Code Generator for Optical ZCZ Sequence with Zero-Correlation Zone $2^z$

Yasuaki Ohira$^1$, Takahiro Matsumoto$^2$, Hideyuki Torii$^3$, Yuta Ida$^4$ and Shinya Matsufuji$^5$

$^1$ School of Sciences and Technology for Innovation, Yamaguchi University, Ube 755-8611, Japan
$^2$ Department of Information Network and Communication, Kanagawa Institute of Technology, Atsugi 243-0292, Japan
$^3$ Graduate School of Sciences and Technology for Innovation, Yamaguchi University, Ube 755-8611, Japan
$^4$ Graduate School of Science and Engineering, Yamaguchi University, Ube 755-8611, Japan
$^5$ Graduate School of Science and Engineering, Yamaguchi University, Ube 755-8611, Japan

E-mail: $^{1,2,4,5}$w003wc, $^2$matugen, $^3$y.ida, $^5$s-matsufj@yamaguchi-u.ac.jp, $^4$torii@nw.kaikanaga-it.ac.jp

Abstract Generation methods of the optical zero-correlation zone (ZCZ) sequence set with ZCZ sizes of 2, 4, and 2$^z$ have been proposed, where $z$ is a natural number. However, the code generator for the sequence with ZCZ $2^z$ has not been considered. In this paper, we propose a new construction of a code generator without ROM. The proposed code generator can suppress the circuit size and operate faster than the conventional code generator with ROM. In addition, the conventional and proposed code generators for the sequences with the ZCZ sizes of 2, 4, and 8 are implemented on a field-programmable gate array (FPGA) and compared. As a result, the proposed code generator can reduce the circuit size and operate faster than the conventional code generator.

Keywords: optical CDMA system, optical ZCZ sequence set, code generator, field-programmable gate array (FPGA)

1. Introduction

The optical direct sequence spread spectrum (DS-SS) system can be expected to enable high-speed communication using a wide band [1–3]. Multiple access interference (MAI) cancellation methods with the orthogonal code (OOC) or modified prime sequence code (MPSC) have been proposed in the optical code division multiple access (CDMA) system [4, 5]. The optical CDMA system using the optical zero-correlation zone (ZCZ) sequence set can detect the desired sequence without the MAI of undesired sequences, even if the synchronization is not perfect between users [6]. The ambiguity of synchronization is dependent on the size of ZCZ. The generation methods of the optical ZCZ sequence set with ZCZ = 1, 4, and 2$^z$ have been proposed, where ZCZ is the size of the zero-correlation zone and $z$ is a natural number [7, 8]. In addition, compact code generators for the optical ZCZ sequence with ZCZ = 1, 4, and 2$^z$ have been proposed [7]. However, the code generator for the sequence with ZCZ = 2$^z$ has not been considered. Although a code generator using ROM is common, the circuit size increases exponentially relative to the sequence length.

In this paper, we propose a new construction of a code generator without ROM. The proposed code generator can have a smaller circuit size and operate faster than the conventional code generator with ROM. The proposed code generators for the sequences with the ZCZ sizes of 2, 4, and 8 are implemented on a field-programmable gate array (FPGA). In Sect. 2, we introduce the definition of the optical ZCZ sequence set and describe the construction of an optical ZCZ sequence set with ZCZ = 2$^z$ using an optical ZCZ sequence set with ZCZ = 1. In Sect. 3, we describe the optical CDMA system using the optical ZCZ sequence set. In Sect. 4, we describe the structures of the conventional code generator using ROM and the proposed code generator without ROM. In Sect. 5, we describe the results of the implementation of the conventional and proposed code generators on FPGA and compare them.

2. Optical ZCZ Sequence Set with ZCZ = 2$^z$

2.1 Definition of optical ZCZ sequence set with ZCZ = 2$^z$

Let $a_N^{iz}$ be a biphase sequence of length $N$, whose elements take 1 or −1, written as

$$a_N^{iz} = (a_N^{iz,0}, a_N^{iz,1}, \ldots, a_N^{iz,N-1}), a_N^{iz,j} \in \{1, -1\} \quad (1)$$

where $z$ is a non-negative integer, $j$ is a sequence number, and $i$ is an order variable and denotes $i$ mod $N$. Similarly, let $\hat{a}_N^{id}$ be a binary sequence of length $N$, whose elements take 1 or 0, written as

$$\hat{a}_N^{id} = (\hat{a}_N^{id,0}, \hat{a}_N^{id,1}, \ldots, \hat{a}_N^{id,N-1}), \hat{a}_N^{id,j} \in \{1, 0\} \quad (2)$$

Let $A^i$ be a set of pairs of the biphase sequence $a_N^{iz}$ and the binary sequence $\hat{a}_N^{id}$, written as

$$A^i = \{(a_N^{i0}, \hat{a}_N^{id}), (a_N^{i1}, \hat{a}_N^{id}), \ldots, (a_N^{iN-1}, \hat{a}_N^{id})\} \quad (3)$$
where $M$ is the number of sequences in a sequence family and is called family size.

A periodic correlation function between the sequences $a^{x}_{N,j}$ and $\tilde{a}^{x}_{N,j}$ at shift $\nu$ is defined by

$$
\rho_{a^{x}_{N,j}, \tilde{a}^{x}_{N,j}} = \sum_{i=0}^{N-1} a^{x}_{N,j} \cdot \tilde{a}^{x}_{N,j+\nu} \mod N
$$

(4)

In this paper, the above correlation function $\rho_{a^{x}_{N,j}, \tilde{a}^{x}_{N,j}}$ is called the autocorrelation function for $j = j'$ and the cross-correlation function for $j \neq j'$. If the periodic auto- and cross-correlation functions satisfy

$$
\rho_{a^{x}_{N,j}, \tilde{a}^{x}_{N,j}} = \begin{cases}
w & : \nu = 0, j = j', d = 0 \\
-w & : \nu = 0, j = j', d = 1 \\
0 & : \nu = 0, j \neq j' \\
0 & : 1 \leq |\nu| \leq 2^z
\end{cases}
$$

(5)

with $w = \sum_{0}^{N-1} a^{x}_{N,j} \mod N$, then the set $A^x$ is called an optical ZCZ sequence set [6] with the ZCZ size $Z_{CZ} = 2^z$. The optical ZCZ sequence sets are bounded by $M \leq N/(Z_{CZ} + 1)$.

2.2 Construction of optical ZCZ sequence set with $Z_{CZ} = 1$

A Hadamard matrix is needed for the construction of the optical ZCZ sequence set with $Z_{CZ} = 1$. The construction method of the optical ZCZ sequence set changes in accordance with the type of Hadamard matrix. For example, the Sylvester-type Hadamard matrix is used here because the generation method is simple. Let $H_{N_1}$ be the Sylvester-type Hadamard matrix of order $N_1 = 2^{n_1}$ with $n_1 \geq 2$, written as

$$
H_{N_1} = \begin{bmatrix} h^0_{N_1} & \cdots & h^z_{N_1} & \cdots & h^{N_1-1}_{N_1} \end{bmatrix}
$$

(6)

$$
h^i_{N_1} = (h^{i,0}_{N_1,0}, \cdots, h^{i,N_1-1}_{N_1,0}), h^i_{N_1, j} \in \{1, -1\}
$$

(7)

where the symbol $T$ denotes the matrix transposition, which is defined by

$$
H_{N_1} = H_{N_1}^T \otimes \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}
$$

(8)

where the operation $\otimes$ denotes the Kronecker product, and $h^i_{N_1}$ is called a Sylvester-type Hadamard sequence.

An element of the biphase sequence $a^{0}_{N,j} \in \{1, -1\}$ of length $N = 2N_1$ is given by

$$
a^{0}_{N,j} = \begin{cases}
h^{i}_{j} & : 0 \leq i \leq \frac{N}{2} - 1 \\
(-1)^{i+1}h^{i}_{j} & : \frac{N}{2} \leq i \leq N - 1
\end{cases}
$$

(9)

where $i$ denotes $i \mod N$. If the Hadamard matrix is not the Sylvester-type Hadamard matrix, the negation position of Eq. (9) changes. The mean value of the biphase sequence $a^{0}_{N}$ is given by $\sum_{i=0}^{N-1} a^{0}_{N,i} = 0$ with $j \neq 0, 1$. Therefore, the biphase sequence $a^{0}_{N,j}$ is called a biphase balanced sequence. On the other hand, an element of the binary sequence $a^{0}_{N,j} \in \{0, 1\}$ of length $N$ is given by

$$
a^{0}_{N,j} = \begin{cases}
1 + (-1)^{j}a^{0}_{N,j} & \text{if } j = 0 \\
\frac{1}{2} & \text{if } j = 1
\end{cases}
$$

(10)

Let $A^0$ be a set of $(N/2 - 2)$ pairs of the biphase sequence $a^{0}_{N}$ and the binary sequence $\tilde{a}^{0}_{N}$, of length $N = 2N_1$ except when $j = 0$ and 1. The periodic correlation function between $a^{0}_{N,j}$ and $\tilde{a}^{0}_{N,j}$ is given by

$$
\rho_{a^{0}_{N,j}, \tilde{a}^{0}_{N,j}} = \begin{cases}
\frac{N}{2} & : \nu = 0, j = j', d = 0 \\
-\frac{N}{2} & : \nu = 0, j = j', d = 1 \\
0 & : \nu = 0, j \neq j' \\
0 & : \nu = \pm Z_{CZ} = \pm 2^z
\end{cases}
$$

(11)

Therefore, the above set of $M$ pairs of the biphase sequence $a^{0}_{N}$ and the binary sequence $\tilde{a}^{0}_{N}$ is called an optical ZCZ sequence set with $Z_{CZ} = 2^z$.

2.3 Construction of optical ZCZ sequence set with $Z_{CZ} = 2^z$

The biphase sequence $a^{z}_{N,j}$ of length $N$ and the sequence number $j = 2k, k = 0, 1, \cdots, \frac{N}{2} - 1$ are generated by

$$
a^{z}_{N,j} = (a^{z}_{N,j,0}, a^{z}_{N,j,1}, \cdots, a^{z}_{N,j,N_2}, a^{z}_{N,j,N_2+1}, \cdots, a^{z}_{N,j,N_2-1}, a^{z}_{N,j,N_2})
$$

$$
= (a^{z-1}_{N,j,0} \cdot \frac{1}{2}, a^{z-1}_{N,j,1}, \cdots, a^{z-1}_{N,j,N_2}, a^{z-1}_{N,j,N_2+1})
$$

$$
\cdots, a^{z-1}_{N,j,N_2-1}, -a^{z-1}_{N,j,N_2+1}, a^{z-1}_{N,j,N_2+2}, \cdots, a^{z-1}_{N,j,N_2+2N_2-2})
$$

(12)

Similarly, the biphase sequence $a^{z}_{N,j}$ of length $N$ and the sequence number $j = 2k+1, k = 0, 1, \cdots, \frac{N}{2} - 1$ are generated by

$$
a^{z}_{N,j} = (a^{z-1}_{N,j,0} \cdot \frac{1}{2}, a^{z-1}_{N,j,1}, \cdots, a^{z-1}_{N,j,N_2}, a^{z-1}_{N,j,N_2+1})
$$

$$
\cdots, a^{z-1}_{N,j,N_2-1}, -a^{z-1}_{N,j,N_2+1}, a^{z-1}_{N,j,N_2+2}, \cdots, a^{z-1}_{N,j,N_2+2N_2-2})
$$

(13)

On the other hand, an element of the binary sequence $\tilde{a}^{z}_{N,j}$ of length $N$ is given by Eq. (10). The periodic correlation function between $a^{z}_{N,j}$ and $\tilde{a}^{z}_{N,j}$, except when $0 \leq j, j' \leq (z + 1) \mod 2$, is given by

$$
\rho_{a^{z}_{N,j}, \tilde{a}^{z}_{N,j}} = \begin{cases}
\frac{N}{2} & : \nu = 0, j = j', d = 0 \\
-\frac{N}{2} & : \nu = 0, j = j', d = 1 \\
0 & : \nu = 0, j \neq j' \\
0 & : 1 \leq |\nu| \leq 2^z
\end{cases}
$$

(14)

Therefore, the above set of $M$ pairs of the biphase sequence $a^{z}_{N}$ and the binary sequence $\tilde{a}^{z}_{N}$ is called an optical ZCZ sequence set with $Z_{CZ} = 2^z$.
2.4 Examples

Let $H_4$ be the Sylvester-type Hadamard matrix of order 4, written as

$$
H_4 = \begin{bmatrix}
  h_{0,0} & h_{0,1} & h_{0,2} & h_{0,3} \\
  h_{1,0} & h_{1,1} & h_{1,2} & h_{1,3} \\
  h_{2,0} & h_{2,1} & h_{2,2} & h_{2,3} \\
  h_{3,0} & h_{3,1} & h_{3,2} & h_{3,3}
\end{bmatrix} = \begin{bmatrix}
  + & + & + & + \\
  + & + & + & + \\
  + & + & + & + \\
  + & + & + & +
\end{bmatrix}
$$

where + and − denote 1 and −1, respectively. Let $a^{i0}_k$ be a biphase sequence in an optical ZCZ sequence set with $ZCZ = 1 (= 2^0)$ and $N = 8$. From Eqs. (8) and (9), the biphase sequence $a^{i0}_k$ is generated by a Hadamard sequence $h^k_4$ and is written as

$$
a^{0,0}_k = (h^0_{0,0}, h^0_{0,1}, h^0_{0,2}, h^0_{0,3}) = (+, +, +, +, +, +, +, +)$$

Let $a^{i1}_k$ be a biphase sequence in an optical ZCZ sequence set with $ZCZ = 2 (= 2^1)$ and $N = 16$. From Eqs. (11) and (12), $a^{i1}_k$ is generated by the biphase sequence $a^{i0}_k$ with $ZCZ = 1$ and $N = 8$ and is written as

$$
a^{0,1}_k = \begin{bmatrix}
  a^{0,0}_{8,0} & a^{0,0}_{8,1} & a^{0,0}_{8,2} & a^{0,0}_{8,3} & a^{1,0}_{8,0} & a^{1,0}_{8,1} & a^{1,0}_{8,2} & a^{1,0}_{8,3} \\
  a^{0,0}_{8,4} & a^{0,0}_{8,5} & a^{0,0}_{8,6} & a^{0,0}_{8,7} & a^{1,0}_{8,4} & a^{1,0}_{8,5} & a^{1,0}_{8,6} & a^{1,0}_{8,7}
\end{bmatrix} = \begin{bmatrix}
  +, +, +, +, +, +, +, +, +, +, +, +, +, +, +, +
\end{bmatrix}
$$

The mean value of the biphase sequence $a^{i1}_k$ is given by

$$
\sum_{k=0}^{15} a^{i1}_{16,k} = 0.
$$

From Eq. (10), the binary sequence $\hat{a}^{i1,d}_k$ of $d = 0, 1$ and the sequence number $j = 1, 2, 3$ are written as

$$
\hat{a}^{i1,0}_k = (+, +, +, +, +, +, +, +, +, +, +, +, +, +, +, +)
$$

The periodic autocorrelation function $\rho_{a^{i1},a^{i1}}$ for $j = 1, 2, 3$ and $d = 0, 1$ is given by

$$
\rho_{a^{i1},a^{i1}}(\ell) = \begin{bmatrix}
  (8, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \\
  (8, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
\end{bmatrix}
$$

The periodic cross-correlation function $\rho_{a^{i1},a^{i1'}}$ is given by

$$
\rho_{a^{i1},a^{i1'}}(\ell) = \begin{bmatrix}
  (8, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \\
  (8, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
\end{bmatrix}
$$

From Eqs. (15) and (16), the periodic auto and cross-correlation functions satisfy Eq. (14).

3. Code Generator for Optical ZCZ Sequence with $ZCZ = 2^n$

3.1 Optical ZCZ-CDMA system

Figure 1 shows an optical ZCZ-CDMA system using an optical ZCZ sequence set. In transmitters, each code generator generates the binary sequence $\hat{a}^{i,j}_{d}$ depending on the input data $d_j$ and the sequence number $j$ and sends it as the optical signal. This conversion from the sequences...
to the optical signals occurs at electrical-to-optical (E/O) converters. The binary sequence elements 1 and 0 correspond to the on-off of light sources of the E/O converters, respectively.

A receiver converts the received optical signal to the electrical signal by an optical-to-electrical (O/E) converter. The received signal is correlated with the biphase sequence \(a_{ij}^{z,2}\) in a matched filter (MF). From Eq. (5), if the difference between the arrival times of signals from each transmitter is within \(2^z\), the received data \(d_j\) can be detected by threshold detection without MAI.

When this system is implemented, as many code generators as light sources are necessary. Thus, reducing the size of the code generator is important.

3.2 Conventional code generator using ROM

Although the code generator can be constructed for any code by using ROM, the size of the circuit increases. A code generator using ROM is called a conventional code generator in this paper. The conventional code generator of the optical ZCZ-CDMA system uses a ROM that stores all the values of the optical ZCZ sequences. Note that the sequences stored in the ROM are binary sequences because these are used for transmitters in the optical ZCZ-CDMA system. The ROM address is generated by the combination of the input data \(d\), the sequence number \(j\), and the order variable \(i\). Now, the order variable \(i\) and the sequence number \(j\) are expressed in a binary notation as:

\[
i = \sum_{k=0}^{n-1} 2^k i_k, i_k \in \{1, 0\}
\]

\[
j = \sum_{k=0}^{m-1} 2^k j_k, j_k \in \{1, 0\}
\]

where \(n = \log_2 N\) and \(m = \log_2 M\). Table 1 shows the memory map of ROM in a conventional code generator for the optical ZCZ sequence \(\hat{a}_{ij}^{z}d\) of sequence length \(N\), sequence number \(j\) and ZCZ = \(2^z\). In addition, the order variable \(i\) can be expressed as an up-counter because the order variable \(i\) needs to be increased by one. From the above, Fig. 2 shows the conventional code generator for the sequence \(\hat{a}_{ij}^{z}d\).

3.3 Proposed code generator without ROM

First, the construction of the code generator with ZCZ = 1, \(z = 0\) is described. From Eq. (9), it can be seen that the binary sequence of the optical ZCZ sequence set is constructed by extending the Sylvester-type Hadamard sequence. Let \(\hat{h}_{ij}^{z}d\) be the Sylvester-type Hadamard sequence whose elements are 1 or 0, written as

\[
\hat{h}_{ij}^{z} = \frac{1 + (-1)^{h_{ij}^{z}}} {2}
\]

Furthermore, the Sylvester-type Hadamard sequence \(\hat{h}_{ij}^{z}d\) of length \(N_1 = 2^n\) is written as the following Boolean expression [7].

\[
\hat{h}_{ij}^{z} = d \oplus \sum_{k=0}^{n-1} \text{XOR}(i_k \cdot j_k)
\]

Table 1 Memory map of ROM in a conventional code generator for optical ZCZ sequence

<table>
<thead>
<tr>
<th>Address</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d)</td>
<td>(j_{m-1} \cdots j_1 j_0 \cdots i_{1} i_{0})</td>
</tr>
<tr>
<td>(\hat{a}_{ij}^{z}d)</td>
<td>(\hat{a}_{ij}^{z}d)</td>
</tr>
<tr>
<td>0 0 0 0 0 0 0 0</td>
<td>(\hat{a}_{ij}^{z}d)</td>
</tr>
<tr>
<td>0 0 0 0 0 1 0 0</td>
<td>(\hat{a}_{ij}^{z}d)</td>
</tr>
</tbody>
</table>

Table 1 Memory map of ROM in a conventional code generator for optical ZCZ sequence

Furthermore, the Sylvester-type Hadamard sequence \(\hat{h}_{ij}^{z}d\) of length \(N_1 = 2^n\) is written as the following Boolean expression [7].

\[
\hat{h}_{ij}^{z} = d \oplus \sum_{k=0}^{n-1} \text{XOR}(i_k \cdot j_k)
\]

Here, the notation \(\sum_{\text{XOR}}\) means

\[
\sum_{k=1}^{n} x_k = x_1 \oplus x_2 \oplus \cdots \oplus x_n
\]

In addition, the operations \(\cdot\), \(\oplus\) and \(\overline{\cdot}\) denote the logic operation AND, exclusive-OR (XOR) and NOT, respectively. The code generator of the binary sequence \(\hat{a}_{ij}^{z}d\) of length \(N = 2N_1\) and \(z = 0\) is given by applying Eq. (20) to Eqs. (9) and (10). Therefore, the code generator is written as the following Boolean expression [7].

\[
\hat{a}_{ij}^{0}d = d \oplus (i_0 \cdot j_{n-1}) \oplus \sum_{k=0}^{n-2} \text{XOR}(i_k \cdot j_k)
\]

The order variable \(i\) of Eq. (22) can be expressed as an up-counter as with the conventional code generator. Thus,
which $i$ and $j$ are even numbers are negated. The sequence $\hat{a}_{N,i}^{j,z,d}$, which is generated by applying the interleave operation $z$ times to the sequence with $Zcz = 1$, is written as the following Boolean expression:

$$\hat{a}_{N,i}^{j,z,d} = d \oplus (i_0 \cdot j_0) \oplus \sum_{k=1}^{N-2z-1} (i_{k+z} \cdot j_k) \oplus (i_{k+z} \cdot j_{k+z-1})$$ (23)

Additionally, the elements of Eq. (23) are negated by the second operation in the case that the following Boolean expression $s_{i,j,z}$ is equal to 1.

$$s_{i,j,z} = (i_0 \cdot j_0) \oplus \sum_{k=1}^{Zcz} \sum_{p=0}^{\lfloor \ell_2 \ell_2 \rfloor} \prod_{p=0}^{\ell_2 \ell_2} \left\{ i_p \oplus \left( \frac{\ell}{2p} \mod 2 \right) \right\}$$ (24)

From Eqs. (23) and (24), the binary sequence $\hat{a}_{N,i}^{j,z,d}$ of length $N$ and $z \geq 1$ is written as the following Boolean expression:

$$\prod_{k=1}^{n} x_k = x_1 \cdot x_2 \cdot \cdots \cdot x_n$$ (25)

The notation $\prod_{k=1}^{n} x_k$ means

![Fig. 1 Optical ZCZ-CDMA system using an optical ZCZ sequence set of length $N$, family size $M$, $Zcz = 2^z$ and input data $d_j$](image1)

![Fig. 2 Conventional code generator for an optical ZCZ sequence of length $N$, sequence number $j$ and $Zcz = 2^z$](image2)
expression:
\[
\hat{a}_{N,j}^{i,z,d} = \hat{a}_{N,j}^{i,z,d} \oplus s_{i,j,z} \\
= d \oplus (i_{n-1} \cdot i_k) \oplus \sum_{k=1}^{n-2} \text{XOR}(i_{k+z} \cdot j_k) \\
\oplus (\overline{i_0} \cdot j_0) \oplus \{i_p \oplus \left(\frac{\ell}{2^p}\right) \text{mod} \ 2\} \\
(26)
\]

As with the code generator of \( \hat{a}_{N,j}^{i_0,d} \), the proposed code generator of the sequence \( \hat{a}_{N,j}^{i,z,d} \) can be constructed of only an up-counter and logic gates. From Eq. (26), the code generator of the sequence \( \hat{a}_{N,j}^{i,z,d} \) is shown in Fig. 3.

For example, the proposed code generator for the optical ZCZ sequences of \( N = 64 \) and \( Z_{CZ} = 4 = 2^2 \) is given by

\[
\hat{a}_{64,j}^{i_2,d} = d \oplus (i_1 \cdot i_2) \oplus (i_1 \cdot j_1) \oplus (i_4 \cdot j_2) \\
\oplus (\overline{i_2} \cdot \overline{i_3}) \oplus (\overline{i_0} \cdot j_0) \\
\oplus \{(\overline{i_0} \oplus 0) \cdot (i_1 \oplus 1)\} \\
(27)
\]

\[
= d \oplus (i_1 \cdot i_2) \oplus (i_1 \cdot j_1) \oplus (i_4 \cdot j_2) \\
\oplus (\overline{i_2} \cdot \overline{i_3}) \oplus (\overline{i_0} \cdot j_0) \\
\oplus (\overline{i_0} \cdot i_1) \\
(28)
\]

from Eq. (26). Note that \((i_n \oplus 0)\) and \((i_n \oplus 1)\) in Eq. (27) are \(i_n\) and \(\overline{i_n}\), respectively. From Eq. (28), the code generator for the sequence \( \hat{a}_{64,j}^{i_2,d} \) of length \( N = 64 \) and \( Z_{CZ} = 4 \) is shown in Fig. 4.

4. Code Generator Implementation on FPGA

The conventional and proposed code generators for the optical ZCZ sequences of length \( N = 64, 128, 256, 512 \) and 1024, and \( Z_{CZ} = 2, 4 \) and 8 have been implemented on a field-programmable gate array (FPGA). The FPGA corresponds to 51,840 logic elements (LEs), which are the basic building blocks of an FPGA, containing a 4-input look-up table, a register, and additional logic. The output bus-width is the number of output signal lines, and its size is 1 bit. Table 2 shows the specifications of code generators. Figures 5 and 6 show the number of LEs and the maximum clock frequency of code generators for the sequence of \( Z_{CZ} = 2, 4 \) and 8, respectively.

The upper limit of the maximum clock frequency depends on the specification of FPGA. However, the maximum clock frequency itself depends on the structure of the circuit. The circuit design to reduce logic gates existing between registers is needed to improve the maximum clock frequency. In other words, although the maximum clock frequency can be improved by increasing the number of registers, the circuit size also increases. In the conventional code generator, a ROM is used, and it is composed of registers and multiplexers. Therefore, the circuit depth of the multiplexers increases as the sequence length increases; then the maximum clock frequency decreases. Furthermore, from Table 1, the number of ROM address patterns increases exponentially as the sequence length increases. Hence, the number of LEs also increases exponentially. In contrast, from Eq. (26) and Fig. 3, the proposed code generator does not use ROM, and the sizes of logic gates increase linearly as the sequence length increases. For this reason, even if the sequence length increases, the number of LEs is small and the maximum clock frequency remains high in comparison with the conventional code generator.

5. Conclusions

In this paper, we propose a new structure of a code generator for the optical ZCZ sequence of \( Z_{CZ} = 2^2 \). The proposed code generator can be constructed using an up-counter and logic gates, without ROM. The conventional and proposed code generators are implemented on an FPGA, and the proposed code generator can reduce the number of logic elements and improve the maximum clock frequency compared with the conventional code generator.
Table 2 Specifications of code generators that are implemented on FPGA

<table>
<thead>
<tr>
<th>Reference sequences</th>
<th>optical ZCZ sequence set</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequence length N</td>
<td>64, 128, 256, 512, 1024</td>
</tr>
<tr>
<td>Zero-correlation zone Zcz</td>
<td>2, 4, 8</td>
</tr>
<tr>
<td>Output bus width</td>
<td>1bit</td>
</tr>
<tr>
<td>FPGA</td>
<td>Altera APEX20KE</td>
</tr>
<tr>
<td></td>
<td>EP20K1500EBC652-1X</td>
</tr>
<tr>
<td>Max. logic elements</td>
<td>51, 840</td>
</tr>
<tr>
<td>Max. pins</td>
<td>488</td>
</tr>
<tr>
<td>Analysis and synthesis tool</td>
<td>Altera QuartusII 8.1(64bit)</td>
</tr>
</tbody>
</table>

Fig. 4 Proposed code generator for the optical ZCZ sequence of length $N = 64$, sequence number $j$ and $Zc_z = 4$

Fig. 5 Number of logic elements of code generators for the optical ZCZ sequence of $Zc_z = 2$, 4 and 8

Fig. 6 Maximum clock frequency of code generators for the optical ZCZ sequence of $Zc_z = 2$, 4 and 8

Acknowledgments

This work was supported by the Japan Society for the Promotion of Science (JSPS), Grant-in-Aid for Scientific Research (C)(15K06068), and the Telecommunications Advancement Foundation (TAF).

References


Yasuaki Ohira received the B.E. degree in information science and engineering from Yamaguchi University, Japan, in 2013. Since 2013, he has been a technical staff member at Yamaguchi University, Japan. Since 2016, he has been a doctoral student in the Graduate School of Sciences and Technology for Innovation, Yamaguchi University. His research interests include optical CDMA systems and their applications.

Takahiro Matsumoto received his B.Eng. and M.Eng. degrees in information and computer science from Kagoshima University, Japan, in 1996 and 1998, respectively, and his Ph.D. degree in engineering from Yamaguchi University, Japan, in 2007. He was a research associate from 1998 to 2007 and an Assistant Professor from 2007 to 2012 at Yamaguchi University, Japan. Since 2012, he has been an Associate Professor at Yamaguchi University. From 2010 to 2011, he was a visiting researcher at the University of Melbourne, Australia. His current research interests include spread spectrum systems and their applications. He is a member of IEEE and IEICE.

Shinya Matsufuji graduated from the Department of Electronic Engineering at Fukuoka University in 1977. He received his Dr. Eng. degree in computer science and communication engineering from Kyushu University, Fukuoka, Japan in 1993. From 1984 to 2002, he was a research associate in the Department of Information Science at Saga University. From 2002 to 2012, he was an Associate Professor of the Department of Computer Science and Systems Engineering at Yamaguchi University. He is currently a Professor of the Graduate School of Sciences and Technology for Innovation, Yamaguchi University. His current research interests include sequence design and spread spectrum systems. He is a member of IEEE, IEICE, and IEEJ.

(Received May 15, 2017; revised July 26, 2017)

(Received May 15, 2017; revised July 26, 2017)