 \) elements. If the first element is selected, the vector \( U \) will be \( U = [7 17 1 33]^{T} \). In the next step, from each element of \( U \), a complete vector of pilot indexes is generated using (11) and the resulting matrix \( Z \) becomes

\[
Z = \begin{bmatrix}
7 & 23 & 39 & 55 \\
17 & 33 & 49 & 1 \\
1 & 17 & 33 & 49 \\
49 & 1 & 17 & 33
\end{bmatrix}
\]

In the final step, the sum of the powers of the symbols having indexes defined by each row of \( Z \) is computed and the row providing the largest sum of powers is considered to comprise the indexes of the true pilot symbols.

3.4 Features of the proposed system

The advantages of the proposed PAPR reduction technique compared with other related techniques are noted below. (1) It does not send any SI to the receiver, unlike in Ref.[8] or reserve any portion of the bandwidth for PAPR reduction, unlike in Ref.[20], hence the whole bandwidth can be used for data transmission, making it a spectrally efficient scheme. (2) It does not cause spectral regrowth, which is usually found in clipping and companding. (3) Since it involves pre-IDFT processing for PAPR reduction, it does not affect the orthogonality of the time-domain signal. (4) It does not require an exhaustive search for the optimum phases of pilots, unlike in Ref.[15], or pilots’ phases and locations, unlike in Ref.[16], or suitable subcarriers for exchanging with null subcarriers, unlike in Ref.[10]. (5) Unlike companding, our proposed system does not affect the average signal power significantly. All these advantages, however, are accompanied by increased computational complexity in the detection of pilot symbols at the receiver. Nevertheless, the advantages of the proposed technique undoubtedly outweigh the demerit of an increased computational overhead.

4. System Evaluation

The performance of the proposed system is investigated from different viewpoints, such as PAPR reduction capability, BER, power saving due to the PAPR reduction, and throughput improvement due to the BER improvement. During the simulation, a baseband system is considered. Since a passband system has twice the PAPR of a baseband system[21], the PAPR reduction performance observed in the baseband system will follow a similar trend in the passband system. The baseband modulations used throughout this paper for the pilot symbols and data symbols
are binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK), respectively, unless otherwise stated. To measure PAPR more precisely, the oversampling factor $L = 4$ is used. Every simulation result in this paper is obtained by sending $1 \times 10^5$ OFDM symbols.

Usually a larger power is allocated to the pilot symbols than to the data symbols. While the use of an extremely large pilot power increases the average signal power, the use of a low pilot power makes channel estimation difficult. Therefore, determination of a suitable pilot power is crucial. The effects of the pilot power on PAPR, caused by an increase in the average power, are investigated and are shown in Fig.4 for a conventional OFDM system with $N = 128$ and $N_p = 4$ in the form of a complementary cumulative distribution function (CCDF). It is evident that PAPR decreases owing to the increased average power caused by the higher pilot power. The pilot power $P = 39$ used in Ref.[16] itself decreases PAPR by roughly 1 dB at a CCDF level of $10^{-3}$. This indicates a significant increase in the average signal power. A pilot power of $P = 9$, however, does not increase the average power significantly, which is revealed by the insignificant reduction of PAPR. Since a higher pilot power is required to facilitate effective pilot detection at the receiver, $P = 9$ will be used throughout this paper, unless otherwise stated.

4.1 PAPR reduction

Figure 5 shows the PAPR reduction capability of the proposed system, where solid lines are used for the PAPR of the conventional OFDM and dashed lines are used for the proposed modified OFDM (M.OFDM). The total number of subcarriers is 128 and the ratio of the numbers of data and pilot subcarriers, $R$, is varied. The simulation result reveals that $R$ has no noticeable effect on the PAPR of the conventional OFDM system. This property, however, is not found for the proposed system. A negative correlation is found between $R$ and PAPR for the proposed system, where a higher PAPR reduction can be achieved for a smaller number of pilot symbols. This is due to the fact that a smaller number of pilots provides a larger search space for a signal with a smaller PAPR, thereby making the probability of achieving a lower PAPR higher. For $R = 32$, the PAPR of the proposed system is 2.2 dB smaller than that of the conventional system at a CCDF level of $10^{-2}$. A slight deterioration of PAPR of roughly 0.2 dB for the proposed system is observed at CCDF levels of $10^{-3}$ and $10^{-4}$. A noticeable difference in the PAPR of the proposed system is observed for different $R$. The PAPR reduction capability is degraded by 5%, 33% and 65% for data-to-pilot subcarrier ratios of 16, 8 and 4, respectively, from the PAPR reduction capability of 2 dB obtained using $R = 32$ at a CCDF level of $10^{-4}$. The loss of the PAPR reduction capability can, however, be considered as beneficial because it allows complexity of the system to be reduced by 87.5% by reducing the required number of IFFT operations from 32 to 4. While up to 1128 and 71 IFFT operations are required to reduce PAPR by 0.5 dB at a CCDF level of $10^{-3}$ by the methods proposed in Ref.[10] and [11], respectively, our proposed method can reduce PAPR by 0.75 dB using only four IFFT operations. This also makes the proposed system suitable for delay-sensitive devices.

A comparison of the PAPR reduction capability of the proposed scheme with three existing schemes is given in Fig.6. The compared existing schemes are PTS based on simulated annealing (PTS-SA)[22], an orthogonal-pilot-sequence-based scheme (OPS)[23] and a null subcarrier-switching-based scheme[11]. For PTS-SA, the values of the simulation parameters are as follows: number of iterations=28, initial annealing temperature=100, attenuation factor=0.98, final annealing temperature=0.00001. In Ref.[11], two null subcarriers are used for PAPR reduction. As is shown, the proposed scheme outperforms the existing schemes. At a CCDF level of $10^{-3}$, the proposed scheme reduces PAPR by 87%, 44% and 17%.
4.2 BER improvement

The robustness of the proposed PAPR reduction system strongly depends on effective detection of the pilot symbols at the receiver because erroneous pilot detection can increase BER degradation. In addition to the pilot symbols’ power and the number of pilots used, the detection performance also depends on the parameter $\gamma$ defined in Eq.(7). The computational complexity of the receiver also depends on $\gamma$.

Determining a suitable value of $\gamma$ is imperative for efficient performance. Figure 7 shows the effect of $\gamma$ on the percentage error rate for various pilot powers and total numbers of subcarriers at 0 dB SNR over an additive white Gaussian noise (AWGN) channel. The error rate is smaller for $N = 128$ than that for $N = 64$. Figure 7 also reveals that a higher pilot power ensures better detection accuracy. The error rate is minimum for $\gamma$ values between 0.6 to 0.85. A soft $\gamma$ with an initial value of 0.66 is used in the rest of this paper. For a specific $\gamma$, the pilot detection accuracy is also affected by the SNR. Table 1 shows the pilot detection accuracy of the proposed system for various SNRs over an AWGN channel. In most cases, the detection accuracy is more than 95%. For $R = 16$, the accuracy is roughly 95% at 0 dB SNR, 99% at 3 dB SNR and almost 100% for SNR values of greater than or equal to 6 dB. The detection accuracy, however, degrades significantly for $R = 8$ at an SNR of less than 6 dB except when $N = 64$. The highest detection accuracy is found at $N = 128$ and $R = 16$, where the detection accuracy is more than 98% for any SNR. The pilot detection accuracy, however, degrades significantly over a known flat fading channel as is revealed in Table 2. Similarly to the cases of an AWGN channel, $R = 16$ provides far better accuracy than $R = 8$ over the fading channel (with the exception of $N = 256$, at which $R = 16$ provides far better detection accuracy than $R = 8$ over the AWGN channel). The accuracies for $R = 16$ and $R = 8$ are very similar over the flat fading channel. While the detection accuracy over the AWGN channel at 0 dB SNR is around 95%, it decreases to roughly 85% over the fading channel, a degradation of 10% in the detection rate. Over the fading channel, a detection rate of around 95% is observed at 5 dB SNR, which is 5 dB higher than that over the AWGN channel. The detection accuracies over the fading channel are around 98% and 99% at SNRs of 10 dB and 20 dB, respectively. It becomes very close to 100% for SNRs of 20 dB or more. According to Tables 1 and 2, the data-to-pilot subcarrier ratio, $R$, plays a noteworthy role in determining the detection accuracy. For this reason, the number of pilots that optimizes the detection accuracy for a particular total number of subcarriers is investigated as shown in Fig.8. The use of a larger number of pilots is detrimental to the detection accuracy. While the lowest detection accuracy is observed when the number of pilots is half the number of data subcarriers ($R = 2$), $R = 16$ results in the highest average accuracy. The optimum numbers of pilot symbols for $N = 64$, $N = 128$, $N = 256$ and $N = 512$ are 8, 4, 8 and 32, respectively.

Table 1 Pilot detection rate (%) over AWGN channel

<table>
<thead>
<tr>
<th>N</th>
<th>$R$</th>
<th>SNR (dB)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>0 dB</td>
<td>3 dB</td>
</tr>
<tr>
<td>64</td>
<td>16</td>
<td>95.16</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>95.48</td>
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<tr>
<td>128</td>
<td>16</td>
<td>98.51</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>89.11</td>
</tr>
<tr>
<td>256</td>
<td>16</td>
<td>95.73</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>83.59</td>
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</table>

Table 2 Pilot detection rate (%) over fading channel

<table>
<thead>
<tr>
<th>N</th>
<th>$R$</th>
<th>SNR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>64</td>
<td>16</td>
<td>75.1</td>
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<td>16</td>
<td>88.6</td>
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<tr>
<td></td>
<td>8</td>
<td>84.8</td>
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<tr>
<td>256</td>
<td>16</td>
<td>88.7</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>87.4</td>
</tr>
</tbody>
</table>
The BER of the proposed system is investigated while considering the nonlinearity effects of the HPA. We employ Rapp’s model[24] of a solid-state power amplifier (SSPA) to simulate the HPA. Since the amplitude modulation (AM)/phase modulation (PM) conversion effect of an SSPA is negligible[25], only the AM/AM conversion of the SSPA will be considered in this paper. Rapp’s HPA model for the AM/AM conversion is simulated using the relation[25]

$$g(x) = \frac{|x|}{(1 + |x|^2)\tau}$$ (13)

where $|x|$ is the amplitude of the scaled version of the input signal and $\tau$ represents the smoothness of the transition from a linear region to a saturation region. A typical value of $\tau$ is 1[26], which is used throughout this paper. For a specific input saturation power $P_{i,\text{sat}}$ and input back-off (IBO), the average power of the input to the SSPA is scaled to set its operating point at a required position using the following formula:

$$P_{i,\text{avg}} = \frac{P_{i,\text{sat}}}{\text{IBO}}$$ (14)

Figure 9 shows the BER of the proposed system over both the AWGN and Rayleigh flat fading channels. For BER investigation, $P = 16$ and BPSK is used as the modulation scheme for the data sequence. The IBOs mentioned in Fig.9 are in dB and $P_{i,\text{sat}} = 3$ dB. The improvement of the BER due to the reduction of PAPR at the transmitter by the proposed system is revealed in Fig. 9. It also shows the effect of the IBO on the BER. The lower the IBO, higher the BER because of the increased clipping by the HPA. The BER performance of the proposed system, however, improves with decreasing IBO. While the SNR of the conventional OFDM system required to obtain a BER of $2 \times 10^{-2}$ is 15 dB with 0 dB IBO over the AWGN channel, the proposed system can achieve the same BER with a 13 dB SNR, resulting in an SNR improvement of 2 dB. A similar SNR gain is observed over the fading channel to achieve the same BER. For IBO = 3 dB, SNR gains of 3 dB and 2.5 dB are found at BER of $5 \times 10^{-3}$ over the AWGN and fading channels, respectively. With further increasing IBO, the SNR improvement decreases significantly over the fading channel, but less significantly over the AWGN channel. A slight degradation in the BER of the proposed system at a very low SNR (< 2 dB) is observed due to the increased erroneous detection of the pilot symbols.

4.3 Impact on network performance

Reducing the PAPR in OFDM improves the system performance in different ways. Power saving and channel capacity improvement are the most prominent impacts of PAPR reduction on a network performance. While the power saving reduces OPEX, the channel capacity improvement increases users’ satisfaction. A linear amplifier has very low efficiency in converting the input dc power to the output ac power. PAPR reduction techniques save power by improving the efficiency of the HPA by making it possible for it to be operated close to its saturation region. The efficiency of an HPA, $\eta$, is given by

$$\eta = \frac{P_{o,\text{avg}}}{P_{\text{dc}}}$$ (15)

where $P_{o,\text{avg}}$ is the average output power and $P_{\text{dc}}$ is the input dc power. A larger $\eta$ allows a greater output power from the same input dc power. For a specific $P_{o,\text{avg}}$ requirement, the only way to save power is to reduce the input dc power consumption, that is, increase $\eta$. PAPR reduction allows power saving by decreasing the dc power consumption. The output back-off (OBO) of an amplifier is given in Ref.[27] as

$$\text{OBO} = \frac{P_{\text{sat}}}{P_{o,\text{avg}}}$$ (16)

$$P_{o,\text{avg}} = \frac{P_{\text{sat}}}{\text{OBO}}$$ (17)
where $P_{sat}$ is the output saturation power. For a linear amplifier such as a Class A amplifier, IBO is equal to OBO[28]. Since IBO is equal to PAPR[6], we can conclude that OBO=PAPR. In addition, for a Class A HPA, $P_{dc}=2P_{sat}$. Under these conditions, Eq.(17) becomes

$$P_{o,avg} = \frac{P_{sat}}{\zeta} = \frac{P_{dc}}{2\zeta}$$

$$P_{dc} = 2\zeta P_{o,avg}$$ (18)

where $\zeta$ represents PAPR. Suppose that the proposed PAPR reduction strategy decreases PAPR from $\zeta_1$ to $\zeta_2$, and that the input dc powers required to produce PAPRs of $\zeta_1$ and $\zeta_2$ are $P_{dc1}$ and $P_{dc2}$, respectively. Since the average output power is considered constant, we obtain from Eq.(18)

$$P_{dc1} = 2\zeta_1 P_{o,avg}$$

$$P_{dc2} = 2\zeta_2 P_{o,avg}$$

The power saving resulting from the proposed PAPR reduction strategy is given by

$$P_{dc1} - P_{dc2} = 2P_{o,avg}(\zeta_1 - \zeta_2)$$ (19)

For a typical PAPR, $P_{o,avg}$ ranges from 63mW to 250mW for wireless communication[6].

The power saving achieved at different clipping probabilities due to the PAPR reduction is shown in Fig.10. The higher the average output power $P_{o,avg}$, the higher the power saving. The power saving increases from a clipping probability of 0.9 and reaches its maximum value at a clipping probability of $10^{-4}$. There is a positive correlation between $R$ and power saving. The highest power saving is obtained for $R = 32$ and $P_{o,avg} = 250$ mW for any clipping probability, in which case the power saving increases steadily from the lowest value of 0.28 W obtained at a clipping probability of 0.9 to its peak value of 2.64 W at a clipping probability of $10^{-4}$, which is 136% higher than that obtained for $R = 4$ at the same clipping probability. The power-saving pattern obtained at $P_{o,avg} = 250$ mW for $R = 4$, with a maximum power saving of about 1.28 W for a clipping probability of $10^{-4}$, is similar to that at $P_{o,avg} = 110$ mW for $R = 32$. At a clipping probability of $10^{-4}$, the lowest power-saving trend for $R = 32$, which is obtained at $P_{o,avg} = 63$ mW, is higher than that for $R = 4$ with $P_{o,avg} = 110$ mW and $P_{o,avg} = 63$ mW. All these investigations reveal the importance of using a smaller number of pilots and a higher average output power for increased power saving.

In addition to power saving, the reduced PAPR improves channel throughput. For example, suppose that the target BER and IBO are $5 \times 10^{-3}$ and 3 dB, respectively. According to Fig. 9, while this target BER can be achieved by an OFDM system without a PAPR reduction scheme at a 14.26 W (11.54 dB) SNR over the AWGN channel, the same BER can be obtained by the proposed system at a 11.19 W (10.49 dB) SNR, i.e., a 3.07 W lower SNR. Since the target BER is achieved with a lower SNR, the additional SNR can be employed to increase the throughput. According to the Shannon capacity formula, the channel throughput is described as $C = \Delta f \log_2(1 + SNR)$, where $\Delta f$ is the subcarrier spacing. In the Long-Term Evolution (LTE) standard, the subcarrier spacing $\Delta f$ is set at 15 kHz. Thus, in the LTE system the additional channel throughput that can be obtained due to the saving of a 3.07 W SNR is 30.4 kbps. The additional channel capacities obtained due to the PAPR reduction by the proposed system over the AWGN and flat fading channels for different IBOs are shown in Fig.11. A larger throughput gain can be achieved over the fading channel than over the AWGN channel. For different IBOs, the throughput gain varies from 20 kbps to 150 kbps for BERs of $10^{-2}$ or more. The greatest capacity gain is achieved at IBO = 0 dB. This is due to the higher clipping rate for which no back-off is provided. The use of the PAPR reduction scheme in this case can further reduce clipping, thereby improving BER and resulting in a larger capacity gain.
5. Conclusion

In this paper, a new approach based on shifting pilot symbols among data symbols to reduce the PAPR in OFDM was proposed. We changed pilot symbols’ locations iteratively in a predefined manner. To avoid sending SI to inform the receiver of the pilot symbols’ locations, a robust pilot subcarrier detection algorithm was proposed. Through simulations, the PAPR reduction capability and the BER of the resulting system were investigated. The PAPR of the proposed system is 2.2 dB smaller than that of conventional systems at a CCDF level of $10^{-2}$ and a significant improvement in BER was observed, especially over an AWGN channel. This approach facilitates considerable power saving as a result of the PAPR reduction. Moreover, an enhanced throughput was also observed. Since the phases of the pilot symbols are not used in the receiver, the phases may also be exploited in the pilot detection in addition to the equidistance and large-power properties. The combined effects of utilizing these three properties will significantly improve the pilot detection accuracy, and hence BER, and we hope to examine this possibility in future.

References

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