Special Topic Article

Quasi-Symmetry in Stellarator Research

3. Quasi-Symmetry Concepts in Helical Systems

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Abstract

Quasi-symmetry concepts in helical systems are briefly explained. The one of the magnetic coordinates, Boozer coordinates, are the essential basis for considering quasi-symmetry concepts in terms of drift trajectories and neoclassical transport properties. Several features of quasi-symmetric (quasi-axisymmetric, helically symmetric and poloidally symmetric) concept are briefly described from the viewpoints of configurational properties. The intuitive classification of these concepts (including (quasi-)isodynamic concept) is also given.

Keywords:

quasi-symmetry (quasi-axisymmetric, helically symmetric and poloidally symmetric) concept, (quasi-)isodynamic concept, Boozer coordinates, drift trajectory, neoclassical transport

3.1 Introduction

The properties of a stellarator configuration are completely determined by prescribing the shape of an outermost flux surface. Parameterizing this shape, and studying the dependence of physical properties on these parameters have led to some innovative stellarator concepts including quasi-symmetric configurations.

The choice of the magnetic topography, i.e., the functional form of the magnetic field strength, B , in magnetic coordinates is one essential issue to determine the physical properties of plasma confinement in stellarators. Information regarding the actual direction of the magnetic field, i.e., $\mathbf{B}$, is of little relevance. This result follows since the equations governing the drift trajectories [1] of charged particles within the plasma depend only on B expressed in terms of Boozer coordinates (s, θ, ζ) [2]. Here, s gives the label of flux surfaces, the poloidal angle θ defines the short way around the torus, and the toroidal angle ζ the long way around the torus.

If the functional forms of B in terms of Boozer

coordinates are similar, those plasma equilibria can have similar neoclassical transport properties. In other words, the difference of transport properties can be evaluated in Boozer coordinates without returning to Euclidean space. For example, as for the properties of drift trajectory and particle confinement, deviation of drift trajectories from a flux surface, fraction of reflected particles and fraction of particles leaving the confining region can be easily estimated in Boozer coordinates even if the shape of drift trajectories differ from those in Euclidean space. The advantage for the analysis of drift trajectories in Boozer coordinates also allows to evaluate the deviation of the distribution function from Maxwellian distribution and resulting neoclassical transport by solving the drift-kinetic equations. Indeed, the DKES code [3] evaluates the particle and heat transport across a flux surface and also the bootstrap current. Thus, the Boozer coordinates are the plausible basis for evaluating particle confinement and neoclassical transport properties in stellarator configurations.

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Historically, the major challenges for stellarators have been done to provide acceptable drift-orbit confinement, allowing low neoclassical transport and adequate fast-ion confinement.

Ideally, it is desirable to find toroidal plasma equilibrium in which B is constant within flux surfaces, i.e., $B(s)$, since their guiding-center drift trajectories would always lie [4] within the flux surfaces. Thus, these toroidal equilibria, if they exist, would have no radial neoclassical transport, to the accuracy of the guiding-center approximation. Palumbo was the first to suggest these “isodynamic” (or “omnigenous”) equilibria. However, it was shown by Bernadin *et al.*, that toroidal isodynamic equilibria can possibly exist only in limits in which B on axis vanishes or the flux surfaces become open. Since neither of these two limits are practical for plasma confinement, toroidal isodynamic equilibria can not be well approximated.

Thus, two strategies were identified to provide adequate drift-orbit and neoclassical confinement, and they have become now practical through numerical optimization.

The first strategy minimizes the poloidal variation of B , which reduces the radial drift of particles to make their deviation from a flux surface smaller. The magnetic field strength is stronger in the direction of the local curvature, and it is weak in the opposite direction [5]. The perfect elimination of the (poloidal) variation of B in practical torus geometry is impossible as mentioned above. Thus, the idea of “quasi-isodynamicity” was raised by Nührenberg, in which the poloidal variation of B is partially vanished in the region of the “straight” section of a torus. This idea has been developed leading to the design of the Wendelstein 7-X [6]. The trapped particles are well localized in this straight section of a torus by adjusting the region of the local minimum of B to this section. This is the origin of a predominant bumpy (or mirror) component of B in W7-X. The bumpy component denotes the toroidal variation of B without the poloidal angle dependence. Since the radial drift (induced by the poloidal variation of B) is well suppressed in the straight section where trapped particles are localized, good particle confinement and low neoclassical transport properties can be realized (cf. [7]).

The magnetic field strength in quasi-isodynamic concept is fully three-dimensional. This provides the additional freedom to control other physical properties. For example, in the W7-X, the bootstrap current is minimized by combining the current components

induced by each Fourier component of B [8]. The three-dimensionality of B keeps knobs to consider some other attractive or innovative stellarator configurations.

It should be noted that the further reduction of the poloidal variation of B is now being explored at low aspect ratio (A_p) in the design of the quasi-poloidally (bumpy) symmetric (QPS) configuration [9].

The second strategy is called “quasi-symmetry”. This concept has been explored to develop stellarator configurations that, while three-dimensional in Euclidean space, have a direction (either helical, toroidal or poloidal) of approximate symmetry of B in Boozer coordinates. Since the particle momentum in the direction of approximate symmetry becomes the constant of motion in such circumstances, the deviation of particle trajectory from a flux surface can be suppressed. Quasi-symmetric configurations have drift-orbits similar to equivalent symmetric configurations, and thus similarly good neoclassical transport properties.

The following quasi-symmetric stellarator configurations have been investigated. Physical aspects of these configurations will be described in detail in the following sections. Thus, several features of these configurations are briefly mentioned here from the viewpoints of configurational properties. This kind of description would be helpful to grasp the existence region of these configurations in wide parameter space of three-dimensional (in Euclidean space) configurations.

3.2 Quasi-Symmetric Configuration

3.2.1 Quasi-Helically Symmetric (QHS)

Configuration:

When we consider a torus geometry, it is hard to imagine for the magnetic topography to have a helical symmetry, that is, the toroidicity of magnetic field strength to be well suppressed. However, it was shown computationally by Nührenberg and Zille [10] and also analytically by Garren and Boozer [11] that a QHS configuration is possible to be realized with introducing the appropriate helical excursion of a torus in Euclidean space. The magnetic field strength of QHS configurations, i.e., $B(s, \alpha)$ in Boozer coordinates, with $\alpha \equiv \theta - M\zeta$ and M an integer, obviously exists for configurations that are perfectly axisymmetric, which is the trivial $M = 0$ case.

For the experimental test of the QHS concept, the Helically Symmetric eXperiment (HSX) was constructed and now being operated at the University of Wisconsin. We will read the recent experimental results

from the HSX in Section 4. The significant feature of the HSX magnetic configuration is a large helical excursion of magnetic axis whose radius of the excursion is comparable to the plasma minor radius. Due to this helical excursion, the toroidicity of B is significantly suppressed to the level of that of a configuration with A_p of about 240, although the real A_p of HSX is about 10. The required helical excursion to almost eliminate the toroidicity of B increases as A_p is decreased due to its inherently larger toroidicity in configurations with smaller A_p [12]. Thus, it is easier to realize QHS configurations with larger A_p (10 is selected for the HSX). It is also noted that the relatively larger helical field contributions in larger number of the toroidal period (M) allows the reduction of the required helical excursion to suppress the toroidicity [12].

3.2.2 Quasi-Axisymmetric Symmetric (QAS) Configuration:

Another quasi-symmetry concept was innovated upon the following question : Can axisymmetric tokamaks be generalized into quasi-axisymmetric tokamaks, i.e., truly three-dimensional configurations in Euclidean space with an axisymmetric structure of B in Boozer coordinates? This question was raised based on the experimental demonstration of the suppression of disruptions and tearing modes by replacing a fraction of the toroidal current by an externally created rotational transform [13].

Nührenberg and Garabedian separately found QAS configurations with different configurational properties such as the rotational transform and plasma shape *etc* [14, 15]. The example approach towards a QAS configuration by superposing modulations of plasma shape is described step by step in Ref. [12], which would be instructive how the non-axisymmetric components of B are controlled and reduced in non-axisymmetric deformation of plasma shape. It is shown there that the straight section of magnetic configuration is not necessary to realize QAS property since the toroidicity should be kept in this case. Also, configurations with smaller A_p are suitable since they inherently have larger toroidicity of B . The smaller number of M (typically 2 or 3 in design study of QAS configurations) is also plausible from this reason. The smaller A_p , that is a “compactness”, of QAS configurations are advantageous to design a “compact” stellarator.

3.2.3 Quasi-Poloidally Symmetric (QPS) Configuration:

The quasi-poloidally symmetry (QPS) concept is also now being investigated as mentioned above [9]. The enhancement of poloidally symmetric components of B does not arise the radial drift [16]. That is the key feature in QPS configuration to achieve good particle confinement and low neoclassical transport. Two distinct configuration types have been considered in Ref. [9]: (1) one which achieves the drift optimization at low beta value and low bootstrap current by appropriate plasma shaping; and (2) one which has a great reliance on plasma beta value and bootstrap current for supplying the rotational transform and obtaining quasi-poloidally symmetry. Due to the requirement of the compactness in this specific design study (for a lower cost near term experiment), low A_p configurations are considered. The first type of configuration has an A_p about 2.5 with $M = 2$, which has an inherently large toroidicity of B if it is an axisymmetric torus. The strong deformation of an axisymmetric torus into a “race-track” torus provides the significant reduction of the toroidicity of B and introduces predominant poloidally symmetric components of B . The A_p of the second type of configuration is also relatively small, about 3.7. It is noted that configurations with higher A_p are easier to realize QPS configuration with its inherently small toroidicity of B [12] if the requirement of the compactness could be removed. Further detailed investigations are being progressed currently and the proposal of an attractive QPS configuration would be anticipated.

Fig. 1 The intuitive classification of isodynamic, quasi-isodynamic and quasi-symmetry concepts is schematically summarized.

Finally, the intuitive classification of these concepts (including (quasi-)isodynamic concept) are schematically summarized in Fig. 1. In this section, quasi-isodynamic and quasi-symmetry concepts are briefly introduced. The global configurational properties of quasi-symmetric configurations are also intuitively described to grasp inter-relationship between these configurations as an introduction to the following sections. Of course, detailed physical properties of plasma confinement depend also on local configurational properties, electric field structure and turbulent properties *etc.* These detailed investigations in quasi-symmetric configurations (QHS and QAS) are described in the following sections.

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