Evaluation of Absorption Rate by Using Full-Wave Maxwell Simulation for Plug ECRH in the GAMMA 10 Tandem Mirror

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The absorption rate of heat microwave power for plug ECRH (electron cyclotron resonance heating) in the GAMMA 10 tandem mirror is calculated in three ways over a wide range of plasma density. These methods include ray-tracing calculation, analytic estimation and full-wave Maxwell simulation. The results correlate very closely with each other. The full-wave Maxwell simulation is found to be a powerful method to evaluate the microwave power deposition for ECRH. It also predicts the power deposition in the region of \( \omega_c < \omega \) for beach heating.

Keywords: electron cyclotron resonance heating, tandem mirror, ray tracing, Maxwell simulation

Fundamental ECRH has been applied to the GAMMA 10 plug region for creation of a plasma confining potential. The schematic configuration of the plug region is shown in Fig. 1. A high power microwave beam of 28 GHz \((\lambda = 1.07 \text{ cm})\) is launched from a Vlasov antenna with linear polarization. It is reflected once by a mirror, and then injected into the resonance layer, where the magnetic field strength is 1T, at an angle \( \theta = 54^\circ \) to the GAMMA 10 axis. The microwave beam is injected into the plasma at almost a right angle to the local magnetic field line and the microwave electric field is perpendicular to the ambient magnetic field just after reflection by the mirror. Thus, it consists of an almost full X-mode component that couples to the R-wave as the wave vector becomes parallel to the magnetic field, and contributes to electron heating. Since the plug plasma is in the tenuous regime satisfying \((\omega_p/\omega)^2 < 2(v_t/c)N \cos \theta\) \([1]\), where \(\omega_p\) is the electron plasma frequency, \(\omega\) is the wave frequency, \(v_t\) is the electron thermal velocity, and \(N\) is the refractive index, X mode injection is favorable for efficient absorption.

A microwave beam is injected from the high field side and beach heating is realized. Since the magnetic scale length along the GAMMA 10 axis, \(L_B\), is about 0.4 m for the plug ECR region, the axial width of the resonance layer, \((kv_t \cos \theta)/\omega) L_B\), is about 2 cm for electrons with \(T_e = 1\) keV. Therefore, very local power deposition is expected. The power absorption rate of the microwave is very large for the plug plasma, and accurately estimating its value is difficult. A 3-D ray tracing calculation has been applied to the evaluation of the absorption rate for 2nd harmonic resonance heating in the GAMMA 10 barrier and central regions \([2,3]\). However, for the fundamental ECRH in the plug region, the absorption rate is very large and the scale length of the absorption is as short as the wave length of the microwave, \(k = k_i\), and thus the condition for WKB approximation is not satisfied. Recently we developed a computational code for full-wave Maxwell simulation \([4]\). It includes the effect of the fundamental ECR absorption through the conductivity tensor. Here, we evaluate the power absorption rate for the plug ECRH in three ways and confirm the efficacy of our full-wave simulation for the evaluation of power absorption by comparison with results obtained by other methods.

The profiles of the electron density and the temperature are assumed as \(n = n_0 \exp(-\psi/\psi_n)\) and \(T = T_0 \exp(-\psi/\psi_T)\), respectively, where \(n_0\) and \(T_0\) denote the on-axial values in the plug region and \(\psi\) is the magnetic flux. Symbols \(\psi_n\) and \(\psi_T\) determine the \(e\)-folding radii of the density and temperature profiles. Absorption rates are calculated for various \(n_0\) with \(T_0 = 1\) keV. Figure 2 plots the absorption rates as functions of \(x_0 = (\omega p/\omega)^2\), which is related to the electron density as \(x_0 \approx n_0(\text{cm}^{-3})/10^{13}\) for the microwave frequency of

\[ x_0 = (\omega p/\omega)^2 = \frac{n_0}{10^{13}} \]

\[ n_0(\text{cm}^{-3}) \]

\[ T_0(\text{keV}) \]

\[ \psi_n(\text{cm}) \]

\[ \psi_T(\text{cm}) \]

\[ k(\text{cm}) \]

\[ k_i(\text{cm}) \]

\[ \omega(\text{rad/s}) \]

\[ v_t(\text{cm/s}) \]

\[ L_B(\text{cm}) \]

\[ (kv_t \cos \theta)/\omega \]

\[ x_0 \approx n_0(\text{cm}^{-3})/10^{13} \]

\[ \text{X mode injection} \]

\[ \omega_c < \omega \]

\[ \text{beach heating} \]

\[ \text{Fundamental ECRH} \]

\[ \text{GAMMA 10} \]

\[ \text{plug region} \]

\[ \text{ray tracing} \]

\[ \text{Maxwell simulation} \]

Fig. 1 Schematic view of the GAMMA 10 plug region.
Absorption Rate by Using Full-Wave Maxwell Simulation

28 GHz. In the typical operation of the GAMMA 10 experiments, the plug density is \( x_0 \approx 0.01 \).

The open squares represent the absorption rate calculated with the full-wave Maxwell simulation. They are determined as the ratio of the microwave power density calculated with the ECR absorption term in the conductivity tensor to that without this term. The ratio is evaluated at the on-axis resonance position. The closed circles represent the results of the ray tracing code for the tenuous regime. The power absorption is estimated from the imaginary part of the wave vector \( k_i \) calculated with the warm plasma dispersion relation, though the WKB approximation does not necessarily hold for strong power deposition.

The solid curve in Fig. 2 shows the analytic estimation of the absorption rate. It is assumed that the density and the temperature are uniform and that the rays straightly propagate. Then, the absorption coefficient is represented as

\[
\alpha_i = \frac{2\pi}{\omega_0^2} \frac{1 + \cos^2 \theta}{\cos \theta} \exp \left( -\frac{\zeta_i}{\omega} \right)
\]

for a tenuous plasma and oblique propagation [1], where \( \zeta_i = (\omega - \omega_0)/\sqrt{k} \left| \cos \theta \right|v, \) and \( \omega_0 \) is the local electron cyclotron frequency. The optical depth of the ray is calculated by the integration of eq. (1) along the ray as \( \tau = \int \alpha_i \, dl \). The integral by the coordinate along the ray, \( l \), is transformed to that by \( \zeta_i \) with

\[
dl = \frac{dl}{dz} \frac{dz}{d\omega \, d\zeta_i} = \frac{1}{\cos \theta} \frac{L_{\omega i}}{\omega_0} \left| \frac{\partial}{\partial \omega} \left( \sqrt{k} \cos \theta \right) \right| \, d\zeta_i .
\]

Along the ray, \( \zeta_i \) varies strongly and the limits of integration are replaced as \( \int_{\zeta_i}^{\infty} d\zeta_i \). Thus we obtain

\[
\tau = \frac{\pi}{4} x_0 L_{\omega_i} \frac{1 + \cos^2 \theta}{\cos \theta} .
\]

Similarly the optical depth for the finite density regime is obtained [1] as

\[
\tau = \pi \left( 1 - \frac{x_0}{2} \right) \left( 1 + \frac{x_0^2}{2} \right) \frac{1}{x_0} \left( \frac{v_i}{c} \right)^2 k L_{\omega_i} \cos \theta .
\]

Figure 3 shows the absorption rate obtained with the full-wave simulation along the line with \( \theta = 54^\circ \) for \( x_0 = 0.001 \) (A1), 0.003 (A2), and 0.01 (A3) against \( z \) parallel to the GAMMA 10 axis. The magnetic field strength along the line is also plotted, where \( z = 0 \) mm is determined as the point of \( B = 1 \)T (see Fig. 1) and the \( z \)-coordinate of the mirror is \(-194 \) mm. The figure shows that the power is absorbed not only in the region of \( B > 1 \)T but also in the region of \( B < 1 \)T.

The results of full-wave simulation and ray tracing agree well with each other though the ray tracing calculation may be out of the applicable range for strong absorption. The absorption rate of the analytic evaluation is slightly larger than the others because it is estimated under the assumption of uniform density and temperature. Thus, our new simulation is effective to calculate the microwave propagation and the ECRH absorption rate. It shows that power is also deposited in the region of \( \omega_c < \omega \) for the beach heating.