LECTURE

Circuit Theory Based on New Concepts and Its Application to Quantum Theory

15. Reconsideration of Einstein's Well-Known Formula $E=mc^2$ in Steady State

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Abstract Circuit theory includes steady-state responses that are obtained using the Laplace transform; however, they are not included in classical physics. By applying the Laplace transform to the Dirac equation as a basic equation of quantum theory, we can examine the steady-state response in quantum theory and determine the spin 1/2 as a characteristic phenomenon in this theory.

Keywords: Riccati differential equation, Dirac equation, Laplace transform, spin 1/2, Einstein's well known formula, voltage and current, steady-state, active and reactive powers, pass-band and stop-band

1. Introduction

Photons and electrons have wave-particle duality, which is a concept of physical phenomena treated in quantum mechanics and cannot be explained by Newtonian mechanics. In circuit theory, there is a steady-state response that is independent of time derivative terms. The steady-state response is not treated in the scope of classical physics, which can specify the position, direction, and velocity of an object [1]. Therefore, we consider that physical phenomena corresponding to the steady-state response exist in quantum mechanics and quantum theory. In Session 14, we assumed that the physical phenomena treated in quantum mechanics and quantum theory, such as renormalization, gauge theory, and reduction of wave packets, are the steady-state responses and attempted to apply circuit theory to quantum mechanics and quantum theory. In this session, we examine a suitable way of treating Einstein's well-known formula $E=mc^2$, included in the Dirac equation, which is derived from the combination of quantum theory and the theory of relativity, in the steady state.

2. Spin 1/2 and $E=mc^2$ in an Ordinary World

In Session 10, the telegrapher's equations were extended using the Riccati differential equation. The reason for this was that quanta have a characteristic property of spin 1/2 and this property cannot be determined from only the

telegrapher's equations. When the Riccati differential equation was used in Session 10, a calculation method that seemed to determine the spin was obtained but no proof of spin 1/2 was obtained. Then, the Dirac equation was used in Sessions 11 and 12.

The Dirac equation is a basic equation of quantum theory that can derive particles with spin 1/2 and antiparticles [2] and can clarify the excited state of electrons. Therefore, it is considered important to assume a steady state for the Dirac equation and apply circuit theory to the state.

In that case, the energy of the waves in the steady state is excessively large, although the Dirac equation uses Einstein's well-known formula $E=mc^2$ obtained from the theory of relativity, as stated in Section 9 of Session 12. When the waves are assumed to have been extended from electromagnetic waves, they are determined as waves in a stopband. Therefore, it is necessary to reconsider the energy given by $E=mc^2$ in the steady state in an ordinary world that is not related to the theory of relativity, as stated in Section 9 in Session 12.

We started this lecture series with a discussion on the reason why spin 1/2 is required in quantum theory. Now, we reconsider the energy given by Einstein's well-known formula, which was derived from the theory of relativity using the Dirac equation, in the steady state. Here, we start to reconsider the energy derived by Einstein from the viewpoint of the application of circuit theory to quantum theory.

3. Energy in Steady State for Dirac Equation

According to Ref. [1], Planck assumed that frequency packets called quanta release or absorb energy expressed by

$$E = h\nu = \hbar\omega \quad (15.1)$$

Here, h is the Planck constant.

In 1905, Einstein published five very important papers, one of which gave the well-known formula derived from the special theory of relativity that is expressed by

$$E = mc^2 \quad (15.2)$$

This equation means that the energy (E) of an object is equivalent to its mass (m). For example, when 1 g of an object is converted to energy, the converted energy is greater than the energy released from the atomic bomb dropped on Hiroshima, meaning that the energy given by Eq. (15.2) is extremely large. Both the conversion from mass to energy and that from energy to mass naturally occur between particles. However, the conversion rate greatly depends on the type of particle. For example, neutrons have a mass of 99.9% after their decay, whereas muons have only 0.5% mass after their decay.

In Session 12, we assumed the Dirac equation in the steady state and showed that the energy given by Eq. (15.2) was extremely large and quantum waves are in a stopband similar to the band of the tunneling effect. To determine a suitable value of the converted (confined) energy in the steady state, we again examine the method of determining the steady-state response from the Dirac equation obtained in Sessions 11 and 12.

As discussed in Session 11, the Dirac equation has an energy given by Eqs. (15.1) and (15.2) and is expressed by four wave functions. By selecting two of the four wave functions that express waves traveling in a single direction spatially, we can obtain the simultaneous equations given by Eqs. (11.9a) and (11.9b), which are given again below.

$$j \frac{\partial}{\partial t} \psi_1 = \frac{mc^2}{\hbar} \psi_1 - jc \frac{\partial}{\partial x_1} \psi_4 \quad (15.3a)$$

$$j \frac{\partial}{\partial t} \psi_4 = -jc \frac{\partial}{\partial x_1} \psi_1 - \frac{mc^2}{\hbar} \psi_4 \quad (15.3b)$$

In circuit theory, time derivatives are converted into $j\omega$ by the Laplace transform and are transposed to the right-hand sides. The left-hand sides of the equations should be given by a derivative with respect to x_1 , rather than time. Therefore, Eqs. (15.3a) and (15.3b) are rewritten as

$$jc \frac{\partial}{\partial x_1} \psi_4 = \frac{mc^2}{\hbar} \psi_1 - j \frac{\partial}{\partial t} \psi_1 \quad (15.4a)$$

$$jc \frac{\partial}{\partial x_1} \psi_1 = -j \frac{\partial}{\partial t} \psi_4 - \frac{mc^2}{\hbar} \psi_4 \quad (15.4b)$$

As described in Session 11, each term in these equations is multiplied by j/c and subjected to the Laplace transform. By replacing ψ_1 and ψ_4 with the current $I(x_1)$ and the voltage $V(x_1)$, respectively, a partial time derivative $\partial/\partial t$ is converted into $j\omega$ and Eqs. (15.4a) and (15.4b) are rewritten as

$$-\frac{d}{dx_1} \begin{pmatrix} V(x_1) \\ I(x_1) \end{pmatrix} = \begin{pmatrix} 0 & \frac{j}{ch}(\omega\hbar + mc^2) \\ \frac{j}{ch}(\omega\hbar - mc^2) & 0 \end{pmatrix} \begin{pmatrix} V(x_1) \\ I(x_1) \end{pmatrix} \quad (15.5)$$

This equation has both the energy in quantum theory given by Eq. (15.1) and that in the theory of relativity given by Eq. (15.2). Hence, the waves in a space where the theory of relativity holds may satisfy Eq. (15.5). However, the waves that satisfy Eq. (15.5) are in a stopband in an ordinary world. Both the [row (r)1, column (c)2] and (r2,c1) components of the matrix on the right-hand side in Eq. (15.5) are imaginary numbers. The former and latter components are positive and negative imaginary numbers, respectively, because of the extremely large energy given by mc^2 . Thus, the two components have opposite signs and the waves are in a stopband.

A straightforward method of research is to combine and develop quantum theory and the theory of relativity. However, let us first examine the ordinary world, that is, quantum theory in the steady state. Namely, we examine how much wave energy should be stored in an ordinary world to allow the existence of waves in a passband. To do this, the constant mc^2 in Eq. (15.5) is replaced with δ to obtain

$$-\frac{d}{dx_1} \begin{pmatrix} V(x_1) \\ I(x_1) \end{pmatrix} = \begin{pmatrix} 0 & \frac{j}{ch}(\omega\hbar + \delta) \\ \frac{j}{ch}(\omega\hbar - \delta) & 0 \end{pmatrix} \begin{pmatrix} V(x_1) \\ I(x_1) \end{pmatrix} \quad (15.6)$$

4. Quantum Waves with Energy

Equation (15.6) indicates that the energy δ is added to quanta such as photons that travel at the light velocity c . We examine the range of δ as a constant in which the quanta can exist as waves.

Both the (r1,c2) and (r2,c1) components of the matrix on the right-hand side of Eq. (15.6) are imaginary numbers. The range of δ in which the signs of the two components are positive is

$$0 \leq \delta < \omega\hbar \quad (15.7)$$

Next, the eigenvalues of the matrix on the right-hand side of Eq. (15.6) are obtained from the eigenequation given by

$$\det \begin{pmatrix} -\gamma & \frac{j}{ch}(\omega\hbar + \delta) \\ \frac{j}{ch}(\omega\hbar - \delta) & -\gamma \end{pmatrix} = 0 \quad (15.8a)$$

$$\therefore \gamma = \pm j \frac{\sqrt{\omega^2 \hbar^2 - \delta^2}}{c\hbar} = \pm j\beta_1 \quad (15.8b)$$

The characteristic impedances corresponding to the thus-obtained phase constant β_1 , i.e., those from the left and right sides of the circuit, are the same and equal to the characteristic resistance R_{01} given by

$$R_{01} = \sqrt{\frac{\omega\hbar + \delta}{\omega\hbar - \delta}} \quad (15.9)$$

Using the β_1 and R_{01} thus obtained and two integral constants N_1 and N_2 , we obtain the voltage and current as

$$V(x_1) = N_1 R_{01} \exp(-j\beta_1 x_1) + N_2 R_{01} \exp(j\beta_1 x_1) \quad (15.10a)$$

$$I(x_1) = N_1 \exp(-j\beta_1 x_1) - N_2 \exp(j\beta_1 x_1) \quad (15.10b)$$

From these equations, the cascade matrix of a circuit element with a length of l is given by

$$\begin{pmatrix} \cos \beta_1 l & jR_{01} \sin \beta_1 l \\ \frac{j}{R_{01}} \sin \beta_1 l & \cos \beta_1 l \end{pmatrix} \quad (15.11)$$

The phase constant of this circuit element is obtained from Eq. (15.8b) and is different by δ from the phase constant of a unit element. Therefore, it is considered that an element with rotation (spin) different from that of a unit element is obtained. However, the circuit element is symmetric and is not considered to have a sufficient spin effect. Assuming quanta to be particles rather than strings (superstrings), however, there is a limit on the wavelength relative to the size of the particles and δ can be considered to be added as the energy independent of the wavelength that is required to exceed the limit. If the energy δ corresponds to radiation, the quanta that satisfy Eq. (15.6) are considered to be X-rays or gamma rays.

5. Use of Riccati Differential Equation

As described in Session 11, we have attempted to determine the quantum spin 1/2 using the Dirac equation. Although Eqs. (15.3a) and (15.3b) were obtained, the left-hand sides of these equations comprise a partial time derivative and the right-hand sides comprise two different wave functions. In circuit theory, the left-hand sides should have a partial derivative with respect to x_1 . Hence, the partial derivatives with respect to x_1 on the right-hand sides were transposed to the left-hand sides. As a result, the obtained Eqs. (15.4a) and (15.4b) have two terms with the same wave function on the right-hand sides. By changing the wave function of the term with a coefficient of mc^2 to the same wave function as on the left-hand side, the Riccati differential equations were obtained. When this approach is applied to Eq. (15.6), we obtain

$$-\frac{d}{dx_1} \begin{pmatrix} V(x_1) \\ I(x_1) \end{pmatrix} = \begin{pmatrix} j\frac{\delta}{c\hbar} & j\frac{\omega}{c} \\ j\frac{\omega}{c} & -j\frac{\delta}{c\hbar} \end{pmatrix} \begin{pmatrix} V(x_1) \\ I(x_1) \end{pmatrix} \quad (15.12)$$

Because the diagonal terms of the matrix on the right-hand side of Eq. (15.12) are not 0, the range of δ cannot be determined from this equation. First, we solve Eq. (15.12). To do this, we determine the eigenvalues and the corresponding eigenvectors of the matrix on the right-hand side by solving

$$\det \begin{pmatrix} j\frac{\delta}{c\hbar} - \gamma & j\frac{\omega}{c} \\ j\frac{\omega}{c} & -j\frac{\delta}{c\hbar} - \gamma \end{pmatrix} = 0 \quad (15.13)$$

From Eq. (15.13), the eigenvalues are obtained as follows. Note that the eigenvalues represent the propagation constant, and if they are purely imaginary numbers, they represent the phase constant. In this case, the eigenvalues and the phase constant β_b are expressed by

$$\therefore \gamma = \pm j \sqrt{\frac{\omega^2}{c^2} + \frac{\delta^2}{c^2 \hbar^2}} = \pm j\beta_b \quad (15.14)$$

The eigenvectors corresponding to the thus-obtained two eigenvalues are given as follows. For the eigenvalue given by

$$\gamma_1 = j\beta_b \quad (15.15a)$$

the corresponding eigenvector is given by

$$\begin{pmatrix} \sqrt{1 + \frac{\delta^2}{\omega^2 \hbar^2}} + \frac{\delta}{\omega\hbar} \\ 1 \end{pmatrix} = \begin{pmatrix} R_{0f} \\ 1 \end{pmatrix} \quad (15.15b)$$

For the eigenvalue given by

$$\gamma_2 = -j\beta_b \quad (15.16a)$$

the corresponding eigenvector is given by

$$\begin{pmatrix} \sqrt{1 + \frac{\delta^2}{\omega^2 \hbar^2}} - \frac{\delta}{\omega\hbar} \\ -1 \end{pmatrix} = \begin{pmatrix} R_{0b} \\ -1 \end{pmatrix} \quad (15.16b)$$

The Riccati differential equation [Eq. (15.12)] shown above agrees with the one-dimensional Dirac equation in Session 11 in terms of the coefficients (real or purely imaginary numbers). Hence, Eq. (15.12) is also called the one-dimensional Dirac equation. The one-dimensional Dirac equation was also examined in Session 10. The circuit elements obtained from this equation are asymmetric, and the characteristic impedances of the forward and backward waves are both resistances and have different values. On the other hand, LC ladder circuits and Pasteur media are also asymmetric, but the characteristic impedances of the forward and backward waves are complex conjugates, meaning that no reflection occurs when two circuit elements are connected. In contrast, reflection is considered to occur when the two circuit elements that satisfy the one-dimensional Dirac equation presented in this session are connected. Therefore, these elements are considered to be independent and are expected to be electrons.

The spin of electrons is 1/2. Assuming that the spin of the element expressed by the above-obtained one-dimensional Dirac equation is 1/2, δ is given by

$$\delta = \frac{1}{2} \hbar \omega \quad (15.17)$$

By substituting Eq. (15.17) into Eq. (15.12), the Riccati differential equation for the component (assumed to be an electron) with spin 1/2 that satisfies the one-dimensional Dirac equation is given by

$$-\frac{d}{dx_1} \begin{pmatrix} V(x_1) \\ I(x_1) \end{pmatrix} = \begin{pmatrix} j\frac{\omega}{2c} & j\frac{\omega}{c} \\ j\frac{\omega}{c} & -j\frac{\omega}{2c} \end{pmatrix} \begin{pmatrix} V(x_1) \\ I(x_1) \end{pmatrix} \quad (15.18)$$

The phase constant β_b of the element of the one-dimensional Dirac equation that satisfies Eq. (15.18) is given by

$$j\beta_b = j\frac{\sqrt{5}\omega}{2c} \quad (15.19)$$

Denoting the characteristic impedances of the forward and backward waves as R_{of} and R_{ob} , respectively, we obtain

$$R_{of} = \frac{\sqrt{5}}{2} + \frac{1}{2} \quad (15.20a)$$

$$R_{ob} = \frac{\sqrt{5}}{2} - \frac{1}{2} \quad (15.20b)$$

Note that we described the atomic model without decay that is required to satisfy the quantum conditions in Section 6 of Session 9. In that session, a unit element was assumed as the model of electrons. However, the element with spin 1/2 that satisfies the one-dimensional Dirac equation is considered to be more suitable as the model of electrons than the unit element. This will be demonstrated in future work.

6. Steady State of Modified Dirac Equation

We have examined the behavior of the Dirac equation in the steady state. The energy in the steady state is related to power, which is classified into active and reactive powers. The energy, that is, the power, used in the Dirac equation is active power. We discussed whether reactive power, instead of active power, can be assumed in the Dirac equation in Session 12. In this session, we resume the discussion in Session 12.

In Session 12, we rewrote mc^2 , which corresponded to active power in the Dirac equation, as jmc^2 and obtained Eqs. (12.10a) and (12.10b). They are again given below.

$$j\frac{\partial}{\partial t}\psi_1 = \frac{jmc^2}{\hbar}\psi_1 - jc\frac{\partial}{\partial x_1}\psi_4 \quad (15.21a)$$

$$j\frac{\partial}{\partial t}\psi_4 = -jc\frac{\partial}{\partial x_1}\psi_1 - \frac{jmc^2}{\hbar}\psi_4 \quad (15.21b)$$

In circuit theory, time derivatives are converted into $j\omega$ by the Laplace transform and are transposed to the right-

hand sides. Moreover, the left-hand sides should have a derivative term with respect to x_1 , rather than time. Therefore, Eqs. (15.21a) and (15.21b) are rewritten as

$$jc\frac{\partial}{\partial x_1}\psi_4 = \frac{jmc^2}{\hbar}\psi_1 - j\frac{\partial}{\partial t}\psi_1 \quad (15.22a)$$

$$jc\frac{\partial}{\partial x_1}\psi_1 = -j\frac{\partial}{\partial t}\psi_4 - \frac{jmc^2}{\hbar}\psi_4 \quad (15.22b)$$

As described in Session 12, each term in Eqs. (15.22a) and (15.22b) is multiplied by j/c and subjected to the Laplace transform. By replacing ψ_1 and ψ_4 with the current $I(x_1)$ and the voltage $V(x_1)$, respectively, the partial time derivative $\partial/\partial t$ can be converted into $j\omega$ and we obtain

$$-\frac{d}{dx_1} \begin{pmatrix} V(x_1) \\ I(x_1) \end{pmatrix} = \begin{pmatrix} 0 & \frac{j}{c\hbar}(\omega\hbar + jmc^2) \\ \frac{j}{c\hbar}(\omega\hbar - jmc^2) & 0 \end{pmatrix} \begin{pmatrix} V(x_1) \\ I(x_1) \end{pmatrix} \quad (15.23)$$

In Eq. (15.23), the waves are in a stopband because the reactive power jmc^2 is extremely large, as described in Session 12. To examine the waves in a passband as described in Section 3, jmc^2 in Eq. (15.23) is rewritten as $j\delta$ to obtain

$$-\frac{d}{dx_1} \begin{pmatrix} V(x_1) \\ I(x_1) \end{pmatrix} = \begin{pmatrix} 0 & \frac{j}{c\hbar}(\omega\hbar + j\delta) \\ \frac{j}{c\hbar}(\omega\hbar - j\delta) & 0 \end{pmatrix} \begin{pmatrix} V(x_1) \\ I(x_1) \end{pmatrix} \quad (15.24)$$

6.1 Quantum waves with reactive power

Equation (15.24) indicates that the reactive power $j\delta$ is added to quanta such as photons that travel at the light velocity c . In this section, we examine the type of quantum wave.

The eigenvalues (γ) of the matrix in Eq. (15.24) are obtained from the eigenequation given by

$$\det \begin{pmatrix} -\gamma & j\left(\frac{\omega}{c} + j\frac{\delta}{c\hbar}\right) \\ j\left(\frac{\omega}{c} - j\frac{\delta}{c\hbar}\right) & -\gamma \end{pmatrix} = 0 \quad (15.25)$$

The eigenvalues and the phase constant β_c are expressed by

$$\gamma = \pm j\sqrt{\frac{\omega^2}{c^2} + \frac{\delta^2}{c^2\hbar^2}} = \pm j\beta_c \quad (15.26)$$

The characteristic impedances corresponding to the thus-obtained phase constant, i.e., those from the left- and right-hand sides of the circuit, are complex and equal to Z_{0a} given by

$$Z_{0a} = \frac{\frac{\omega}{c} + j\frac{\delta}{c\hbar}}{\sqrt{\frac{\omega^2}{c^2} + \frac{\delta^2}{c^2\hbar^2}}} \quad (15.27)$$

The circuit element expressed by the modified Dirac equation [Eq. (15.24)] has complex characteristic impedances. The characteristic impedances of the forward and backward waves are equal, that is, they are not complex conjugates. Thus, the circuit element has the features that have not been considered before. For the complex characteristic impedance Z_{0a} given by Eq. (15.27), a large δ means a large value of the imaginary part, that is, the reactive power of the quantum wave is large. Hence, the value that satisfies Eq. (15.7) is considered to be appropriate for δ . Moreover, the circuit element is reciprocal but not lossless. Therefore, the quantum wave should be considered as neutrinos, as described in Session 12, although an explanation of this is a future task.

6.2 Use of Riccati differential equation

To determine the quantum spin 1/2 using the Dirac equation, the method described in Section 5 is applied to Eq. (15.24) to obtain the Riccati differential equation given by

$$-\frac{d}{dx_1} \begin{pmatrix} V(x_1) \\ I(x_1) \end{pmatrix} = \begin{pmatrix} -\frac{\delta}{c\hbar} & j\frac{\omega}{c} \\ j\frac{\omega}{c} & \frac{\delta}{c\hbar} \end{pmatrix} \begin{pmatrix} V(x_1) \\ I(x_1) \end{pmatrix} \quad (15.28)$$

The eigenvalues (γ) of the matrix in Eq. (15.28) are obtained from the eigenequation given by

$$\det \begin{pmatrix} -\frac{\delta}{c\hbar} - \gamma & j\frac{\omega}{c} \\ j\frac{\omega}{c} & \frac{\delta}{c\hbar} - \gamma \end{pmatrix} = 0 \quad (15.29)$$

The square of the eigenvalues obtained from Eq. (15.29) is the propagation constant given by

$$\gamma^2 = -\left(\frac{\omega^2}{c^2} - \frac{\delta^2}{c^2\hbar^2}\right) \quad (15.30)$$

If the eigenvalues are purely imaginary numbers, they are the phase constant. The range of δ in which the eigenvalues are expressed by the phase constant β_d is similar to that in Eq. (15.7) and given by

$$0 \leq \delta < \omega\hbar \quad (15.31)$$

When δ satisfies this condition, the phase constant β_d is given by

$$\gamma = \pm j\sqrt{\frac{\omega^2}{c^2} - \frac{\delta^2}{c^2\hbar^2}} = \pm j\beta_d \quad (15.32)$$

The characteristic impedances of the forward and backward waves corresponding to the thus-obtained phase constants are complex conjugates and are denoted as Z_{0f} and Z_{0b} , respectively, given by

$$Z_{0f} = \sqrt{1 - \frac{\delta^2}{\omega^2\hbar^2}} + j\frac{\delta}{\omega\hbar} \quad (15.33a)$$

$$Z_{0b} = \sqrt{1 - \frac{\delta^2}{\omega^2\hbar^2}} - j\frac{\delta}{\omega\hbar} = Z_{0f}^* \quad (15.33b)$$

The Riccati differential equation [Eq. (15.28)] agrees with the equation for Pasteur media shown in Session 10 in terms of the coefficients (real or purely imaginary numbers). Thus, the element expressed by Eq. (15.28) can also be called the Pasteur medium. As discussed in Session 10, the circuit elements obtained from the equation for Pasteur media are asymmetric, and the characteristic impedances of the forward and backward waves are complex conjugates. Thus, two circuit elements can be image-connected or iteratively connected. Namely, the circuit elements can be assumed to be quanta that are connected to each other similarly to protons and neutrons in an atomic nucleus. We assume quanta with spin 1/2 and set the value of δ as

$$\delta = \frac{1}{2}\omega\hbar \quad (15.34)$$

By substituting Eq. (15.34) into Eq. (15.28), the Riccati differential equation for the Pasteur medium with spin 1/2 (assumed to be a proton or neutron) is given by

$$-\frac{d}{dx_1} \begin{pmatrix} V(x_1) \\ I(x_1) \end{pmatrix} = \begin{pmatrix} -\frac{\omega}{2c} & j\frac{\omega}{c} \\ j\frac{\omega}{c} & \frac{\omega}{2c} \end{pmatrix} \begin{pmatrix} V(x_1) \\ I(x_1) \end{pmatrix} \quad (15.35)$$

The phase constant β_d of the Pasteur medium that satisfies Eq. (15.35) is given by

$$j\beta_d = j\frac{\sqrt{3}\omega}{2c} \quad (15.36)$$

The characteristic impedances of the forward and backward waves are denoted as Z_{0f} and Z_{0b} , respectively, and given by

$$Z_{0f} = \frac{\sqrt{3}}{2} + j\frac{1}{2} \quad (15.37a)$$

$$Z_{0b} = \frac{\sqrt{3}}{2} - j\frac{1}{2} \quad (15.37b)$$

7. Future Tasks

Circuit theory includes steady-state responses that are obtained using the Laplace transform; however, they are not included in classical physics. With the assumption that a steady-state response may exist in quantum mechanics, we examined the steady-state response of the Dirac equation as a basic equation of quantum theory. However, it was shown in Session 12 that the quanta that satisfy the Dirac equation cannot exist as waves because the energy given by Einstein's well-known formula $E=mc^2$ included in the Dirac equation is extremely large in the steady state. In this session, we examined a suitable value of the energy given by Einstein's well-known formula that enables the quanta to

exist as waves in the steady state. As a result, the active or reactive power of the quanta with spin $1/2$ was found to be suitable, that is, they are electrons, neutrons, and protons with spin $1/2$. However, the quanta have the light velocity c when the Dirac equation is used, and we need to obtain the quanta with a velocity lower than c , such as electrons, neutrons, and protons, in future work.

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