Numerical Study for Positional Control of ECCD by the Ordinary Wave in a Tokamak Plasma

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(Received 5 August 1998/Accepted 18 December 1998)

Abstract
The locality and spatial controllability of ECCD are numerically analyzed by changing the direction and location of EC beam injection. The optimum angles of beam injection are obtained for the most localized current drive and the maximum drive efficiency. The profile of the driven current with the maximum efficiency is very broad, i.e., not suitable for localized current drive. When the location of driven current is controlled by the beam injection from the equatorial plane, the locality is extremely destroyed due to the refraction of the ray according to the location of cyclotron resonance layer. The locality is well kept by the injection from an upper or lower side of the equatorial plane.

Keywords:
ECCD, ordinary wave, fundamental resonance, positional control, numerical study, tokamak

1. Introduction
The current drive by electron cyclotron (EC) waves is regarded as one of efficient control methods for steady-state operation in a tokamak fusion reactor. In the analysis of the EC current drive (ECCD), the propagation and absorption of EC wave and the deformation of electron velocity distribution due to wave absorption are the key processes as same as the other current drive schemes by radio frequency waves. The propagation and absorption of EC wave are usually analyzed by the ray tracing method [1] and the driving mechanism is described by the quasi-linear Fokker-Planck equation [2]. Nowadays many numerical codes have been developed to solve the Fokker-Planck equation combined with the ray tracing analysis [3] and used for the study and analysis of ECCD [4].

As compared with other methods, i.e., lower hybrid wave, neutral beam injection and so on, one of the advantages of ECCD is the possibility of localized current drive, which is hoped to stabilize MHD instabilities, especially the neoclassical tearing mode [5]. A positional controllability of driven current is also considered as the advantage of ECCD. The wave with large wave number along the magnetic field has a potential to drive large current. However, the region of cyclotron resonance is spatially broadened by the Doppler effects with the increase of wave number along the magnetic field. As for the non-inductively driven current, the total driven current is usually discussed by using the current drive efficiency which is a ratio of the total driven current to the total absorbed power. In order to make the best use of ECCD, it is needed to clarify the difference between the driven current with maximum efficiency and the most localized driven current. Within a local analysis, it is generally accepted that the cyclotron resonance in the high field side is favorable to high efficient current drive, since the effect of magnetic trapping becomes week [2]. When the driven current is evaluated along the ray trajectory determined by the injection condition, we can not simply expect a efficient current drive in the high field side. Because, the following processes must be taken into

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account: (1) The ray trajectory is affected by the profile of electron density. (2) The local rate of wave damping and local efficiency of current drive are dependent on the magnetic field and electron temperature and density. (3) The absorbed power density which is related to the density of driven current strongly depends on the geometrical relation between a ray trajectory and a magnetic surface. It is much complicated to obtain analytically the best condition of wave injection to control a location and profile of driven current. Therefore, we search numerically the best condition of wave injection.

In this paper, the ECD using the fundamental resonance of the ordinary (O) wave is examined in the JT-60U plasma as an example. We employ a plasma profile considered to be rather difficult to control a position of driven current, i.e., the profiles of electron density and temperature have steep gradient in an internal region, which models a plasma with the internal transport barrier [6,7]. As for the numerical method, the ray trajectory is numerically traced by using the standard method [1] and the driven current is calculated by using the adjoint method along the formulation by Taguchi [2]. This numerical method is capable to examine a large number of conditions of wave injection, since a computation time for each condition is reduced by the adjoint method. In order to examine the difference between the driven current with maximum efficiency and the most localized driven current, we find the optimum direction of beam injection for these two cases by scanning the injection angles in both the toroidal and poloidal directions. Next we analyze a spatial controllability of driven current with a fixed wave frequency by changing the toroidal magnetic field. The optimum direction and better localization of wave injection are obtained with taking account of beam divergence.

This paper is organized as follows. In the next section, the numerical model and procedure are briefly explained. The results of numerical analysis are shown in section 3. In section 4, summary and discussions are presented.

2. Numerical Model and Procedure
2.1 Numerical model
The density of driven current is evaluated by the product of absorbed power density and current drive efficiency. To obtain the density profile of absorbed power of ECD waves, a standard ray tracing method [1] is employed. Ray trajectories are numerically traced by using the dispersion relation for cold plasma. The wave damping is estimated under the assumption of weakly relativistic plasma, where the dielectric tensor is expressed by the generalized Shkarofsky function [8] and its imaginary part is evaluated by the formula in Ref. [9]. The absorbed power and wave vector are calculated at each point along the ray.

When the EC power is weak, the deformation of velocity distribution from the Maxwellian distribution is assumed to be sufficiently small. Then the driven current is obtained from the steady-state linearized Fokker-Planck equation. In this study, the current drive efficiency is calculated by using an adjoint method along the formulation by Taguchi [2]. Modified points are follows: (1) a shape of magnetic surface is changed to non-circular from circular. (2) The adjoint equation is numerically solved by using Kerney's method [10] without an expansion by the Sonine polynomial.

We track the EC waves in cylindrical coordinates $(R, \varphi, z)$, where the system is assumed to be axially symmetric, i.e., $\partial/\partial \varphi = 0$. The magnetic field can be expressed as $B = B_\psi R_0 \nabla \psi + \nabla \psi \times \nabla \psi$, where $B_\psi$ is the toroidal magnetic field at the magnetic axis, $R = R_0$, and $\psi$ is the poloidal flux function. The poloidal flux is numerically obtained as a solution of the Grad-Shafranov equation. Spatial profiles of the plasma density and temperature are assumed to be functions of normalized variable $\rho = \sqrt{V(\psi)}/V(\psi_0)$, where $V(\psi)$ is volume within a flux surface and $\psi_0$ shows the plasma surface. The ray equation is integrated from the initial condition given by the position $r_i$ and the wave vector $k_i$. The starting position $r_i$, the direction of ray injection $k = k_i/|k_i|$ and wave frequency $\omega$ are given as input parameters. The wave number $|k|$ is determined by the local dispersion relation. The direction of ray injection is expressed by angles in the toroidal and poloidal directions, i.e., $\theta_0 = \pi/2 - \cos^{-1}(e_\psi \cdot k)$ and $\theta_\psi = \pi/2 - \cos^{-1}(e_\psi \cdot k)$, where $e_\psi$ and $e_\psi$ are the unit vectors along the $\varphi$- and $z$-axes, respectively. The initial position is set just inside of the most outer magnetic surface. The location on the magnetics surface is indicated by $\chi$, defined by $\tan^{-1}(e_\psi/(R - R_0))$.

2.2 Numerical Procedure
In order to discuss an efficient current drive, we examine the driven current density and total driven current. The density profile of driven current is obtained as a function of $\rho$ by averaging over magnetic surfaces. On the numerical code, the magnetic surface average is performed in shells, which are divided with same interval of $\rho$. We introduce the notation $J(\rho, \theta_0, \theta_\psi; \rho_{\text{avg}})$ as the driven current density, where $\rho_{\text{avg}}$ expresses the peak position of the current density as a label. Here, we
\[ I(\theta, \rho; \rho_{\text{max}}) = 2\pi \int_0^1 \rho d\rho I(\rho; \theta, \rho; \rho_{\text{max}}) \] (1)

To obtain the maximum current density and maximum total current, the direction of wave injection is scanned on the \((\theta, \rho)\)-plane. Then we employ the following numerical procedure. First, the poloidal injection angle \(\theta_p\) is changed in the range of \(\theta_{p1} \leq \theta_p \leq \theta_{p2}\) with a fixed \(\theta_t\). The maximum density of driven current \(J_{\text{max}}\) is defined as

\[ J_{\text{max}}(\rho_{\text{max}}, \theta) = \max_{\theta_0 \leq \theta_2 \leq \theta_1} I(\rho_{\text{max}}, \theta, \theta; \rho_{\text{max}}) \]

where \(\theta_0\) is a function of \(\rho_{\text{max}}\) and \(\theta_1\). The maximum total driven current \(J_{\text{max}}\) is also defined as

\[ J_{\text{max}}(\rho_{\text{max}}, \theta) = \max_{\theta_0 \leq \theta_2 \leq \theta_1} I(\rho_{\text{max}}, \theta, \theta; \rho_{\text{max}}) \]

where \(\theta_0\) is a function of \(\rho_{\text{max}}\) and \(\theta_1\). Secondly, the toroidal injection angle \(\theta_t\) is scanned in the range of \(\theta_t \leq \theta_t \leq \theta_{t1}\). The maximum current density \(J_{\text{max}}\) and the maximum total driven current \(J_{\text{max}}\) are obtained as functions of \(\rho_{\text{max}}\):

\[ J_{\text{max}}(\rho_{\text{max}}, \theta_t) = \max_{\theta_0 \leq \theta_2 \leq \theta_1} J_{\text{max}}(\rho_{\text{max}}, \theta) \]

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where \(\theta_0\) and \(\theta_1\) are functions of \(\rho_{\text{max}}\). Here, \(\theta_0\) and \(\theta_1\) can be expressed as functions of single variable \(\rho_{\text{max}}\). Hereafter, \(\theta_0\), \(\theta_1\), \(\theta_t\), \(\theta_{t0}\), and \(\theta_{t1}\) are called the optimum angles. In the following section, the total driven current is reported by using the conventional index, i.e., the current drive efficiency \(\gamma = \langle n_e \rangle R_0 \sin \chi / P_{\text{in}}\) where \(\langle n_e \rangle\) is the volume averaged electron density and the total input power \(P_{\text{in}}\) is used as the denominator. As for the maximum total current \(J_{\text{max}}\) and \(J_{\text{max}}\), the notations \(J_{\text{max}}\) and \(J_{\text{max}}\) are respectively used.

3. Numerical Results

We employ a model configuration: \(R_0 = 3.4\) m, \(B_0 = 3.3 T / 3.9 T\) and the minor radius is about 1 m, which simulates the JT-60U plasma. The electron density and temperature profiles are modeled by the typical discharge of JT-60U with the internal transport barrier (ITB) in the reversed magnetic shear profile. When the ITB is formed in the reversed shear profile, density and temperature profiles have steep gradient near the ITB [6,7]. In the numerical analysis, the density and temperature profiles are assumed by the same one shown in Fig. 1. The center density and temperature are assumed to be \(10^{20} m^{-3}\) and 10 keV, respectively. Further a flat \(Z_{\text{eff}}\) profile is assumed with \(Z_{\text{eff}} = 3\). The power deposition of EC wave depends strongly on the local magnetic field strength. However, we do not mention a \(q\)-profile, since the poloidal field is fairly unimportant in the total field. The O wave is launched from two positions, \(\chi = 0\) and 45°. The wave frequency \(\omega / 2\pi\) is chosen to be 110 GHz, which corresponds to the fundamental cyclotron resonance frequency of \(B_{\text{T0}} \approx 3.9 T\). In following numerical analyses, an input power \(P_{\text{in}}\) is fixed at 1 MW.

3.1 Comparison between driven currents for \(J_{\text{max}}\) and \(J_{\text{max}}\)

In order to investigate the optimum injection for efficient current drive, the difference between driven currents for \(J_{\text{max}}\) and \(J_{\text{max}}\) is analyzed. In this subsection, we examine the case of \(B_{\text{T0}} = 3.3 T\), where the cyclotron resonance layer locates in the higher field side far from the magnetic axis. The EC beams are injected from the position of \(\chi = 45°\). The divergence of injected beam is neglected, that is an analysis by using a single ray.

The maximum current density \(J_{\text{max}}(\rho_{\text{max}}, \theta)\) and optimum injection angles \(\theta_{i0}\) and \(\theta_{i1}\) are shown in Fig. 2(a). In the middle part of figure, contours of \(J_{\text{max}}\) are drawn and optimum angle \(\theta_{i0}\) is plotted by the thick curve. The upper part shows \(J_{\text{max}}(\rho_{\text{max}}, \theta = 42°)\), where the line \(\theta_i = 42°\) is shown by the dotted one in the middle part. The optimum angle \(\theta_{i0}(\rho_{\text{max}})\) is also...
indicated in the lower part. In Fig. 2(b), the maximum efficiency $\gamma^{\max}(\rho_{\max}, \theta_{\gamma})$ and the optimum angles $\theta_{\gamma}$ and $\theta_{\rho}$ are shown by the same manner of Fig. 2(a). Many corrugation and some spikes are appeared in these figures, since $J^{\max}$ and $\gamma^{\max}$ are obtained by scanning $\theta$, and $\theta_{p}$ with same intervals. To drive the current near the magnetic axis, the toroidal injection angle should be chosen as about 40°, which is determined by the Doppler shift. The current is not driven in the region of $0.6 \leq \rho_{\max} \leq 0.8, 20° \leq \theta_{\gamma}$, since the beam is strongly retracted near the region of steep density gradient; $\rho \sim 0.6$. Figure 3 shows ray trajectories projected on the poloidal plane, where $\theta_{p}$ is changed from -30° to 15° at an interval on $S$, and $\theta_{p}$ is fixed at 42°. Tracing of each ray is stopped, when a half of wave energy is absorbed or the ray reaches the plasma boundary. The dashed line indicates the cyclotron resonance layer $\omega = \Omega_{c}$, where $\Omega_{c}$ is evaluated by the rest mass and the toroidal magnetic field.

Figure 4 shows $J^{\max}(\rho_{\max})$ and $\gamma^{\max}(\rho_{\max})$. In Fig. 4(a), the full curve shows $J^{\max}(\rho_{\max})$ and the dashed curve shows $J(\rho_{\max}, \theta_{\gamma}, \theta_{p}, \rho_{\max})$, which is the peak current density for $\gamma^{\max}$. The peak density for $\gamma^{\max}$ is extremely reduced from $J^{\max}$. The current drive efficiency is shown in Fig. 4(b). The dashed curve shows $\gamma^{\max}$ and the full curve shows $\gamma(\rho_{\max}, \theta_{\gamma}, \theta_{p}, \rho_{\max})$, which means the current drive efficiency for $J^{\max}$. The efficiency for $J^{\max}$ is not reduced so much. The absorbed powers $P_{abs}$ for $J^{\max}(\rho_{\max})$ and $\gamma^{\max}(\rho_{\max})$ are drawn by the full and dashed curves in Fig. 4(c), respectively. The input power is perfectly absorbed in the both case, except for the cases where $\rho_{\max}$ is located near the plasma periphery. The difference between the behavior of $J^{\max}$ and $\gamma^{\max}$ is due to the radial profile of driven current shown in Fig. 5, where $J(\rho_{\max}, \theta_{\gamma}, \theta_{p}, \rho_{\max})$ and $J(\rho, \theta_{\gamma}, \theta_{p}, \rho_{\max})$ are plotted as functions of $\rho$ for $\rho_{\max} = 0.2, 0.4, 0.6$ and 0.8. In the region of $0.5 \leq \rho_{\max} \leq 1$, the driven current for $\gamma^{\max}$ is spatially broadened by the Doppler
effect, since $\theta_1$ is larger than $\theta_2$ as shown in Fig. 2. In
the region of $0.1 < \rho < 0.5$, however, $\theta_1$ and $\theta_2$ are
in the similar range. The profile of driven current for
$\gamma_{\max}$ may be broadened by the geometrical effects due
to the difference of poloidal injection angle. These
tendencies are common with the injection from $\chi_s = 0$.

From the view point of the localization control of
toroidal current, we investigate the optimum injection
for the most peaked current drive in the following sub-
sections.

3.2 Position control of driven current

In actual plasma operations the strength of toroidal
magnetic field is often changed. The wave frequency,
however, can be hardly changed so much. We analyze
the maximum current density in the cases of $B_{T0} = 3.3$
T and $B_{T0} = 3.9$ T with the fixed wave frequency. And
also the difference of $\rho_{\max}$ between the waves injected
from $\chi_s = 0$ and $45^\circ$ is examined.

Fig. 4 The current density and current drive efficiency are
comparing between the cases for $J_{\max}$ and $\gamma_{\max}$. (a)
shows $J_{\max}(\rho_{\max})$ and $\gamma_{\max}$ by the full and
dashed curves. (b) shows $\gamma(\theta_1, \theta_2, \rho_{\max})$ and
$\gamma(\rho_{\max})$ by the full and dashed curves. (c) shows the
absorbed powers $P_{\text{abs}}$ for $\beta_{\max}$ and $\gamma_{\max}$ by the full
and dashed curves, respectively.

Fig. 5 The density profiles of $J(\rho, \theta_1, \theta_2, \rho_{\max})$ and
$J(\rho, \theta_1, \theta_2, \rho_{\max})$ are plotted by the full and dotted curves,
respectively.

Fig. 6 The maximum current densities $\beta_{\max}(\rho_{\max})$ are
plotted, when the magnetic field and location of
wave injection are changed.

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The maximum current density \( J_{\text{max}}(\rho_{\text{max}}) \) is drawn in Fig. 6, where the full and dotted curves show the cases of \( \chi_r = 45^\circ \) and of \( \chi_r = 0 \), respectively. The case of \( B_{\text{Te}} = 3.3 \) T is shown in Fig. 6(a). In the region of \( 0.1 \leq \rho_{\text{max}} \leq 0.5 \), the maximum current density of \( \chi_r = 0 \) is reduced to 20% - 50% in comparison with the case of \( \chi_r = 45^\circ \). Figure 6(b) shows the case of \( B_{\text{Te}} \approx 3.9 \) T, where the resonance layer locates near the magnetic axis. The maximum current density of \( \chi_r = 0 \) becomes about a half of the case of \( \chi_r = 45^\circ \) in the region of \( 0.6 \leq \rho_{\text{max}} \leq 1 \). The wave injection from \( \chi_r = 45^\circ \) drives larger current density than the injection from \( \chi_r = 0 \).

Figure 7 shows the optimum angles \( \theta_r' \) and \( \theta_p' \) which are obtained in the calculation of Fig. 6. Figure 7(a) shows the case of \( B_{\text{Te}} \approx 3.3 \) T with \( \chi_r = 0 \). Here, \( \theta_p' \) is scanned in the upper side of the equatorial plane, since the up-down asymmetry is weak in the considered configuration. The case of \( B_{\text{Te}} \approx 3.3 \) T with \( \chi_r = 45^\circ \) has been already shown in the Fig. 2(a). The cases of \( \chi_r = 45^\circ \) and \( \chi_r = 0 \) in \( B_{\text{Te}} \approx 3.9 \) T are shown in Fig. 7(b) and (c), respectively. Since the cyclotron resonance layer of \( B_{\text{Te}} \approx 3.9 \) T locates near the magnetic axis, the optimum angle \( \theta_r' \) is reduced to about 20' at \( \rho_{\text{max}} = 2.0 \). The Doppler broadening is reduced by decreasing \( \theta_r' \), and then the current density is larger than the case of \( B_{\text{Te}} \approx 3.3 \) T around the magnetic axis as shown in Fig. 6.

### 3.3 Effects of beam divergence

The EC ray is strongly refracted around the region of steep density gradient according to the condition of beam injection as shown in Fig. 3. Then the refraction enhances the beam divergence, which can not be suppressed perfectly due to a technical problem. The locality of driven current is strongly affected by the beam divergence. In this subsection, we analyze the effects of beam divergence for the optimum angles \( \theta_r' \) and \( \theta_p' \) obtained in the previous subsections. The divergence of beam cone is modeled by Gaussian power distribution with half angle \( \gamma' \) as the initial condition, which is simulated by a bundle of 29 rays. Then the launched power of the beam cone is set at 1 MW.

Figure 8 shows typical 10 beam cones projected on the poloidal plane for four cases of optimum angles in Fig. 6. Here, each ray is stopped when a half of wave energy is deposited. The beam cones injected from the position \( \chi_r = 45^\circ \) are drawn in Fig. 8(a) and (b), where the injection angles are changed by using the results of Fig. 2(a) and Fig. 7(b) respectively. The beam cones from \( \chi_r = 0 \) are also shown in Fig. 8(c) and (d), where the injection angles are also determined by using Fig. 7(a) and (c). Figure 8 (a) and (c) are the cases of \( B_{\text{Te}} \approx 3.3 \) T. Figure 8 (b) and (d) are the cases of \( B_{\text{Te}} \approx 3.9 \) T. The cyclotron resonance layer is indicated by the dashed line. Figure 9 shows the radial profiles of driven currents corresponding to the beam cones in Fig. 8, where two profiles near the magnetic axis are omitted. In Fig. 9, the dashed curves show the maximum current densities \( J_{\text{max}}(\rho_{\text{max}}) \) without consideration of beam divergence, which are same to the curves in Fig. 6. In the case of \( B_{\text{Te}} \approx 3.3 \) T, broadening of beam cones near resonance points is not so much dependent on the difference between \( \chi_r = 0 \) and \( 45^\circ \) (Compare Fig. 8(a) with (c)). Then the peaked density of driven current of \( \chi_r = 45^\circ \) is larger than that of \( \chi_r = 0 \) shown in Fig. 9(a) and (c). In the case of \( B_{\text{Te}} \approx 3.9 \) T, the beam cone from \( \chi_r = 0 \) is extremely expanded outside the region of steep density gradient (\( \rho > 0.6 \)) (See Fig. 8(d)).
Fig. 8 Typical beam cones projected on the poloidal plane corresponding to Fig. 6. The cyclotron resonance layer is indicated by the dashed line.

Fig. 9 The radial profiles of driven currents corresponding to the beam cones in Fig. 8. The dashed curves show the maximum current densities $J^{\text{max}}(\rho = \rho^{\text{max}})$, which are same as the curves in Fig. 6.
profile of driven current becomes very broad in the same region shown in Fig. 9(d). On the other hand, the beam come from $\chi_a = 45^\circ$ is scarcely affected by the density gradient (See Fig. 8(b)) and the locality of driven current is well kept in the whole region (See Fig. 9(b)).

It is the better condition for efficient current drive that the cyclotron resonance layer is located near the magnetic axis and the beam is injected from $\chi_a = 45^\circ$. When the beam is injected from $\chi_a = 45^\circ$, the maximum driven current density and its locality can be kept for the change of toroidal magnetic field.

4. Summary and Discussions

The ECCD by the fundamental O wave has been analyzed numerically in the case of low power input. The optimum angles for the most localized current drive are different from those for the maximum current drive efficiency. The profile of driven current with maximum efficiency is much broader than that of most localized current drive. The current drive efficiency for the most localized current drive is not reduced so much from the maximum drive efficiency. As for the spatial controllability of the most peaked current drive, the wave injection from $\chi_a = 45^\circ$ can keep the locality of driven current. When the cyclotron resonance layer locates in the high field side of the magnetic axis, the position of driven current is controlled in the almost whole plasma column. On the other hand, in the case of the injection from the equatorial plane, the profile of driven current is considerably expanded by the refraction according to the location of resonance layer.

In this article, we have not so much physical explanations of the obtained numerical results. The problem is entangled by many processes as described in the introduction. We may need more consideration and more numerical analyses to explain it. However, the obtained numerical results summarized in the previous paragraph is very important to make the best use of ECCD in experiments and machine designing.

We choose the plasma profile with steep gradient in an internal region. The tendency of the obtained results does not qualitatively change in normal profiles. It is noted that the scanning range of $\theta_p$ in the case of $\chi_a = 45^\circ$ is almost double the case of $\chi_a = 0$. This may be closely related to technical restrictions. As for the location of beam injection, we examine the injection from the upper side of equatorial plane. When the up-down symmetry is not broken so much, the similar results may be obtained by the injection from the lower side. The adjacent method we used does not include the relativistic effect in the Coulomb collision term and calculation of the electron orbit. Therefore the electron temperature is chosen to 10 keV. To analyze the ECCD in a plasma with higher electron temperature, an extension to the full relativistic adjacent method is needed and is left for future study.

Acknowledgments

The author would like to thank Dr. Y. Ikeda and Dr. S. Ide, for useful discussions and comments.

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