Neoclassical Radial Electric Field in a Plasma with a Flow

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Abstract

Neoclassical radial electric fields are calculated in a tokamak plasma with a flow by using a δf Monte Carlo particle simulation code "FORTEC-S". The calculations are restricted to the plasma with a constant temperature. The distribution function related to the given flow is treated as a perturbation to the background Maxwellian distribution function in the simulation. The time development of the radial electric field is analyzed by a simple model and the average radial electric field is estimated in the long time limit. It is shown that the δf simulation results are well in agreement with those of the analytical estimation.

Keywords:

neoclassical transport, radial electric field, Monte Carlo method, particle simulation, δf method, plasma flow, geodesic acoustic mode

1. Introduction

The radial electric field plays an important role for the plasma transport. It is interesting to examine what mechanism determines the radial electric field. In standard neoclassical theories [1,2], which have been constructed in the limit of small orbit width (SOW) under the assumption of $\varepsilon = \rho_p / L \ll 1$ (ρ_p the poloidal Larmor radius and L the characteristic length of the plasma), the particle transport is intrinsically ambipolar. Therefore, to determine the neoclassical radial electric field E_r in an axisymmetric tokamak, the fourth order $O(\varepsilon^4)$ of particle flux must be calculated solving the drift kinetic equation up to the second order $O(\varepsilon^2)$. Assuming the constant temperature ($\nabla T = 0$), Rosenbluth *et al.* [3] calculated the fourth order particle flux and obtained the radial electric field at the steady state. Novakovskii *et al.* [4] developed a numerical simulation and analytical

theory for the radial electric field dynamics in low collisional tokamak plasmas using the standard neoclassical ordering $\varepsilon \ll 1$.

On the other hand, the assumption for the standard neoclassical transport theory breaks down for the plasma near the magnetic axis where particles draw complicated orbits different from standard banana and passing particles [5,6]. The width of this peculiar region is approximately given by $\Delta_p \approx (2q^2\rho^2R_0)^{1/3}$ [6] (q the safety factor, ρ the Larmor radius, and R_0 the major radius of the plasma). This peculiar region is wide and important for the plasma with a high value of the safety factor at the axis as seen in the reversed magnetic shear operation. Since the widths of trapped particle orbits are comparable with the radial positions of the particles near the magnetic axis, the effect of finite orbit width (FOW) must be considered to understand transport in this

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region. Recently, neoclassical transport has been explored for the near-axis plasma using the Lagrangian method including the FOW effect accurately [7]. However, the determination of the radial electric field in this region has not yet been considered. Another case when the condition for the standard neoclassical theory breaks down is the plasma with a steep ∇p (p is the plasma pressure) as is seen in the internal transport barrier mode in tokamaks. When ρ_p becomes comparable to L , the ordering of ε can not be used and some different methods are required to study neoclassical transport and determine the radial electric field.

To include comprehensively the FOW effect in studying neoclassical transport, the δf Monte Carlo method has been developed [8-13]. Recently, Wang *et al.* [11] calculated the neoclassical radial electric field consistently by using a δf simulation code. Since the FOW effect is included, their method is general and applicable even to the plasma with a steep density gradient. However, their calculations are restricted to the case with a constant safety factor ($q = \text{const}$) and no toroidal rotations. Actually, the plasma may have toroidal rotations due to momentum input by, e.g. NBI heating. Lewandowski *et al.* [14] have developed a method to calculate the neoclassical radial electric field in stellarator plasmas, which is based on the variation of the combined parallel and perpendicular pressures on the magnetic surface.

In the present paper, we present the calculation of the radial electric field in a plasma with a flow or a toroidal rotation in an axisymmetric tokamak. The calculations are carried out by using the δf Monte Carlo particle simulation code “FORTEC-S”. The parallel flow is initially given to the plasma and the distribution function related to the flow is treated as a perturbation (δf) to the background Maxwellian distribution function (f_M). The calculations are restricted to the plasma having a constant temperature in space and time. The equation for the radial electric field is analyzed assuming that the distribution function is a shifted-Maxwellian distribution function f_{SM} . The radial electric field interacts with moving particles to excite the geodesic acoustic mode [15]. The radial electric field averaged in the long time limit after this mode damps out is estimated and compared to the simulation result.

We briefly describe the method of δf Monte Carlo particle simulation in Sec. 2 and the equation for radial electric field is presented in Sec. 3. The time development of the radial electric field is analyzed by a simple model and the averaged electric field in the long time

limit is estimated to compare with the simulation results in Sec. 4. Simulation results are shown in Sec. 5 and the results are compared with the simple analytical estimation. Finally, concluding discussions are given in Summary.

2. δf Monte Carlo Simulation

Only ions are treated in the present paper. The density n and the temperature T are assumed to be functions of the radial position and constant with time. On the other hand, the electric potential Φ is assumed to depend on the position and time. We introduce the variables of the total energy $\mathcal{E} = v^2/2 + e\Phi/m$ and the magnetic moment $\mu = v_\perp^2/(2B)$, where v is the particle speed, $v_\perp$ is the perpendicular velocity, e is the ionic charge, and m is the ion mass. We consider a plasma in an axisymmetric tokamak and all the variables depend only on the radial position r and the poloidal angle θ . In the coordinates $(\mathcal{E}, \mu, r, \theta)$, the drift kinetic equation is given by [1]

$$\frac{\partial f}{\partial t} + \frac{e}{m} \frac{\partial \Phi}{\partial t} \frac{\partial f}{\partial \mathcal{E}} + (\bar{v}_\parallel + \bar{v}_d) \cdot \nabla f = C(f, f), \quad (1)$$

where $\bar{v}_\parallel$ is the parallel velocity, $\bar{v}_d$ is the drift velocity, and C is the collision operator. We split the distribution function f into

$$f = f_M + \delta f \quad (2)$$

with the assumption $|\delta f| \ll f_M$. The function f_M is a Maxwellian distribution function and given by

$$f_M = \frac{n}{(\pi v_{th}^2)^{3/2}} \exp\left(-\frac{e\Phi}{T} - \frac{2\mathcal{E}}{v_{th}^2}\right), \quad (3)$$

where $v_{th} = \sqrt{2T/m}$ is the thermal velocity. The drift kinetic equation (1) can be decomposed into

$$\frac{\partial f_M}{\partial t} + \frac{e}{m} \frac{\partial \Phi}{\partial t} \frac{\partial f_M}{\partial \mathcal{E}} + \bar{v}_\parallel \cdot \nabla f_M = 0 \quad (4)$$

$$\begin{aligned} & \frac{\partial \delta f}{\partial t} + \frac{e}{m} \frac{\partial \Phi}{\partial t} \frac{\partial \delta f}{\partial \mathcal{E}} + \bar{v}_d \cdot \nabla f_M + (\bar{v}_\parallel + \bar{v}_d) \cdot \nabla \delta f \\ & = C(\delta f, f_M) + C(f_M, \delta f). \end{aligned} \quad (5)$$

The term $\bar{v}_d \cdot \nabla \delta f$ on the left hand side of Eq. (5) represents the FOW effect. The collision operator is linearized neglecting $C(\delta f, \delta f)$. $C_{TP} \equiv C(\delta f, f_M)$ is a test particle collision operator with a background Maxwellian distribution function f_M involving drag,

pitch angle scattering, and energy scattering processes. $Pf_M \equiv C(f_M, \delta f)$ is a collision operator to compensate excess momentum and energy produced by C_{TP} . Pf_M is modelled so as to satisfy momentum and energy conservations during the collision. In the present paper, we employ the collision model for C_{TP} by Lin *et al.* [8] and Pf_M developed by Wang *et al.* [9]. The decomposed drift kinetic equations can be written as

$$\frac{Df_M}{Dt} = \bar{v}_d \cdot \nabla f_M \quad (6)$$

$$\frac{D\delta f}{Dt} = -\bar{v}_d \cdot \nabla f_M + C(f_M, \delta f), \quad (7)$$

where D/Dt operating an arbitrary function A is defined by

$$\frac{DA}{Dt} \equiv \frac{\partial A}{\partial t} + \frac{e}{m} \frac{\partial \Phi}{\partial t} \frac{\partial A}{\partial \mathcal{E}} + (\bar{v}_{\parallel} + \bar{v}_d) \cdot \nabla A - C(A, f_M). \quad (8)$$

We consider a function $g(\vec{x}, \vec{v}, t)$ which satisfies

$$\frac{Dg}{Dt} = 0. \quad (9)$$

This equation is the homogeneous equation corresponding to Eqs. (6) and (7). To extract f_M and δf from g , we introduce the weight fields $W(\vec{x}, \vec{v}, t)$ and $P(\vec{x}, \vec{v}, t)$ which should satisfy

$$f_M = Pg \quad (10)$$

$$\delta f = Wg. \quad (11)$$

From Eqs. (6) to (11), the weight fields should satisfy

$$\frac{DP}{Dt} = \frac{P}{f_M} \bar{v}_d \cdot \nabla f_M \quad (12)$$

$$\frac{DW}{Dt} = \frac{P}{f_M} [-\bar{v}_d \cdot \nabla f_M + C(f_M, \delta f)]. \quad (13)$$

We solve the set of equations (9) to (13) by a Monte Carlo particle simulation. Many test particles loaded in the simulation domain are followed solving the guiding center equations and giving collisions by use of random numbers. The function g is the distribution function for these test particles and is given by

$$g(\vec{x}, \vec{v}, t) = \sum_i \delta[\vec{x} - \vec{x}_i(t)] \delta[\vec{v} - \vec{v}_i(t)], \quad (14)$$

where $\vec{x}_i(t)$ and $\vec{v}_i(t)$ are the position and the velocity of i -th particle at time t . We assign two weights to each test particle

$$w_i(t) = W[\vec{x}_i(t), \vec{v}_i(t), t] \quad (15)$$

$$p_i(t) = P[\vec{x}_i(t), \vec{v}_i(t), t]. \quad (16)$$

Once w_i and p_i are determined for all particles, the objective distribution functions can be obtained by

$$f_M(\vec{x}, \vec{v}, t) = \sum_i p_i(t) \delta[\vec{x} - \vec{x}_i(t)] \delta[\vec{v} - \vec{v}_i(t)] \quad (17)$$

$$\delta f(\vec{x}, \vec{v}, t) = \sum_i w_i(t) \delta[\vec{x} - \vec{x}_i(t)] \delta[\vec{v} - \vec{v}_i(t)]. \quad (18)$$

Considering weight spreading and its reduction method [10], we obtain the weight equations for i -th test particle as

$$\begin{aligned} \frac{dw_i}{dt} &= \frac{p_i}{f_{Mi}} [-\bar{v}_d \cdot \nabla f_M + C(f_M, f_1)]_i \\ &- \eta \left[w_i - \bar{W}(\vec{x}_i, \vec{v}_i, t) \right] \end{aligned} \quad (19)$$

$$\frac{dp_i}{dt} = \frac{p_i}{f_{Mi}} [\bar{v}_d \cdot \nabla f_M]_i - \eta \left[p_i - \bar{P}(\vec{x}_i, \vec{v}_i, t) \right]. \quad (20)$$

The operator d/dt means the derivative along the particle orbits and the quantities with the suffix i must be evaluated at i -th particle's position and velocity. The functions $\bar{W}$ and $\bar{P}$ are field weights averaged over particles in a small phase space element and η is a damping factor. The weights are averaged so as to conserve the particle number, momentum, and energy in the small phase space element.

3. Equation for the Radial Electric Field

From Poisson equation and the continuity equation, we obtain

$$\frac{\partial E_r}{\partial t} = -4\pi J_r, \quad (21)$$

where J_r is the radial current and consists of

$$J_r = J_{rp}^C + J_{rp}^{NC} + J_{rd}^{NC}. \quad (22)$$

Since electron current is much smaller than the ion current ($|J_{re}| \ll |J_{ri}|$), only ion current $J_r = J_{ri}$ is considered. J_{rp}^C is the classical polarization current given by

$$J_{rp}^C = \frac{nmc^2}{B^2} \frac{\partial E_r}{\partial t}, \quad (23)$$

where c is the light speed and B is the magnetic field strength. This current is not calculated in the simulation because gyro-motions are not followed. J_{rp}^{NC} is the neoclassical polarization current and J_{rd}^{NC} is the drift

current, which are calculated in the simulation. In the general coordinates, the evolution of the radial electric field is given by

$$\left[\left\langle |\nabla\psi|^2 \right\rangle + 4\pi n m c^2 \left\langle \frac{|\nabla\psi|^2}{B^2} \right\rangle \right] \frac{\partial^2 \Phi}{\partial t \partial \psi} = 4\pi e \Gamma_i. \quad (24)$$

In this equation, ψ is the toroidal flux and the second term on the left hand side is attributed to the classical polarization current. The bracket $\langle \dots \rangle$ means the flux average. The ion particle flux Γ_i is defined by

$$\Gamma_i = \left\langle \int d\vec{v} (\vec{v}_d \cdot \nabla \psi) f \right\rangle. \quad (25)$$

Equation (24) with (25) holds independently of the magnitude of pressure gradients. That is, Eqs. (24) and (25) are unrelated to the ordering of $\varepsilon = \rho_p/L$. In the standard neoclassical theory, like-particle collisions produce no particle flux in the SOW limit, and hence Γ_i vanishes. However, in the present simulation, Γ_i remains finite even if the pressure gradient is very small, because the effect of FOW is rigorously taken into account by solving guiding center equations. If the FOW effect is small, that is, if Δ/L_p is small (Δ is the average deviation of particle orbits from the magnetic surface and L_p is the pressure gradient scale length), a small Γ_i is generated and if FOW effect is large (Δ/L_p is relatively large), a large Γ_i is generated. The radial electric field evolves so as to annihilate Γ_i to maintain the charge neutrality property of the plasma. Accordingly, it is expected that a small pressure gradient produces a small radial electric field and a large gradient produces a large radial electric field.

In the simulation code “FORTEC-S”, we solve the distribution function according to the method described in the Sec. 2. The simulation domain is divided into N_r^c parts in the r -direction and N_θ^c parts in the θ -direction. We calculate Γ_i in each cell divided by $N_r^c \times N_\theta^c$ from Eq. (25) using the obtained distribution function. The radial electric field is then calculated from Eq. (24) by the predictor-corrector scheme. The obtained radial electric field $E_\psi = -\partial\Phi/\partial\psi$ is interpolated in each test particle position to solve the guiding center equations. This procedure is repeated every time step.

4. Simple Analytic Estimation

In this section, we consider the oscillation of the radial electric field and its average value in the long time limit using a simple analytical model. The plasma

considered here has a flow parallel to the magnetic field $u_\parallel(r)$, which has a radial profile. The flow is assumed to be small compared to the thermal velocity ($\bar{u}_\parallel \equiv u_\parallel/v_{th} \ll 1$). We assume that the ion temperature T is constant in space and time ($\nabla T = 0$) for simplicity. The equation for the radial electric field E_r can be written from Eq. (24) as

$$\left(1 + \frac{c^2}{v_A^2} \right) \frac{\partial E_r}{\partial t} = -4\pi e \left\langle \int d^3v f v_{dr} \right\rangle, \quad (26)$$

where $v_A = B/(\sqrt{4\pi n m})$ is the Alfvén speed. The radial drift velocity is given by

$$v_{dr} = -\frac{v_\parallel^2 + v^2}{2\Omega_0 R_0} \sin \theta, \quad (27)$$

where R_0 is the major radius and $\Omega_0 = eB_0/mc$ with B_0 the magnetic field at the axis. Taking the time derivative of Eq. (26), we have

$$\varepsilon_\perp \frac{\partial^2 E_r}{\partial t^2} = -4\pi e \left\langle \int d^3v v \frac{\partial f}{\partial t} v_{dr} \right\rangle \quad (28)$$

with

$$\varepsilon_\perp \equiv 1 + \frac{c^2}{v_A^2} \simeq \frac{c^2}{v_A^2}. \quad (29)$$

From the drift kinetic equation (1)

$$\frac{\partial f}{\partial t} = -\frac{e}{m} \frac{\partial \Phi}{\partial t} \frac{\partial f}{\partial \mathcal{E}} - (\bar{v}_\parallel + \bar{v}_d) \cdot \nabla f + C. \quad (30)$$

We assume that the distribution function f on the right hand side in this equation is approximated by a shifted-Maxwellian distribution function f_{SM} expressed as

$$f_{SM} = \frac{n}{(\pi v_{th}^2)^{3/2}} \exp \left(\frac{e\Phi}{T} - \frac{2\mathcal{E}}{v_{th}^2} - \frac{-2u_\parallel v_z + u_\parallel^2}{v_{th}^2} \right), \quad (31)$$

where $u_\parallel$ is the parallel flow velocity which has a profile in the r -direction. It is noted that $v_z = v_\parallel$ is a function of $\mathcal{E}$, μ , r , and θ . The shifted-Maxwellian distribution function f_{SM} annihilates off the collision term C . Hence, we assume $C = 0$ in Eq. (30). Inserting Eq. (30) together with Eq. (31) into Eq. (28) yields, up to the order of $O(\varepsilon_\perp = r/R_0)$ and $O(\bar{u}_\parallel^2)$,

$$\frac{\partial^2 E_r}{\partial t^2} + \frac{e}{T} G E_r = H \quad (32)$$

with

$$G = \frac{T}{e} \frac{v_{th}^2}{4R_0^2} (I_0 - \bar{u}_{||} I_1) \quad (33)$$

$$H = \frac{T}{e} \frac{v_{th}^2}{4R_0^2} \left[\frac{1}{n} \frac{dn}{dr} I_0 + 2 (I_2 - \bar{u}_{||} I_0) \frac{d\bar{u}_{||}}{dr} \right] + \frac{T}{e} \frac{v_{th}^2}{4R_0^2} \frac{2r}{q R_0 \rho_0} I_3 \bar{u}_{||}, \quad (34)$$

where $\rho_0 \equiv v_{th}/\Omega_0$. We have used the relation $\langle \sin^2 \theta \rangle = 1/2$ and $\langle \sin^2 \theta \cos^2 \theta \rangle = 1/2$. The integrals I_0 , I_1 , I_2 , and I_3 , which contain Gaussian function and v_{dr} , are calculated as $I_0 = 7 + 16\bar{u}_{||}^2$, $I_1 = 14\bar{u}_{||}$, $I_2 = 23\bar{u}_{||}$, and $I_3 = 3 + 2\bar{u}_{||}^2$. Equation (32) suggests that the radial electric field interacts with moving particles to generate an oscillation with a frequency $\omega_G = (e/T)G$. This oscillation is the geodesic acoustic mode (GAM) [15]. The solution to Eq. (32) is $E_r = \bar{E}_r + A e^{-i\omega_G t}$. The amplitude A is Landau damped [4]. The investigation of the damping rate requires more detailed kinetic analysis, which is outside the scope of the present paper. In the long time limit after the GAM oscillation damps out or after time averaging, the radial electric field $\bar{E}_r = (T/e)(H/G)$ becomes, up to $O(\bar{u}_{||}^2)$,

$$\bar{E}_r = \frac{T}{e} \left[\frac{1}{n} \frac{dn}{dr} (1 + 2\bar{u}_{||}^2) + 4.57 \frac{d\bar{u}_{||}}{dr} \right] + 0.857 \frac{T}{e} \frac{r}{q R_0 \rho_0} \bar{u}_{||}. \quad (35)$$

Since E_r oscillates with the frequency ω_G , it is required in the simulation that Eq. (26) must be solved at a smaller time step than ω_G^{-1} . Equation (35) indicates that Boltzman relation $\bar{E}_r = (T/e)(1/n)(dn/dr)$ (or $n = n_0 \exp(-e\Phi/T)$) holds if $u_{||} = 0$ and $\nabla T = 0$. This relation is a result from the particle flux of the toroidal angular momentum diffusion at the steady state, which was calculated by solving the drift kinetic equation up to the second order in $\varepsilon = \rho_p/L$ [3]. It is noted that this Boltzman relation can hold even for the plasma with a steep density gradient [11].

5. Calculation Results

Using the δf Monte Carlo particle simulation code “FORTEC-S” the simulations are carried out for a simple magnetic field geometry in the large aspect ratio limit given by

$$\vec{B} = \frac{B_0}{1 + \varepsilon_i \cos \theta} \left[\vec{\zeta} + \frac{\varepsilon_i}{q(r)} \vec{\theta} \right], \quad (36)$$

where $\varepsilon_i = r/R_0$ is the inverse aspect ratio. The term with $\vec{\zeta}$ is the toroidal magnetic field and another term with $\vec{\theta}$ is the poloidal magnetic field. The density is given by

$$n(r) = n_0 \left\{ A_n + B_n \exp \left[-\alpha_n \left(\frac{r}{a_p} \right)^{\beta_n} \right] \right\} \quad (37)$$

where n_0 , A_n , B_n , α_n , and β_n are specified. We take $A_n + B_n = 1.0$ so that n_0 is the central density. Many particles are initially loaded in the simulation domain to form a local shifted-Maxwellian distribution with T in each cell divided in the r -direction. In the present simulation, the effect of the parallel flow is treated as a perturbation in δf according to the procedure described in Sec. 2. The flow is initially given by

$$\bar{u}_{||}(x, t = 0) = \bar{u}_0 \exp[-\alpha_u(x - x_0)^{\beta_u}]. \quad (38)$$

Here, $x = r/a_p$, $\bar{u}_0 = u_{||}(x = x_0)/v_{th}$, α_u , β_u , and x_0 are input parameters. In the present paper, three cases of $\bar{u}_0 = 0$, and ± 0.1 are examined. The parameters throughout the calculations are $B_0 = 3$ T (magnetic field strength), $R_0 = 3$ m (plasma major radius), $a_p = 0.5$ m (plasma minor radius), $n_0 = 5.0 \times 10^{19} \text{ m}^{-3}$ (central density), $T = 1.5$ keV, $N_r^c = 10$, $N_\theta^c = 10$, $\alpha_u = 2$, $\beta_u = 2$, $x_0 = 0$, and $N_{\text{test}} = 1,000,000$ (number of test particles).

We show the calculation results for two cases. One is the Case-A with a small density gradient and positive shear of $q(r)$. Another is the Case-B with a steep density gradient and negative shear of $q(r)$. Calculation parameters are $A_n = 0.04$, $B_n = 0.96$, $\alpha_n = 2.78$, $\beta_n = 2.01$, $q(0) = 1.0$, and $q(a_p) = 3.0$ for Case-A and $A_n = 0.1$, $B_n = 0.9$, $\alpha_n = 400.0$, $\beta_n = 10.0$, $q(0) = 8.0$, $q_{\min}(0.6a_p) = 1.0$, and $q(a_p) = 3.0$ for Case-B.

The profiles of the density n and the safety factor q are illustrated in Fig. 1 and Fig. 2, respectively. The solid lines are Case A and the dotted lines are Case-B. The maximum values of $\varepsilon_n \equiv \rho_p/L_n$ ($L_n^{-1} = (1/n)dn/dr$) are 0.16 in Case-A and 0.6 in Case-B, respectively. The potato region of ions is $\Delta_p \approx (2q^2\rho^2R_0)^{1/3}$. In Case-A, $\Delta_p/a_p = 0.1$ and ions lie in the banana regime in $x = r/a_p = 0.2$ to 1.0 . In other region ions are in the plateau regime. In Case-B, steep density gradient case, $\Delta_p/a_p = 0.4$ and ions lie in the banana regime in other region.

Time behaviours of Γ_i and E_r at $r = 0.55a_p$ are shown in Fig. 3 and Fig. 4 for Case-A and in Fig. 5 and Fig. 6 for Case-B, respectively. Both cases show clearly

Fig. 1 Profile of the density. Solid line is for Case-A and dotted line is for Case-B.

Fig. 2 Profile of the safety factor. Solid line is for Case-A and dotted line is for Case-B.

Fig. 3 Time development of Γ_i at $r = 0.55a_p$ for Case-A (arbitrary unit).

Fig. 4 Time development of E_r at $r = 0.55a_p$ for Case-A (kV/m).

Fig. 5 Time development of Γ_i at $r = 0.55a_p$ for Case-B (arbitrary unit).

Fig. 6 Time development of E_r at $r = 0.55a_p$ for Case-B (10 kV/m).

that the particle flux Γ_i vanishes in a short time and the radial electric field E_r reaches a constant value synchronizing with the behaviour of Γ_i . As predicted in Sec. 4, E_r oscillates first but converges in a short time at the fixed value which is $\bar{E}_r$. Once Γ_i vanishes and E_r reaches a fixed value, the collision operator in “FORTEC-S” ensuring momentum and energy conservations keeps $\Gamma_i = 0$ and constant E_r .

Figures 7 and 8 demonstrate the radial profiles of radial electric fields for Case-A and Case-B, respectively. Simulation results marked with symbols $\square$, $\times$, and $+$ are obtained by time averaging from $t = 4.8\tau_i$ to t

$= 5.0\tau_i$, where τ_i is the ion-ion collision time. The marks of $\square$, $\times$, and $+$ indicate the simulation results for $\bar{u}_0 = 0$, $\bar{u}_0 = -0.1$, and $\bar{u}_0 = +0.1$, respectively, and the corresponding lines (solid, chained, and dotted lines) are calculation results of Eq. (35). Since the last term with $\bar{u}_{||}$ on the right hand side of Eq. (35) has a maximum near $x = 0.5$, the effect of the flow can be seen in the region near $x = 0.5$ in Fig. 7 (Case-A). In Case-B, the steep density case, the first term with $(1/n)dn/dr$ dominates in Eq. (35) and the flow effect is relatively small as seen in Fig. 8.

6. Summary

The Monte Carlo particle simulation code “FORTEC-S” based on the δf method has been developed and extended to calculate neoclassical radial electric fields in a tokamak plasma with a simple magnetic field configuration. Radial electric fields have firstly been calculated for the plasma with a parallel flow. In the simulation, the distribution function associated to the flow was treated as a perturbation while the background plasma was subjected to the Maxwellian. Analysis of the time development equation for the radial electric field was performed and the dynamic equation of the radial electric field was obtained up to the order of $O(r/R_0)$ and $O(\bar{u}_{||}^2)$. The simulations by “FORTEC-S” were carried out for two cases. One is the case when the density gradient is small and another is the case when the density gradient is steep, which is outside the scope of the standard neoclassical theory. For both cases, simulation results and analytical results are well in agreement.

The calculations have been restricted to the plasma with no temperature gradients ($\nabla T = 0$). The purpose of this restriction is to compare simulation results to the analytical estimations. The purpose was successfully achieved. Simulations for plasmas with temperature gradients are in progress. It could be possible to understand what mechanism determines the generation of the radial electric field in tokamaks by comparing experimental observations to simulations including temperature gradients.

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Fig. 7 Profile of $E_r(r)$ for Case-A. Simulation results are marked with $\square$ for $\bar{u}_0 = 0$, $\times$ for $\bar{u}_0 = -0.1$, and $+$ for $\bar{u}_0 = +0.1$. Analytical results are solid line for $\bar{u}_0 = 0$, chained line for $\bar{u}_0 = -0.1$, and dotted line for $\bar{u}_0 = +0.1$.

Fig. 8 Profile of $E_r(r)$ for Case-B. Simulation results are marked with $\square$ for $\bar{u}_0 = 0$, $\times$ for $\bar{u}_0 = -0.1$, and $+$ for $\bar{u}_0 = +0.1$. Analytical results are solid line for $\bar{u}_0 = 0$, chained line for $\bar{u}_0 = -0.1$, and dotted line for $\bar{u}_0 = +0.1$.

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