LECTURE

Circuit Theory Based on New Concepts and Its Application to Quantum Theory

13. Reconsideration of Transmission Circuit Theory for Applying It to Quantum Theory

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Abstract To apply circuit theory to quantum theory, we review the problems of circuit theory described in the past sessions. In concrete, the physical properties of circuits, such as losslessness, steady-state response, and reactive power, are reexamined. It is shown that the Laplace transform is more advantageous than Heaviside's operational calculus and that active power can be conserved when a lossless circuit is used. It is also demonstrated that a forward wave $\exp(-j\beta x)$ travels clockwise because $\exp(j\omega t)$ in the Laplace transform is required for separating the variables of partial differential equations related to waves.

Keywords: Hamiltonian operator, Heaviside's operational calculus, Laplace transform, lossless, active and reactive powers, cascade matrix, steady-state, resonance, Schrödinger equation, transmission circuit theory

1. Introduction

In Session 8, we showed that circuit theory can be applied to the Schrödinger equation, a basic equation in quantum mechanics. Then, we considered the possibility of expanding the application range of circuit theory to quantum theory and elementary particle theory. In Sessions 11 and 12, we applied circuit theory to the Dirac equation, a basic equation in quantum theory. As a result, we found that the waves satisfying the Dirac equation are those in the stopband in an ordinary world with the same properties as the waves in the tunneling effect. This is because the Dirac equation was derived by discussing how to integrate quantum theory with the special theory of relativity and uses Einstein's well-known formula $E = mc^2$, meaning that the energy is excessively large.

In this session, we review and reconsider the waves treated in circuit theory used in an ordinary world. We focus on the properties that were not treated in quantum theory, that is, losslessness, reactive power, and resonance, which are fully discussed below.

2. Waves in Mechanics

Quanta, such as photons and electrons, are considered to have both particle and wave nature simultaneously. In quantum mechanics, however, the particle nature is mainly

focused on. For example, the Schrödinger equation is known as a basic equation in quantum mechanics and is considered to represent wave mechanics in quantum mechanics.[1] However, the handling of waves in conventional quantum mechanics is unsatisfactory compared with that in circuit theory. In this section, we start our discussion with the reason why Maxwell's equations are handled in mechanics.

In the law of energy conservation in mechanics, the sum of kinetic and potential energies is constant. According to Ref. [2], the law of energy conservation in mechanics is applied to Maxwell's equations because the magnetic and electric energies in Maxwell's equations are similar to the kinetic and potential energies, respectively. Thus, Maxwell derived his equations using scalar and vector potentials. When electrons exist, electric charges exist and so electric energy also exists, which is expressed by a scalar potential. When the electrons move, a magnetic field is generated and its potential is expressed by a vector potential. Namely, the existence and transfer of electrons are expressed by scalar and vector potentials, respectively.

Maxwell's equations are spatially expressed using three-dimensional functions and are analyzed in analytical mechanics,[3] which starts from Newtonian mechanics. Problems that are difficult to solve with Newtonian mechanics were elegantly solved by Lagrange. The Lagrange equation is a generalized form of Newton's

equation and does not change its form whichever coordinates are used.[3] In Ref. [3], Soda stated the following: “Starting from the Lagrange equation, the most important formulation of mechanics in modern physics is by Hamilton’s principle, which allows a very compact and beautiful formulation of classical mechanics. However, a more important point of this principle is that classical electromagnetics can be formulated by applying the same variational principle to an appropriate Lagrange function or that quantum mechanics can be formulated by extending Hamilton’s principle.”

The above-mentioned Hamilton’s principle can be extended to the formulation of the Schrödinger equation.[4] This will be briefly explained in Section 4.

For the extended theory of quantum mechanics, such as the quantum theory of fields and elementary particle theory, Muta stated that elementary particles can all be considered to be quantized fields and satisfy different equations of fields (wave equations) according to their properties, e.g., mass and spin. The equations of fields include Maxwell’s equations, the Klein–Gordon equation, the Dirac equation, and the Yang–Mills equation.[5]

As above, quantum mechanics and its extended theories, such as the quantum theory of fields and elementary particle theory, are considered to be formulated by the Hamiltonian operator, which is related to the law of energy conservation in mechanics.

3. Waves in Circuit Theory

As described in Section 2, in physics, Maxwell’s equations are expressed using scalar and vector potentials and formulated by applying Hamilton’s principle. In contrast, Heaviside removed the scalar and vector potentials by using rotation (rot) in vector analysis and proposed the handling of waves in a theoretical system using voltage and current.[6]

Research fields involving voltage and current include circuit theory, in which researchers examine which properties of waves can be obtained using circuit elements such as coils and capacitors and to what the properties can be applied. As a result, it has been found that magnetic energy stored in coils and electric energy stored in capacitors are reactive powers and, strictly speaking, are not energies even though they are called energies. By using circuit theory, it has been shown that coils and capacitors are lossless circuit elements that store reactive power.

Maxwell’s equations are spatially expressed using three-dimensional functions. Therefore, circuit theory with a one-dimensional theoretical spatial system is not always applicable to Maxwell’s equations. To compensate for this disadvantage, circuit theory was applied to microwave circuits and a commensurate transmission line circuit was developed as an element applicable to ICs using telegrapher’s equations.[7] A unit element was defined in

the commensurate transmission line circuit. It was shown in Session 3 that active and reactive powers can be stored in a circuit constructed using unit elements, meaning that the unit elements are lossless circuit elements.

As mentioned above, only the law of energy conservation is assumed in physics, whereas circuit theory involves properties that are not considered in physics, such as losslessness. Moreover, circuit theory is a theoretical system that has physical properties making it effective for addressing problems such as the differences between Heaviside’s operational calculus and the Laplace transform, the discrimination of transient and steady-state responses, and the differences between active power (energy) and reactive power. The solutions to these problems based on physical properties are shown in the following sections.

3.1 Heaviside’s operational calculus or Laplace transform

In circuit theory, researchers sometimes discuss the differences between Heaviside’s operational calculus and the Laplace transform, which is more appropriate for circuit theory. The differences are discussed here.

In Ref. [6], Nahin stated: “Heaviside introduced a new and radical mathematical attack (operational calculus) on physical problems which was very powerful but also very obviously full of holes.”

In addition, he mentioned Heaviside’s operational method and the Laplace transform as follows: “When the second edition appeared in 1960, after calling both the Laplace transformation method and Heaviside method ‘expeditious,’ Agnew went on to say that ‘The Laplace transformation method seems to have completely won the battle for attention because it is more interesting, it is easier to apply to routine problems, and ... it is more versatile.’”[6]

The mathematical advantages and disadvantages of the two methods seem to be completely interpreted in the above. However, Nahin described Heaviside’s method from the viewpoint of physical phenomena in Ref. [6]: “...it [the theory of wave propagation along wires] has been worked out in a very complete manner by Mr. Oliver Heaviside; and Mr. Heaviside has pointed out and accentuated this result of his mathematical theory – that electromagnetic induction is a positive benefit: it helps to carry the current. It is the same kind of benefit that mass is to a body shoved along a viscous resistance ...”

Moreover, the features of Heaviside’s work are well described in Ref. [6]; this description is slightly long but quoted verbatim as follows: “There is strange paradox involved in trying to understand just what it was Oliver Heaviside did. To understand Einstein’s special theory of relativity, for example, one needs mathematics no more advanced than algebra. To truly delve into Heaviside’s major works, however, requires a bit more, indeed some fairly heavy mathematical artillery. The paradox in this is that Einstein’s theory applies almost everywhere in the

universe (one notable exception is the inside of a black hole where nobody knows what happens), while Heaviside's efforts are mostly applicable to much more restrictive situations, e.g., the insides of an electric cable. It seems superficially that more mathematics is required to understand the details of a telephone circuit than to comprehend the most intimate secrets of Mother Nature.

Another part of the paradox is that no matter how much theoretical physicists may grumble about it, Oliver Heaviside's work on how to make a decent telephone cable plays a vastly greater role in our everyday lives than does Einstein's work (knowledge of the equation behind The Bomb, $E=mc^2$, is not essential for its making, i.e., Hiroshima and Nagasaki would have been doomed even had Einstein never lived). We all make several telephone calls nearly every day, and we expect those on the other end of the line to be able to understand the sounds that emerge from their receivers. How often, on the other hand, does the conduct of your life depend on the nuances of relativity?"

From another viewpoint, the above sentences can be interpreted as follows: The physical phenomena required to realize a telephone cable were understood by Heaviside alone, and physicists could not explain some physical phenomena even though they are necessary in daily lives. According to Ref. [6], Heaviside might have carried out an abstract study on the basics of quantum mechanics in the beginning of the 20th century. Therefore, he would be studying quantum mechanics and quantum theory if he lived now.

Anyway, Heaviside was a researcher who lived 100 years ago. In those days, the treatment of loss terms for the inside of a telephone cable was considered to be very difficult, unlike that for current telephone cables. Today, the interpretation of losslessness has been established and the transient and steady-state responses can be distinguished. By reconsidering circuits on the basis of such theory, the physics of telephone cables can be understood. This will settle the battle between Heaviside's operational calculus and the Laplace transform, as discussed in the following sections.

3.2 Losslessness and steady state

Many parts of circuit theory are based on lumped constant circuits and treat positive real functions and matrices.[8] In this case, lossless circuits are expressed as reactance circuits. However, reactance circuits are mathematically defined and considered inappropriate for the physical interpretation of losslessness. Thus, we first reexamine losslessness.

A definition that enables the appropriate physical interpretation of losslessness is given in Ref. [9]. We sometimes find articles that cite Ref. [9] but rarely find articles that give an accurate interpretation of the physical phenomena written in the reference. By referring to Ref. [10], we examine the physical phenomena written in Ref. [9] below.

In Ref. [9], a cascade matrix is used to derive the condition for lossless circuits. Hence, we first define a cascade matrix as

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix} \quad (13.1)$$

Here, the positive direction of current I_2 is from left to right.

When port 1 is assumed to be the input terminal of a circuit with a cascade matrix given by Eq. (13.1), twice the active power determined by the voltage and current at the input terminal is equal to twice the real part of the complex power and is given by

$$\begin{aligned} 2\text{Re}\{V_1 * I_1\} &= V_1 * I_1 + I_1 * V_1 \\ &= (V_1 * I_1) \begin{pmatrix} I_1 \\ V_1 \end{pmatrix} \\ &= (V_2 * I_2) \begin{pmatrix} A^* & C^* \\ B^* & D^* \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix} \end{aligned} \quad (13.2)$$

Here, $\text{Re}\{\}$ represents the real part in $\{\}$.

Similarly, twice the active power determined by the voltage and current at the output terminal (port 2) is given by

$$V_2 * I_2 + I_2 * V_2 = (V_2 * I_2) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix} \quad (13.3)$$

The active power at the input terminal must be equal to that at the output terminal when the two-port circuit is lossless. This condition is summarized as follows.

[Condition for cascade matrix of lossless circuits]

The cascade matrix of a two-port lossless circuit should be given by

$$\begin{pmatrix} A^* & C^* \\ B^* & D^* \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (13.4)$$

The cascade matrix that satisfies Eq. (13.4) is called a J-unitary matrix.[9]

As mentioned above, the condition for two-port lossless circuits given in Ref. [9] is expressed using a cascade matrix and active power. Power can be divided into active and reactive powers when the circuit is in the steady state. Therefore, the condition for lossless circuits is determined when the circuit is in the steady state.

In physics, energy is considered to be an important physical quantity; it is discussed in terms of potential and is related to movement. In electronics and electromagnetics, the product of voltage and current is defined as power, and the power in the steady state can be divided into active and reactive powers. In circuit theory, there are two types of response: the transient response caused by the movement of electrons and the steady-state response as seen inside a telephone cable. The steady-state response is slightly different from a physical phenomenon in mechanics and is considered to be unique to circuit theory.

Here, we examine the physical interpretation of the condition for lossless circuits given by Eq. (13.4). Although the effectiveness of the cascade matrix was partly explained in Session 6, we need to reexamine the importance of Eq. (13.4) and the equations used to derive Eq. (13.4), i.e., Eqs. (13.2) and (13.3). As expressed by Eq. (13.4), the active powers at the input and output terminals of a lossless circuit are equal, meaning that the active power is conserved. By using a cascade matrix, a circuit can be simply divided into two circuits, that is, a circuit on the side of the input terminal has another terminal, which is assumed to be the second output terminal. The active power at this output terminal is also equal to that at the input terminal. Thus, the active power at the input terminal is maintained at any points inside the lossless circuit. This means that the inside of the lossless circuit in the steady state maintains the same active power. When an active power is applied to the input terminal, it is immediately transferred to the output terminal and output from the output terminal. Therefore, even though a circuit has a certain size and it takes time for the voltage and current waves to travel inside the circuit, the active power is input and output at the same time, and the steady-state response is independent of time. In other words, there is no time derivative term in the steady state.

The action in a telephone cable is that in the steady state. Heaviside attempted to show the steady-state response of a telephone cable, but he had difficulty in explaining the steady state because telephone cables in those days had a large amount of loss and were not assumed to be lossless.

Considering that the steady-state expression has no time derivative term, we can explain the difference between Heaviside's operational calculus and the Laplace transform. When the Laplace transform is performed, the obtained equation has no time derivative term. For Heaviside's operational calculus, however, we find the following equation in Ref. [6].

$$p^{1/2} \cdot 1 = \frac{0!}{\left(-\frac{1}{2}\right)!} t^{-1/2} = 1/\sqrt{\pi t} \quad (13.5)$$

This equation still has the time variable t and is different from the solution in the steady state. Therefore, it is concluded that the Laplace transform should be used to discuss the steady state and losslessness.

3.3 Resonance in steady state

In this section, we examine the steady-state response. Such response is not compatible with the theory of relativity because there is no time derivative term in the steady state. This may be undesirable when aiming to integrate quantum theory and the theory of relativity in the realm of physics. However, the oscillation of a crystal is resonance, that is, a physical phenomenon in the steady state. If the oscillation of an atomic nucleus composed of neutrons and protons can be explained by the oscillation of a crystal, the analysis of

the steady state may be useful. Therefore, we discuss the steady state below.

The power source of a circuit, together with its internal resistance, supplies energy defined as the maximum available power. When the power source is combined with a lossless circuit, a certain amount of active power is determined within the lossless circuit in the steady state. When the active power is equal to the maximum available power, resonance occurs, and its frequency is called the resonance or eigen-oscillation frequency. Waves with a frequency other than the resonance frequency carry less active power than the maximum available power within the lossless circuit. Thus, even if a power source that can supply the active power defined as the maximum available power is used, the maximum available power is supplied to a load only at the resonance frequency. At the other frequencies, an active power smaller than the maximum available power is supplied.

In mechanics, there are two important laws of conservation, i.e., the laws of energy and momentum conservation. As described in Session 4, when two moving objects collide and coalesce, heat should be generated to satisfy the two laws of conservation.

No heat is considered to be generated in lossless circuits. Then, the following question arises when two lossless circuits are connected that corresponds to the above-mentioned mechanical phenomenon: What occurs at the connection point of transmission lines with different characteristic impedances?

In the above case, waves are instantaneously reflected and transmitted, which are specific phenomena in circuits, and a transient phenomenon occurs. Here, the reflection specific to circuits is explained as follows. The reflected voltage and current waves have a sign (plus or minus), and a phase difference between voltage and current waves is caused by the reflection of the waves. The only method of accurately representing a transient phenomenon is the z -transform. The z -transform can be applied to commensurate transmission line circuits[7] as well as to the circuits of resonant tunneling diodes (RTDs) described in Session 7. Standing waves exist in the steady state, resonance occurs, and a certain amount of active power is stored in the RTDs. In addition, reactive power is also stored in the RTDs. When two lossless lines (or circuits) are connected, reactive power is stored in the circuit and corresponds to heat in mechanics. It is also understandable that transient phenomena called multiple reflections are required for the circuit to become resonant. In conclusion, even circuits that cannot be represented by transmission lines, such as LC ladder circuits, could reach the steady state after multiple reflections occur as transient phenomena, and reactive power is generated in the lossless circuits. This indicates that reactive power is stored also in filters and resonant circuits.

3.4 Distortionless circuit

Here, we examine a distortionless circuit described in Ref. [6], which is considered to be not lossless but to show a steady-state response.

In Ref. [6], Nahin discussed the work of Heaviside, a researcher who lived over 100 years ago, and derived a condition for distributed constant lines that would give a distortionless circuit using loaded coils for a telephone cable. The characteristic impedance $Z(p)$ of the distributed constant line is expressed by

$$Z(p) = \sqrt{\frac{R + Lp}{G + Cp}} \quad (13.6)$$

Assuming that $L/R = C/G$, we obtain $Z(p) = \sqrt{R/G}$, which is independent of p . This is the condition for distortionless circuits proposed by Heaviside.

However, the distributed inductance L_0 of an actual telephone cable is smaller than L , which satisfies the condition for distortionless circuits.

$$L_0 < L \quad (13.7)$$

The characteristic impedance of an actual telephone cable is complex, and the ratio of the imaginary part to the real part becomes greater as the length of the cable increases. When such a telephone cable is connected to a load, significant reflection occurs.

For the transmission of waves along a telephone cable, we should consider the loss caused by resistance and the attenuation related to reflection and transmission caused by impedance mismatching. Improving the transmission characteristics by loading coils in the cable is a method of impedance matching and uses parameters of the image connection,[11] which is considered to have been proposed by Heaviside.

The method of loading coils in a cable described in Ref. [11] is modified and described here. A telephone cable with a complex characteristic impedance is cut into $2n$ cables with an equal length. Each cable is assumed to be a half-section circuit and its cascade matrix $[F_c]$ is determined. Next, each cable represented with cascade matrix $[F_c]$ is cascade-connected (loaded) by a series coil of inductance L_x to determine the cascade matrix $[F_1]$, which is represented by the connected style of circuit elements as $[F_c]-L_x$. Two image impedances of the circuit represented with cascade matrix $[F_1]$ are determined. The image impedances of the circuit are complex but become real at a certain angular frequency ω_0 . The inductances L_{x1} and L_{x2} of the loaded coil when the image impedances become real are determined.

We obtain two sorts of basic symmetric loaded cables with real image impedances at ω_0 represented by $[F_c]-L_{x1}$ - $[F_c]$ and L_{x2} - $[F_c]$ - $[F_c]$ - L_{x2} , when each telephone cable loaded with a coil is image-connected. Thus, we obtain two sorts of n basic loaded telephone cables. By cascade-connecting these n basic loaded cables, impedance mismatching is addressed and a loaded telephone cable with

improved transmission characteristics is obtained. Note that the improved characteristics is exhibited in the steady state.

In this method, the actual loaded telephone cable is not pure distributed constant line because lumped constant circuits are series-connected to distributed constant circuits. Therefore, reflection and transmission instantaneously occur at the connection points between the telephone cable and the coil. As a result, a transient response occurs and then the circuit reaches the steady state. The loaded telephone cable should not satisfy the conditions for distortionless circuits, that is, the loaded telephone cable is not a distortionless circuit. Therefore, this loaded telephone cable cannot be used as a digital circuit and has a frequency pass-band around the angular frequency ω_0 .

Thus, circuits in which elements with different impedances are connected are inappropriate as digital circuits. Circuits with a resonance frequency imply that multiple reflections have taken place and they cannot be distortionless circuits.

As mentioned above, apparently mysterious physical phenomena may be explained by considering the conditions for lossless circuits and the steady state. In the future, we will discuss quantum theory while examining these aspects.

4. Circuit Elements Obtained from Partial Differential Equation

Circuit theory in general assumes lumped constant circuits having coils and capacitors as circuit elements, and has a theoretical system represented by differential equations. One may think that circuit theory cannot be applied to quantum mechanics because the Schrödinger equation, a basic equation in quantum mechanics, is given by a partial differential equation. However, transmission line circuits, which are covered by circuit theory, are given by partial differential equations called the telegrapher's equations. Therefore, it is considered that the theory of transmission line circuits could be applied to quantum mechanics.

The condition for lossless lumped constant circuits is that of reactance, which is mathematically defined. If lossless condition is assumed to be defined for transmission line circuits, as described in Section 3.3, the lossless condition can be interpreted as a physical condition that the active power is maintained throughout the circuit in the steady state. Thus, some physical phenomena are considered to be unexplainable for lumped constant circuits but can be easily understood for transmission line circuits. Therefore, we first examine the physical phenomena of transmission line circuits and then attempt to apply the theory of transmission circuits to partial differential equations such as the Schrödinger equation.

4.1 Unit element

The physical properties of circuits, such as losslessness, are mostly given by a cascade matrix. For transmission lines, the forward and backward waves can be determined for both voltage and current waves. We first discussed the use of these waves by referring to the solutions of the lossless telegrapher's equations discussed in Session 2.

By denoting the instantaneous voltage and current as $v(x,t)$ and $i(x,t)$, respectively, the lossless telegrapher's equations are given by the following pair of linear partial differential equations:

$$-\frac{\partial}{\partial x}v(x,t) = L \frac{\partial}{\partial t}i(x,t) \quad (13.8a)$$

$$-\frac{\partial}{\partial x}i(x,t) = C \frac{\partial}{\partial t}v(x,t) \quad (13.8b)$$

The steady-state solutions of the lossless telegrapher's equations are determined by the Laplace transform. Namely, the only term dependent on time t is $\exp(j\omega t)$, and voltage and current are respectively given by

$$v(x,t) = V(x)\exp(j\omega t) \quad (13.9a)$$

$$i(x,t) = I(x)\exp(j\omega t) \quad (13.9b)$$

As an eigenvalue problem of the above differential equations, the propagation constant γ and the characteristic resistance R_0 (as the characteristic impedance) are respectively given by

$$\gamma = j\beta = j\omega/c = j\omega\sqrt{LC} \quad (13.10a)$$

$$R_0 = \sqrt{L/C} \quad (13.10b)$$

Here, c is the velocity of light and β is the phase constant.

Using R_0 and β and denoting the constants of integration as N_1 and N_2 , voltage and current are expressed by

$$V(x) = N_1 R_0 \exp(-j\beta x) + N_2 R_0 \exp(j\beta x) \quad (13.11a)$$

$$I(x) = N_1 \exp(-j\beta x) - N_2 \exp(j\beta x) \quad (13.11b)$$

The first and second terms of the right-hand sides represent the forward and backward waves, respectively. Using Eqs. (13.9a) and (13.9b), the forward and backward waves are given as

$$\text{Forward wave:} \quad \exp\left(j\omega t - j\frac{\omega}{c}x\right) \quad (13.12a)$$

$$\text{Backward wave:} \quad \exp\left(j\omega t + j\frac{\omega}{c}x\right) \quad (13.12b)$$

The forward wave given by Eq. (13.12a) is rewritten as

$$\exp\left(j\omega t - j\frac{\omega}{c}x\right) = \exp\left[j\frac{\omega}{c}(ct - x)\right] \quad (13.13)$$

In physics, the forward wave is generally expressed by a function given by

$$f(x - ct) \quad (13.14)$$

In circuit theory, the wave given by Eq. (13.14) has a term of $\exp(-j\omega t)$, that is, the wave has a negative angular frequency ($-\omega$), and the sign of x is positive, meaning that the wave is the backward wave.

In physics, moving particles expressed as a forward wave are considered to rotate clockwise. Hence, the wave is considered to be expressed by

$$\exp(-j\omega t) \quad (13.15)$$

In circuit theory, the forward wave is given by $\exp(-j\beta x)$ and interpreted as rotating clockwise. Thus, circuit theory only expresses one-dimensional (x direction) movement, which can represent rotation and spin.

4.2 Schrödinger equation

In Session 8, we demonstrated that circuit theory can be applied to the Schrödinger equation specific to quantum mechanics. As shown by Eq. (8.5) in Session 8, the steady-state response of the Schrödinger equation is expressed using the Hamiltonian operator H as

$$\begin{aligned} -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi(x) + U\psi(x) &= H\psi(x) \\ &= \hbar\omega\psi(x) = E\psi(x) \end{aligned} \quad (13.16)$$

This equation does not include a time derivative term. In physics, this equation can be extended to an equation having a time derivative term as

$$\left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U\right) \Psi(x,t) = j\hbar \frac{\partial}{\partial t} \Psi(x,t) \quad (13.17)$$

When the variables of Eq. (13.17) are separated, the solution is given by

$$\Psi(x,t) = \psi(x)\exp(-j\omega t) = \psi(x)\exp\left(-j\frac{E}{\hbar}t\right) \quad (13.18)$$

As described in the previous section, the time derivative term is given by $\exp(j\omega t)$ in circuit theory. Therefore, the Schrödinger equation having a time derivative term is given by Eq. (13.19) as explained in Session 8.

$$\left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U\right) \Psi(x,t) = -j\hbar \frac{\partial}{\partial t} \Psi(x,t) \quad (13.19)$$

Quantum mechanics started from the following quantum hypothesis proposed by Planck: electromagnetic waves inside a black body are emitted and absorbed by oscillators composed of electrons and ions.[1] According to Ref. [1], Planck assumed the oscillators inside a material to be the carriers of energy elements, whereas Einstein considered that the electromagnetic waves themselves can be bunches of energy.

The quanta in the above-mentioned Planck's theory are the oscillators of molecules, which are considered to satisfy the thermal (heat) conduction equation. On the other hand, the electromagnetic waves in Einstein's theory satisfy Maxwell's equations. As shown in Session 4, a lossy circuit element that generates heat is obtained from the heat conduction equation, whereas a lossless circuit element that generates no heat is obtained from the electromagnetic waves. When a circuit element that satisfies the

Schrödinger equation is obtained, judging whether the circuit element is lossless or not may determine the direction of quantum theory. On the basis of this idea, in Session 8, we showed that the circuit element obtained from the Schrödinger equation is lossless.

We had a question: Is it possible to apply circuit theory to the Schrödinger equation used in quantum mechanics? As an example of an application of circuit theory, RTDs were discussed in Session 7.[12] RTDs are circuit elements that function as a switch but are quantum elements with a size. In the OFF state, resonance does not occur at the potential in an RTD. By applying bias voltage to change the potential structure, the RTD reaches the ON state and becomes resonant. That is, RTDs are considered to be time-variable circuit elements that achieve resonance, and the transient response of the circuit should be required. Thus, RTDs are transmission circuit elements that use resonance in the steady state of a lossless circuit.

5. Future Tasks

In this lecture series, we have treated unit elements as important elements (quanta). In the case of actual light, the refractive index is a function of frequency for media other than vacuum, and light is divided into the colors of the rainbow via a prism, showing the variational principle. In contrast, unit elements are idealized circuit elements. The characteristic resistance corresponding to the refractive index of the unit elements is constant. The phenomena of a prism cannot be realized using unit elements, but the z -transform can be applied to the unit elements, and thereby the transient phenomena caused by the reflection and transmission of waves can be demonstrated.

Losslessness is an idealized physical property. Synthesis theory in circuit theory uses losslessness and scattering matrices. Some of the difficult problems solved by Heaviside over 100 years ago can be easily explained by using losslessness. However, losslessness is discussed using voltage and current, and it is not considered to be directly used in physics.

In this session, we attempted to apply one-dimensional transmission lines, which can use losslessness, as well as their cascade matrices, steady state, and resonance to quantum theory, and discussed some problems in which circuit theory can be applied to quantum theory. Here, transmission lines are spatially represented in one dimension but can explain the spin and rotation of the elementary particles expressed by three-dimensional spatial movement. By idealizing such physical properties, circuit theory can be applied to quantum theory and is expected to easily solve difficult problems. One of the difficult problems is spin, which will be mainly discussed in the following sessions.

[Acknowledgments] We are deeply grateful to Professors Kazuhisa Watanabe and Hirofumi Sanada at Hokkaido University of Science for their continued cooperation in our discussion. We are deeply grateful to Professor Hiroshi Tanimoto at Kitami Institute of Technology for his continued cooperation in our discussion via e-mail. We also thank Dr. Jie Ren for her continued support in the development of the figures and calculations.

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