Global full-wave simulation on electromagnetic wave propagation in toroidal plasma with an external magnetic field imaging a tokamak configuration is performed in two dimensions. The temporal behavior of an electromagnetic wave launched into plasma from a wave-guiding region is obtained.

**Keywords:** electromagnetic wave, microwave, millimeter-wave, toroidal plasma, plasma diagnostics

Electromagnetic wave propagation in a plasma is one of the basic problems in plasma physics. The wave trajectory in the microwave and millimeter-wave regimes is very important from the viewpoint of electromagnetic wave-based plasma diagnostics such as interferometry, reflectrometry, and ECE imaging in magnetic confinement devices[1, 2]. In addition, the present study is a response to the inadequacy of usual geometrical optics for the ray-tracing of an electromagnetic wave beam in the above-mentioned frequency ranges. Recently, striking differences between geometrical optics and wave optics based on Maxwell equations have been reported[3].

In this paper, we study the propagation of electromagnetic waves in the microwave and millimeter-wave regimes in toroidal plasma. Figure 1 shows the simulation box in which the white-colored region indicates the plasma and wave-guiding region to be computed, and the gray-colored region indicates a region unnecessary for the computation of the wave propagation. In order to separate these two regions, we introduce an artificial conductivity $\sigma$ and the real part of dielectric constant $\varepsilon$, normalized by $\varepsilon_0$ (i.e., $\varepsilon = \text{Re}(\varepsilon)/\varepsilon_0$). The basic equations for simulations are

$$\frac{\partial}{\partial t} \mathbf{B} = - \nabla \times \mathbf{E} \quad (1)$$

$$\frac{\partial}{\partial t} \mathbf{E} = - \frac{c^2}{\varepsilon(r)} \nabla \times \mathbf{B} - \frac{1}{\varepsilon_0 \varepsilon(r)} [J + \sigma(r) \mathbf{E}] \quad (2)$$

where $\mathbf{E}$ and $\mathbf{B}$ are electromagnetic wave fields, $\mathbf{J}$ the plasma current, $c$ the speed of light, $\omega_{pe}$ the electron plasma frequency, and $B_0$ an external magnetic field. The last term of the right-hand side of eq.(2) is an artificial one introduced to separate the above two regions already discussed. We here assume that $\sigma/\omega \varepsilon_0 = 0$, $\varepsilon_r = 1$ for plasma and the wave-guiding region (white-colored); otherwise (gray-colored) $\sigma/\omega \varepsilon_0 = 10$, $\varepsilon_r = 10$. In this case, the electromagnetic wave becomes strongly damped in the gray-colored region, and the interface...
between the two regions plays the role of a wall boundary creating wave reflections. The details regarding this numerical scheme will be reported elsewhere. If we assume a density profile \( n(r) \) and the external magnetic field \( B_0(r) \) in the above equations, we can perform a simulation run for wave propagation under the initial condition for an incident electromagnetic wave. In the present simulation, we assume a tokamak-like magnetic field profile for \( B_0 \) given by

\[
B_x = B_0 \frac{R}{r} \frac{z - z_0}{r},
\]

\[
B_z = -B_0 \frac{R}{r} \frac{x - x_0}{r},
\]

\[
B_y = B_0 \sqrt{2} \frac{r - R}{d} \exp\left(-\frac{r - R^2}{d}\right)
\]

where \( r^2 = (x - x_0)^2 + (z - z_0)^2 \), \( R \) is the major radius, and \( B_0 \) is the value at \( r = R \). The toroidal field is then given by \( B_t = (B_x^2 + B_y^2)^{1/2} = 1/r \), and \( B_y \) corresponds to the poloidal field. We also assume a Gaussian density profile for \( n(r) \) given by

\[
n = n_0 \exp\left[-\frac{(r - \frac{R}{a})^2}{a}\right]
\]

The number of grids in a simulation box is \( 3,000 \times 3,000 \). In the simulation, the time step \( \Delta t = 0.1\omega_0^{-1} \), and the mesh size is \( \Delta x = \Delta z = 0.1c/\omega_0 \), where \( \omega_0 \) is a reference frequency. The electron plasma, electron cyclotron, and incident wave frequencies are also normalized by \( \omega_0 \), and the plasma and other simulation parameters can then be scaled by \( \omega_0 \). The following parameters are used: \( R = 107 \) cm, \( a = 18 \) cm, \( B_0 = 0.43 \) T, and \( n_0 = 0.6 \times 10^{12} \) cm\(^{-3}\). The incident wave is expressed as \( E_z(x, t) = \exp\left[-(z - z_1)^2/L^2\right]\sin(\omega t) \) on the lower boundary in \( x \), where \( \omega = 8 \) GHz, \( z_1 = 107 \) cm, and \( L = 7.2 \) cm when we set \((x_0, z_0) = (0, 0)\).

Fig. 2 The snap shot of the electromagnetic wave field at \( t = 23.5 \) ns, \( E_x, E_y, E_z \) (from left to right in upper side) and \( B_x, B_y, B_z \) (from left to right in lower side).

We show the result of a simulation run. In the present parameters, there is no cutoff and resonance in the plasma region. Figure 2 shows a snap shot of the electromagnetic wave field at \( t = 23.5 \) ns, where \( E_x, E_y, E_z \) (from left to right in upper side) and \( B_x, B_y, B_z \) (from left to right in lower side) are shown in the absolute values. We see that a part of the electromagnetic wave propagates in the toroidal direction while repeating the reflection with the wall, which is shown by two solid circles.

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