Circuit Theory Based on New Concepts and Its Application to Quantum Theory

12. Modified Dirac Equation Exchanging Reactive Power for Mass of Particles

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Abstract In Session 10, we showed that the telegrapher’s equations can be extended using the Riccati differential equation and that a new circuit element can be obtained. In Session 11, we showed that the Dirac equation, which is used in quantum theory, can be expressed as the extended telegrapher’s equations and that circuit theory can be applied to the Dirac equation. In this session, we demonstrate that a new circuit element can be obtained by applying reactive power to the Dirac equation. In this case, energy given by Einstein’s well-known formula $E = mc^2$ should be considered for the discussion on an ordinary world that does not require the theory of relativity.

Keywords: Dirac equation, Riccati differential equation, reactive power, modified Dirac equation, extended telegrapher’s equation, voltage and current, Laplace transform

1. Introduction

In this lecture series, we have attempted to apply circuit theory to quantum mechanics and quantum theory. As a reason for this, circuit theory is discussed using the wave functions expressed by two complex functions, voltage and current, in the steady state, and lossless circuits and reactive power, which are not discussed in quantum theory, can be defined. Therefore, resonance as a phenomenon with both particle and wave natures can be examined using circuit theory; however, resonance is not considered in quantum theory.

In Session 8, we showed that circuit theory can be applied to the Schrödinger equation, a basic equation in quantum mechanics. In Session 11, we showed that circuit theory can also be applied to the Dirac equation, a basic equation in quantum theory.

In this session, we clarify the properties specific to the paired equations of voltage and current obtained by slightly modifying the Dirac equation using reactive power. In addition, we examine whether the paired equations can be applied to the quantum world.

2. Extended Telegrapher’s Equations

The extended telegrapher’s equations were obtained using the Riccati differential and Dirac equations in Sessions 10 and 11, respectively. From the extended telegrapher’s equations, equations for characteristic transmission lines and circuit elements were obtained.

In the quantum world, characteristic quanta, such as electrons, protons, neutrons, and neutrinos, are assumed. According to Ref. [1], however, all elementary particles are generated from one type of string. In physics, even characteristic physical phenomena tend to be discussed to pursue a unified phenomenon rather than to focus on their individual characteristics. For example, Maxwell’s equations have the terms of electric and magnetic fields, but physicists consider that both fields can be determined by solving the wave equations because these fields satisfy the wave equations.

In circuit theory, in contrast, both voltage and current satisfy the same telegrapher’s equations, but voltage and current are separately calculated. The ratio of the two functions is obtained as impedance, whereas the product of the two functions is obtained as power. There are two types of power, i.e., active and reactive powers. Only the active power can be energy.

In this lecture series, we have focused on the theoretical structure of using two functions of voltage and current. If the physical properties obtained from circuit theory can correspond to the characteristic quantum properties, these properties may be effectively used to clarify the quantum world.
3. Dirac Equation

The Dirac equation consisting of four wave functions was proposed by Dirac as a means of combining quantum theory and the special theory of relativity. In Session 11, we applied the Dirac equation to circuit theory.

The Dirac equation consisting of four wave functions is expressed as

$$j \frac{\partial}{\partial t} [\psi(x)] = \left( -jc \sum_{\alpha} \frac{\partial}{\partial x_{\alpha}} + \beta \frac{mc^2}{\hbar} \right) [\psi(x)] \quad (12.1)$$

Here, $[\psi(x)]$ is a vector with four elements (wave functions), and $\alpha_1, \alpha_2, \alpha_3, \beta$ are $4 \times 4$ matrices given by

$$\alpha_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad (12.2a)$$

$$\alpha_2 = \begin{pmatrix} 0 & 0 & 0 & -j \\ 0 & 0 & j & 0 \\ 0 & -j & 0 & 0 \\ j & 0 & 0 & 0 \end{pmatrix} \quad (12.2b)$$

$$\alpha_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad (12.2c)$$

$$\beta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (12.2d)$$

In Session 11, a circuit element suitable for circuit theory was obtained using Eq. (12.1). In the following sections, we demonstrate that characteristic circuit elements suitable for circuit theory can be obtained by modifying the Dirac equation using reactive power.

4. Reactive Power Applied to the Dirac Equation

In Ref. [3], the relationship between energy $E$ and momentum $p$ for a quantum with a mass of $m$ is described as in the theory of relativity and given by

$$E = \sqrt{(pc)^2 + (mc^2)^2} \quad (12.3)$$

Here, $c$ is the speed of light.

In quantum theory, $E$ and $p$ are replaced as follows.

$$E \to jh \frac{\partial}{\partial t} \quad (12.4a)$$

$$p \to -j h \frac{\partial}{\partial x} \quad (12.4b)$$

Here, $h = \hbar / 2\pi$ ($h$: the Planck constant). By rewriting Eq. (12.3) using Eqs. (12.4a) and (12.4b) and assuming that $pc$ and $mc^2$ are orthogonal and expressed by different coordinates, we obtain

$$j \hbar \frac{\partial}{\partial t} \psi(x,t) = -j \hbar \frac{\partial}{\partial x} \psi(x,t) + mc^2 \psi(x,t) \quad (12.5)$$

Here, $mc^2$, the second term in the square root of the right-hand side of Eq. (12.3), is explained in Ref. [4] as follows: Einstein’s fourth paper written in 1905, the miracle year, is an important contribution made by Einstein himself to the special theory of relativity. In this paper, he proposed the idea of equality of mass and energy, which was later expressed by the formula

$$E = mc^2 \quad (12.6)$$

Following this idea, an object with a mass of $m$ has energy equal to $mc^2$ even if it is not moving. This is highly significant in the theoretical aspect. It was clarified that a huge amount of energy is obtained through nuclear reactions, such as the nuclear fission of uranium, that involve a change in mass, paving the way to the realization of nuclear weapons and nuclear power generation.

As stated above, Einstein explained that any object with a mass of $m$ has energy given by $mc^2$ [Eq. (12.6)]. Note that the sum of kinetic and potential energies is kept constant in mechanics.

In circuit theory, complex power in the same dimensions as energy can be separated into active and reactive powers. Only the active power corresponds to the actual energy, and the reactive power is expressed as purely imaginary power or energy. Considering this property, we examine the energy given by $mc^2$ [Eq. (12.6)]. This is the energy of an object with a mass of $m$ even if it is not moving and is different from potential energy. On the basis of circuit theory, let us assume that the energy given by $mc^2$ is the reactive power if a non-moving object with a mass of $m$ emits no radiation. That is, we assume that a quantum with a mass of $m$, discussed with Eq. (12.3), has a complex energy given by

$$E = pc + j \cdot mc^2 \quad (12.7)$$

A relationship similar to the theory of relativity holds between $E$ and $p$.

By rewriting Eq. (12.7) using Eq. (12.4) and assuming that $pc$ and $jmc^2$ are orthogonal, we obtain

$$j \hbar \frac{\partial}{\partial t} \psi(x,t) = -j \hbar \frac{\partial}{\partial x} \psi(x,t) + jmc^2 \psi(x,t) \quad (12.8)$$

Assuming that independent components in the three directions are assigned to the part given by $\partial/\partial x$ in the first term of the right-hand side of Eq. (12.8), the Dirac equation given by Eq. (12.1) is modified using the reactive power as follows.
Equation (12.9) is referred to as the modified Dirac equation. On the basis of this equation, we examine a paired equation of voltage and current by a method similar to that in Session 11.

5. Application of Circuit Theory to Modified Dirac Equation

The equations used in circuit theory are spatially one-dimensional and are expressed using two functions, voltage and current. In the following, we extract equations given by the two functions from the modified Dirac equation given by Eq. (12.9).

5.1 Wave functions in \( x_1 \)-axis

In circuit theory, spatially one-dimensional functions are examined. Here, we focus on the wave function in Eq. (12.9) in the \( x_1 \)-axis only. Namely, matrices \( a_2 \) and \( a_3 \) in Eq. (12.9) are assumed to be zero matrices. The first and fourth rows of the matrix equation are given by

\[
\begin{align*}
\frac{\partial}{\partial t} \psi_1 &= \frac{jmc^2}{h} \psi_1 - jc \frac{\partial}{\partial x_1} \psi_4 \\
\frac{\partial}{\partial t} \psi_4 &= -jc \frac{\partial}{\partial x_1} \psi_1 - \frac{jmc^2}{h} \psi_4
\end{align*}
\]

(12.10a, 12.10b)

As described in Session 1, a response to an input vibration \([e.g., \exp(j\omega t)]\) is examined in circuit theory. Hence, a derivative with respect to time is converted into \(j\omega \) for the Laplace transformation. That is, the derivative on the left-hand sides of Eqs. (12.10a) and (12.10b) should be with respect to \( x_1 \), rather than time, and these equations are rewritten as

\[
\begin{align*}
jc \frac{\partial}{\partial x_1} \psi_4 &= \frac{jmc^2}{h} \psi_1 - j \frac{\partial}{\partial t} \psi_4 \\
jc \frac{\partial}{\partial x_1} \psi_1 &= -j \frac{\partial}{\partial t} \psi_4 - \frac{jmc^2}{h} \psi_4
\end{align*}
\]

(12.11a, 12.11b)

For example, assuming that \( \psi_1 \) and \( \psi_4 \) in these equations are current and voltage, respectively, simultaneous differential equations of two functions, voltage and current, are obtained. Therefore, circuit theory can be applied to the equations.

5.2 Wave functions in axes other than the \( x_1 \)-axis

In Session 11, we also derived the wave functions in the axes other than the \( x_1 \)-axis. However, the wave function in the \( x_2 \)-axis gives a resistance circuit and does not represent a transmission line. The wave function in the \( x_3 \)-axis gives the same circuit as in the case of the \( x_1 \)-axis. Therefore, the circuits obtained from the wave functions in the \( x_2 \)- and \( x_3 \)-axes are not discussed here.

6. Circuit Element Obtained from the Modified Dirac Equation

In Section 5, paired differential equations of the two wave functions (voltage and current) were obtained from the modified Dirac equation. Here, we attempt to obtain a circuit element from a set of these equations.

When each term in Eqs. (12.11a) and (12.11b) in Section 5.1 is multiplied by \(j/c\) and Laplace transformed, and then \( \psi_1 \) and \( \psi_4 \) are replaced with current \( I(x_1) \) and voltage \( V(x_1) \), respectively, the partial time derivative \( \partial/\partial t \) can be rewritten as \( j\omega \) and we obtain

\[
-\frac{d}{dx_1} \begin{pmatrix} V(x_1) \\ I(x_1) \end{pmatrix} = \begin{pmatrix} 0 & j\left(\frac{\omega}{c} + \frac{jmc}{h}\right) \\ j\left(\frac{\omega}{c} - \frac{jmc}{h}\right) & 0 \end{pmatrix} \begin{pmatrix} V(x_1) \\ I(x_1) \end{pmatrix}
\]

(12.12)

The eigenvalues (\( \gamma \)) of the matrix in Eq. (12.12) are obtained from the eigenequation given by

\[
\det \begin{pmatrix} -\gamma & j\left(\frac{\omega}{c} + \frac{jmc}{h}\right) \\ j\left(\frac{\omega}{c} - \frac{jmc}{h}\right) & -\gamma \end{pmatrix} = 0
\]

(12.13)

Namely, the eigenvalues and the phase constant \( \beta_a \) are expressed by

\[
\gamma = \pm j \sqrt{\frac{\omega^2}{c^2} + \frac{m^2c^2}{h^2}} = \pm j \beta_a
\]

(12.14)

The characteristic impedances corresponding to the thus-obtained phase constant, i.e., those from the left and right sides of the circuit, are complex and equal to \( Z_{oa} \) given by

\[
Z_{oa} = \frac{\omega}{c} + j\frac{mc}{h}\sqrt{\frac{\omega^2}{c^2} + \frac{m^2c^2}{h^2}}
\]

(12.15)

Using the \( \beta_a \) and \( Z_{oa} \) thus obtained and two integral constants \( N_1 \) and \( N_2 \), the voltage and current are given by

\[
\begin{align*}
V(x_1) &= N_1 Z_{oa} \exp(-j\beta_a x_1) + N_2 Z_{oa} \exp(j\beta_a x_1) \\
I(x_1) &= N_1 \exp(-j\beta_a x_1) - N_2 \exp(j\beta_a x_1)
\end{align*}
\]

(12.16a, 12.16b)

From these equations, the cascade matrix of a circuit element with a length of \( l \) is given by

\[
\begin{pmatrix} V(l) \\ I(l) \end{pmatrix} = \frac{1}{Z_{oa}} \begin{pmatrix} \cos \beta_a l & j \beta_a l \\ j \beta_a l & \cos \beta_a l \end{pmatrix} \begin{pmatrix} V(0) \\ I(0) \end{pmatrix}
\]

(12.17)

The circuit element with a cascade matrix given by Eq. (12.17) has complex characteristic impedances. The characteristic impedances of the forward and backward waves are equal and not complex conjugates. Therefore, the circuit element has new properties that have not been examined in the conventional circuit elements. In addition, the circuit element is reciprocal but not lossless.
The equation of the circuit element obtained using the Riccati differential equation is given by Eq. (12.12) and is not in the form of the extended telegrapher’s equations. In the following section, we attempt to express the circuit element in the form of the extended telegrapher’s equations.

7. New Extended Telegrapher’s Equations

Equation (12.12) is considered to be different from the extended telegrapher’s equations. We slightly modify Eq. (12.12) to obtain new extended telegrapher’s equations.

As shown in Eq. (11.7b) in Session 11, the speed of light $c$ is given by

$$c^2 = \frac{1}{LC} \quad (12.18)$$

Moreover, the lossless telegrapher’s equations are Laplace transformed and are given by

$$\frac{d}{dx} (V(x)) + \begin{pmatrix} 0 & j\omega L \end{pmatrix} V(x) = \begin{pmatrix} 0 \end{pmatrix} \quad (12.19)$$

When the telegrapher’s equations are extended using the Riccati differential equation shown in Session 10, zeros in the diagonal elements of the matrix in Eq. (12.19) are replaced with nonzero values, to obtain the Pasteur and Tellegen media in Session 10, as well as to obtain the one-dimensional Dirac equation given by Eq. (12.9). Therefore, we call the circuit elements given by Eqs. (12.17) and (12.25) neu-trinos and discuss them in detail later.

8. Application of Riccati Differential Equation to Add Diagonal Elements

In Session 11, we adopted an extension method in which the partial derivative with respect to the position on the right-hand side of the Dirac equation is moved to the left-hand side and the wave function with a coefficient of $mc^2/h$ is moved to the diagonal element. This extension method is applied to the modified Dirac equation given by Eq. (12.9) in this section. In concrete terms, the extension method is first applied to Eq. (12.11) directly obtained from the modified Dirac equation. Then, the obtained equation is rewritten in the form of the telegrapher’s equations and is extended using the Riccati differential equation. Thus, we discuss the equation with nonzero diagonal elements.

8.1 Moving of wave function in Eqs. (12.11a) and (12.11b)

When the wave functions with a coefficient of $jmc^2/h$ on the right-hand sides of Eqs. (12.11a) and (12.11b) are replaced with those in the left-hand sides, we obtain

$$jc \frac{\partial}{\partial x} \psi_4 = \frac{jmc^2}{h} \psi_4 - j \frac{\partial}{\partial t} \psi_4 \quad (12.26a)$$

$$jc \frac{\partial}{\partial x} \psi_1 = -j \frac{\partial}{\partial t} \psi_1 - \frac{jmc^2}{h} \psi_1 \quad (12.26b)$$

Each term in Eqs. (12.26a) and (12.26b) is multiplied by $j/c$ and Laplace transformed, and then $\psi_1$ and $\psi_4$ are replaced with current $I(x)$ and voltage $V(x)$, respectively. As a result, the partial time derivative $\partial \psi_4 / \partial t$ can be written as $j\omega$ and we obtain

$$-\frac{d}{dx} V(x) = \begin{pmatrix} \frac{mc}{h} & j \omega c \\ j \omega c & \frac{mc}{h} \end{pmatrix} \begin{pmatrix} V(x) \\ I(x) \end{pmatrix} \quad (12.27)$$

The eigenvalues ($\gamma$) of the matrix in Eq. (12.27) are obtained from the eigenequation given by

$$\left[ \begin{array}{cc} \cos \beta_i l & jZ_{0i} \sin \beta_i l \\ jZ_{0i} \sin \beta_i l & \cos \beta_i l \end{array} \right]$$

From these equations, the cascade matrix of a circuit element with a length of $l$ is given by

$$\begin{pmatrix} \cos \beta_i l & jZ_{0i} \sin \beta_i l \\ jZ_{0i} \sin \beta_i l & \cos \beta_i l \end{pmatrix} \quad (12.25)$$

The circuit element with a cascade matrix given by Eq. (12.25) should be given a name because it has new properties, similar to the circuit element given by Eq. (12.17). In addition, the name should be related to quantum theory because the circuit element was obtained from the modified Dirac equation given by Eq. (12.9). Therefore, we discuss the equation with nonzero diagonal elements.

8.2 Moving of wave function in Eqs. (12.11a) and (12.11b)
which could represent a circuit, is given by

\[
\det \left( \begin{array}{cc}
\frac{mc}{h} - \gamma & j\omega c \\
\frac{mc}{h} & j\omega - \gamma
\end{array} \right) = 0
\]  

(12.28a)

Namely, the eigenvalues and the phase constant \( \beta_b \) are expressed by

\[
\gamma = \pm j\sqrt{\frac{1}{c^2} - \frac{m^2c^2}{h^2}} = \pm j\beta_b
\]  

(12.28b)

The characteristic impedances of the forward and backward waves corresponding to the thus-obtained phase constants are complex conjugates and are denoted as \( Z_{0f} \) and \( Z_{0b} \), respectively, given by

\[
Z_{0f} = \sqrt{1 - \frac{m^2c^4}{\omega^2h^2} + j\frac{mc^2}{\omega h}}
\]  

(12.29a)

\[
Z_{0b} = \sqrt{1 - \frac{m^2c^4}{\omega^2h^2} - j\frac{mc^2}{\omega h}} = Z_{0f}^* \quad \text{or } Z_{0f}^*
\]  

(12.29b)

Using the \( \beta_b \) [Eq. (12.28b)], \( Z_{0f} \) [Eq. (12.29a)], and \( Z_{0b} \) [Eq. (12.29b)] thus obtained and two integral constants \( N_1 \) and \( N_2 \), the voltage and current that satisfy Eq. (12.27) are respectively given by

\[
V(x) = N_1Z_{0f}\exp(-j\beta_b x) + N_2Z_{0b}\exp(j\beta_b x)
\]  

(12.30a)

\[
I(x) = N_1\exp(-j\beta_b x) - N_2\exp(j\beta_b x)
\]  

(12.30b)

From these equations, the cascade matrix of a transmission line with a length of \( l \) that satisfies Eq. (12.27), which could represent a circuit, is given by

\[
\frac{1}{Z_{0f} + Z_{0b}} \begin{bmatrix}
Z_{0f}e^{\beta_b l} + Z_{0b}e^{-\beta_b l} & Z_{0f}Z_{0b}(e^{\beta_b l} - e^{-\beta_b l}) \\
e^{\beta_b l} - e^{-\beta_b l} & Z_{0b}e^{\beta_b l} + Z_{0f}e^{-\beta_b l}
\end{bmatrix}
\]  

(12.31)

The circuit element given by this equation is reciprocal and lossless.

### 8.2 Application of telegrapher’s equations to Eq. (12.27)

Equation (12.27) is obtained from the modified Dirac equation given by Eq. (12.9) and has a matrix whose nondiagonal elements are given by \( j\omega / c \). However, the nondiagonal elements of the lossless telegrapher’s equations given by Eq. (12.19) are given by \( j\omega L \) and \( j\omega C \). When the nondiagonal elements of the matrix in Eq. (12.27) are replaced with those in the lossless telegrapher’s equations, we obtain

\[
\frac{d}{dx} \begin{bmatrix}
V(x) \\
I(x)
\end{bmatrix} + \begin{bmatrix}
\frac{mc}{h} & j\omega L \\
\frac{mc}{h} & j\omega C
\end{bmatrix} \begin{bmatrix}
V(x) \\
I(x)
\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]  

(12.32)

The eigenvalues \( \gamma \) of the matrix in Eq. (12.32) are obtained from the eigenequation given by

\[
\det \left( \begin{array}{cc}
\frac{mc}{h} - \gamma & j\omega L \\
\frac{mc}{h} & j\omega C - \gamma
\end{array} \right) = 0
\]  

(12.33a)

Namely, the eigenvalues and the phase constant \( \beta_c \) are expressed by

\[
\gamma = \pm j\sqrt{\frac{1}{c^2} - \frac{m^2c^2}{h^2}} = \pm j\beta_c
\]  

(12.33b)

The characteristic impedances of the forward and backward waves corresponding to the thus-obtained phase constants \( \beta_c \) are complex conjugates and denoted as \( Z_{0f} \) and \( Z_{0b} \), respectively, given by

\[
Z_{0f} = \sqrt{\frac{L}{C} - \frac{m^2c^4}{\omega^2c^2h^2} + j\frac{mc}{\omega Ch}}
\]  

(12.34a)

\[
Z_{0b} = \sqrt{\frac{L}{C} - \frac{m^2c^4}{\omega^2c^2h^2} - j\frac{mc}{\omega Ch}} = Z_{0f}^* \quad \text{or } Z_{0f}^*
\]  

(12.34b)

Using the \( \beta_c \) [Eq. (12.33a)], \( Z_{0f} \) [Eq. (12.34a)], and \( Z_{0b} \) [Eq. (12.34b)] thus obtained and two integral constants \( N_1 \) and \( N_2 \), the voltage and current that satisfy Eq. (12.32) are given by

\[
V(x) = N_1Z_{0f}\exp(-j\beta_c x) + N_2Z_{0b}\exp(j\beta_c x)
\]  

(12.35a)

\[
I(x) = N_1\exp(-j\beta_c x) - N_2\exp(j\beta_c x)
\]  

(12.35b)

From these equations, the cascade matrix of a transmission line with a length of \( l \) that satisfies Eq. (12.32), which could represent a circuit, is given by

\[
\frac{1}{Z_{0f} + Z_{0b}} \begin{bmatrix}
Z_{0f}e^{\beta_c l} + Z_{0b}e^{-\beta_c l} & Z_{0f}Z_{0b}(e^{\beta_c l} - e^{-\beta_c l}) \\
e^{\beta_c l} - e^{-\beta_c l} & Z_{0b}e^{\beta_c l} + Z_{0f}e^{-\beta_c l}
\end{bmatrix}
\]  

(12.36)

The circuit element given by this equation is reciprocal and lossless.

### 9. Notes on Coefficient in Dirac Equation and Future Tasks

In Session 8, we demonstrated that circuit theory can be applied to the Schrödinger equation, a basic equation in quantum mechanics. We consider that the characteristic quanta, such as electrons, protons, and neutrons, in quantum mechanics and quantum theory can be expressed using characteristic circuit elements. In Session 10, we showed that the telegrapher’s equations can be extended using the Riccati differential equation and that characteristic circuit elements such as Pasteur media can be obtained.

The Dirac equation has also been proposed as a means of combining quantum theory and the special theory of relativity. If circuit theory can be applied to this Dirac equation, the importance of circuit theory in quantum theory will further increase. We believe that the possibility of applying circuit theory to the Dirac equation has been
shown in Session 11 and this session; however, an important problem still remains.

This problem is related to energy. Because the Dirac equation is related to the special theory of relativity, energy that matches the theory of relativity is assumed and expressed by Einstein’s well-known formula $E = mc^2$ [Eq. (12.6)].

In Ref. 5, it is stated that Einstein’s theory is correct as far as we know and is thereby considered important, but it is not holy. Frankly speaking, the theory will be replaced with something better if it appears. Similarly, the great scientist should be esteemed as a diligent contributor who helped us understand the nature of the Universe, rather than a prophet.

Assisted by the above statement in Ref. 5, we attempt to reexamine the energy in the ordinary world. As a reason for this, a huge amount of energy can be obtained by the nuclear fission of condensed uranium in the ordinary world, but the energy is considered to be much lower than $E = mc^2$. Therefore, we denote the energy as $\delta$ and discuss the value of $\delta$

For example, to make the square root in Eq. (12.28b) positive, $\delta$ should be smaller than $\omega \delta h$. As described before, $\delta$ is $mc^2$, which is found in Eqs. (12.1) and (12.9). In this session and Session 11, each “$mc^2$” used in the Dirac equation is replaced with “$\delta$”, which should be smaller than $\omega \delta h$.

Here, $\delta$ is considered to be related to spin in quantum mechanics. In future work, we will verify that $\delta$ for spin 1/2 electrons, neutrons, and protons can be set as

$$\delta = \frac{1}{2} \omega \delta h$$  \hspace{1cm} (12.37)

In addition, we will also show that it is appropriate to consider that the element given by Eq. (12.36) is a Pasteur medium, described in Session 10, and represents a proton or neutron. In the future, we will also explain that the circuit element that satisfies the one-dimensional Dirac equation in Session 11 should represent electrons.

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References


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