On the plasma confinement with travelling magnetic field

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Abstract

The method of plasma confinement with travelling magnetic field was investigated both from magnetohydrodynamic and from charged particle orbit theory. The eminent advantages of applying travelling magnetic field to plasma confinement is thought to consist in its highly gradiented field configuration to provide a completely stable magnetic bottle and in its restoring action to drive a particle back to the plasma region. Two facts will be made clear in this paper; one is the need for superposition of two oppositely travelling fields, and the other is that the gradiented travelling field has the effects of driving the charged particles toward the direction where the field strength decreases, and thus eliminates the particle losses from the plasma region. These facts would suggest the possible realization of stable confinement of thermonuclear plasma. The need of superposing static magnetic induction on this scheme is also discussed.

1. Introduction

The possibility of the plasma confinement with travelling magnetic field has early been proposed by S. M. Osovets,⁽¹⁾ but the efforts to apply the method practically has not yet been made. This method, however, has some essential advantages comparing with that to utilize a conventional H.F. field, and seems to be promising for realization of controlled fusion. These advantages consist of the fact that the confining field is highly gradiented in its strength; which is not only quite ideal to prohibit the growth of the magnetohydrodynamic instabilities of the plasma column, but also is effective to drive the charged particle toward the direction where the

field strength decreases, which eliminates the drift losses of the particles.

In this paper, a new scheme of plasma confinement with the rotating magnetic field which has r and θ components is proposed. The rotating axis of the field coincides with the axis toward which the plasma is driven, and the plasma is confined in a cylindrical shape around the axis. The electromagnetic field and the current distribution in the plasma column is analytically calculated by assuming the ideal multiphase alternating currents in the axial direction applied from the external source, and the plasma to be a solid cylinder of metal whose conductivity is finite and uniform throughout the column.

In order to understand the behavior of a single charged particle which runs away from the plasma region, the orbits of the charged particles in the vacuum region were also calculated by using a digital computer for several cases of certain initial conditions, which stimulated the theoretical treatment of the particle motion in such a highly gradiented travelling field.

2. The proposed confinement scheme

The proposed scheme consists of a superposition of two travelling magnetic fields, each of which travels oppositely in both directions parallel to the plasma surface. The field strength becomes weaker at the point nearer to the axis of the plasma column. One example of such confinement scheme is shown in fig. 1. Here, cylindrical coordinates r , θ and z

fig. 1

proposed scheme of plasma confinement with travelling field

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are adopted, and assumed that the configuration is uniform along the z axis from $-\infty$ to $+\infty$. If the z axis is selected to be an endless curve such as a circle, this would provide a plasma confinement in a toroidal shape. The disposition is axisymmetric around the z axis, and the ideal multiphase currents

$$i_{z_1} = I \sin(\omega t + m\theta) \text{ and } i_{z_2} = I' \sin(\omega' t + m'\theta) \dots \dots \dots (1)$$

$$\omega\omega' > 0, m m' < 0, \omega \neq \omega' \quad \left. \begin{array}{l} |m| \\ |m'| \end{array} \right\} = 3, 4, 5, \dots \dots \dots$$

are supplied in the axial direction in the hollow cylindrical region $b < r < c$. The currents may be approximated by the finite phase alternating currents in practice. The frequencies of both currents are selected to be different lest the resultant electromagnetic field become only a single standing field. The magnetic field produced by the currents (1) is as depicted in the figure, and the plasma is expected to be confined around the z axis. The magnetohydrodynamic version of the plasma confinement is similar to that of the conventional H. F. confinement, and the analytical estimation of the confining force realized in the plasma column may be possible, if a proper model of the column is adopted.

As will be shown in the followings, the electromagnetic field utilized in this scheme is highly gradiented which would be quite favorable for confinement.

Another example of the proposed scheme conceivable is shown in fig. 2, where the currents supplied in the region $b < r < c$

fig. 2

Another example of plasma confinement with travelling field

are given as

$$i_{\theta_1} = I \sin (\omega t + \frac{2\pi}{\lambda} z) \text{ and } i_{\theta_2} = I' \sin (\omega' t + \frac{2\pi}{\lambda'} z) \dots\dots\dots (2)$$

$$\omega\omega' > 0, \lambda\lambda' < 0, \omega \neq \omega'$$

and the magnetic field is composed of r and z components while the field given in the previous example has r and θ components. The disposition of this scheme is similar to the previous one, except the travelling direction of the fields in this case is selected to be along the z axis. The latter case may not be discussed in detail in this paper, because the principles developed for the former case may directly be applied to the latter.

3. Calculation of the electromagnetic field

Since no interactions will be caused between both frequency components included in (1), both currents may be treated separately. This section will be concerned with the electromagnetic field produced by the current

$$i_{z_1} = I \sin (\omega t + m\theta) \dots\dots\dots (3)$$

$$m=3, 4, 5, \dots\dots\dots b < r < c$$

In order to find the resultant field of both frequency-components of the currents given in (1), it is enough to simply add two final results which follow from the consideration on each current component given as (3).

The plasma column is assumed to be confined around the axis cylindrically in $0 < r < a$, and the plasma is approximated by a solid metal whose conductivity and permeability are σ and μ_0 throughout the column. For convenience, the regions separated concentrically by the boundary surfaces $r=a$, b and c are designated as region (I), (II),... (IV) from inside to out as shown in fig. 3. Region (I) is occupied by the solid column of metal to simulate the plasma, which is assumed for the moment to be fixed against rotation. (II) and (IV) are thought as

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fig. 3

The model
for calculation

vacuum regions. The region (IV) is not necessarily a vacuum, if the region (II) is enclosed by a hermetically sealed vessel. This is only for the simplicity of calculation, and the electromagnetic constants in the region (IV) is assumed to be the same as those in a vacuum. Though the region (III) is installed with the current carrying conductors for the current distribution (3) to be established, the electromagnetic constants of the region is also assumed to be the same as a vacuum.

In calculating the electromagnetic field realized by the application of the current (3), we can put,

$$\frac{\partial}{\partial z} X = 0 \quad \dots \dots \dots (4)$$

where X represents any physical quantity, such as magnetic induction or electric field strength. As the current supplied in (III) has only Z component, magnetic field will have no z component, namely,

$$H_z = 0 \quad \dots \dots \dots (5)$$

The equations to be solved in each of the regions (I) to (IV) can be written down as follows, the displacement currents being ignored.

(Region (I))

$$\text{rot } \vec{H} = \vec{j} \dots\dots\dots (6)$$

$$\vec{j} = \sigma \vec{E} \dots\dots\dots (7)$$

$$\text{rot } \vec{E} = -\mu_0 \left(\frac{\partial \vec{H}}{\partial t} \right) \dots\dots\dots (8)$$

(Region (II))

$$\text{rot } \vec{H} = 0 \dots\dots\dots (9)$$

(Region (III))

$$\text{rot } \vec{H} = i_z \vec{k} \dots\dots\dots (10)$$

(Region (IV))

$$\text{rot } \vec{H} = 0 \dots\dots\dots (11)$$

In addition to the above, the following relation should be satisfied in all regions.

$$\text{div } \vec{H} = 0 \dots\dots\dots (12)$$

In the above equations, vectors $\vec{H}$, $\vec{E}$ and $\vec{j}$ represent magnetic field, electric field and current density respectively, and $\vec{k}$ is the unit vector in the z direction. These equations are to be solved under the subconditions

$$\vec{H}(0, \theta, z) = 0 \quad \vec{H}(\infty, \theta, z) = 0 \dots\dots\dots (13)$$

$$\left. \begin{aligned} \vec{H}_I(a, \theta, z) &= \vec{H}_{II}(a, \theta, z) \\ \vec{H}_{II}(b, \theta, z) &= \vec{H}_{III}(b, \theta, z) \\ \vec{H}_{III}(c, \theta, z) &= \vec{H}_{IV}(c, \theta, z) \end{aligned} \right\} \dots\dots\dots (14)$$

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where the subscripts added to H indicate that they are the solutions obtained in each region.

Some lengthy calculations are included⁽²⁾ in deriving the following results on the electromagnetic field in the regions (I) and (II).

(Region (I))

$$H_z = E_r = E_\theta = 0 \dots\dots\dots (15)$$

$$H_r = A \{ \alpha(x) + i\beta(x) \} e^{i\xi} + B \{ \alpha(x) - i\beta(x) \} e^{-i\xi} \dots\dots (16)$$

$$H_\theta = iA \{ \gamma(x) + i\delta(x) \} e^{i\xi} - iB \{ \gamma(x) - i\delta(x) \} e^{-i\xi} \dots\dots (17)$$

$$j_z = \sigma E_z = -\frac{\sqrt{\sigma\mu_0\omega}}{m} Ax \{ \alpha(x) + i\beta(x) \} e^{i\xi} \\ - \frac{\sqrt{\sigma\mu_0\omega}}{m} Bx \{ \alpha(x) - i\beta(x) \} e^{-i\xi} \dots\dots (18)$$

(Region (II))

$$H_r = (C r^{m-1} + D r^{-m-1}) e^{i\xi} + (E r^{m-1} + F r^{-m-1}) e^{-i\xi} \dots\dots (19)$$

$$H_\theta = i(C r^{m-1} - D r^{-m-1}) e^{i\xi} - i(E r^{m-1} - F r^{-m-1}) e^{-i\xi} \\ \dots\dots\dots (20)$$

$$E_z = \frac{\mu_0\omega}{m} (C r^m + D r^{-m}) e^{i\xi} + \frac{\mu_0\omega}{m} (E r^m + F r^{-m}) e^{-i\xi} \dots\dots (21)$$

where,

$$\xi = \omega t + m\theta, \quad x = \sqrt{\sigma \mu_0 \omega} r \dots\dots\dots (22)$$

$$\left. \begin{aligned} \alpha(x) &= \text{ber}_{m-1}(x) + \text{ber}_{m+1}(x) \\ \beta(x) &= \text{bei}_{m-1}(x) + \text{bei}_{m+1}(x) \\ \gamma(x) &= \text{ber}_{m-1}(x) - \text{ber}_{m+1}(x) \\ \delta(x) &= \text{bei}_{m-1}(x) - \text{bei}_{m+1}(x) \end{aligned} \right\} \dots\dots\dots (23)$$

$$A = \frac{a^{m-1} Q}{\{\alpha(x_0) + \gamma(x_0)\} + i \{\beta(x_0) + \delta(x_0)\}} \dots\dots\dots (24)$$

$$B = \frac{a^{m-1} Q}{\{\alpha(x_0) + \gamma(x_0)\} - i \{\beta(x_0) + \delta(x_0)\}} \dots\dots\dots (25)$$

$$C = E = \frac{1}{2} Q \dots\dots\dots (26)$$

$$D = \frac{\{\alpha(x_0) - \gamma(x_0)\} + i \{\beta(x_0) - \delta(x_0)\}}{2 \{ \{\alpha(x_0) + \gamma(x_0)\} + i \{\beta(x_0) + \delta(x_0)\} \}} a^{2m_Q} \dots\dots (27)$$

$$F = \frac{\{\alpha(x_0) - \gamma(x_0)\} - i \{\beta(x_0) - \delta(x_0)\}}{2 \{ \{\alpha(x_0) + \gamma(x_0)\} - i \{\beta(x_0) + \delta(x_0)\} \}} a^{2m_Q} \dots\dots (28)$$

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here,

$$x_0 = \sqrt{\sigma \mu_0 \omega} a \dots\dots\dots (29)$$

$$Q = \frac{I}{2(m-2)} (b^{2-m} - c^{2-m}) \dots\dots\dots (30)$$

4. Electromagnetic force exerted to the plasma

As the electromagnetic force which the plasma column experiences can be expressed as

$$\vec{f} = \vec{j} \times \vec{B} \dots\dots\dots (31)$$

where $\vec{j}$ and $\vec{B}$ are current and magnetic induction in the column, it is directly calculated by making use of (16) to (18). If the oscillating frequency of the field ω is too high for the mechanical inertia of the plasma material to follow, the force may well be represented by its time average. The results are,

$$f_r = -\mu_0 \overline{j_z H_\theta}$$

$$= \frac{\mu_0 \sqrt{\sigma \mu_0 \omega} x a^{2m-2} Q^2 \{ \text{bei}_{m-1}(x) \text{bei}_{m+1}(x) - \text{ber}_{m+1}(x) \text{bei}_{m-1}(x) \}}{m \{ \text{ber}_{m-1}^2(x_0) + \text{bei}_{m-1}^2(x_0) \}} \dots\dots\dots (32)$$

$$f_\theta = \mu_0 \overline{j_z H_r}$$

$$= \frac{-\mu_0 \sqrt{\sigma \mu_0 \omega} x a^{2m-2} Q^2 \{ \{ \text{ber}_{m-1}(x) + \text{ber}_{m+1}(x) \}^2 + \{ \text{bei}_{m-1}(x) + \text{bei}_{m+1}(x) \}^2 \}}{2m \{ \text{ber}_{m-1}^2(x_0) + \text{bei}_{m-1}^2(x_0) \}} \dots\dots\dots (33)$$

f_r is always negative at every point which provides the confining force to the plasma. If we assume a simple magnetohydrodynamic relation

$$(\vec{\nabla} p)_r = f_r \dots\dots\dots (34)$$

then the pressure distribution realized in the plasma column can be expressed as

$$p = - \int_r^a f_r dr$$

$$= \frac{\mu_0 a^{2m-2} Q^2}{m \{ \text{ber}_{m-1}^2(x_0) + \text{bei}_{m-1}^2(x_0) \}} \int_x^{x_0} x \{ \text{ber}_{m+1}(x) \text{bei}_{m-1}(x) - \text{ber}_{m-1}(x) \text{bei}_{m+1}(x) \} dx \dots\dots\dots (35)$$

In order to get a numerical concept,

$$x \{ \text{ber}_{m-1}(x) \text{bei}_{m+1}(x) - \text{ber}_{m+1}(x) \text{bei}_{m-1}(x) \} \dots\dots\dots (36)$$

and

$$\int_x^{x_0} x \{ \text{ber}_{m+1}(x) \text{bei}_{m-1}(x) - \text{ber}_{m-1}(x) \text{bei}_{m+1}(x) \} dx \dots\dots\dots (37)$$

which appear in the equations (32) and (35) are plotted in fig. 4, for the case when $m=4$, $x_0=6.5$. If we denote the pressure at the center of the column $P_0 = (\text{atm})$, it is in case of fig. 4,

$$P_0 \simeq 0.22 \times 10^{12} a^6 J^2 \left(\frac{1}{b^2} - \frac{1}{c^2} \right)^2 (\text{atm}) \dots\dots\dots (38)$$

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fig. 4 an example of numerical evaluation of (36) and (37)

If we assume $b^2 \ll c^2$, $a^6 \simeq b^4$ (i.e. $a \simeq 1.6m$ $b \simeq 2m$)

$$I \simeq 2.1 \times 10^7 \frac{A}{m^2} \dots\dots\dots (39)$$

would be enough to confine the plasma whose pressure P_0 is approximately 100 atm, which is thought to be the lowest pressure of the plasma of thermonuclear interest. The value obtained in (39) may be realized by applying an efficient water cooling system to the conventional conductors.

The column has so far been assumed to be fixed against rotation. The application of both currents given in (1) at a time may brake the rotation of the column, and the confining effects are added. It would be effective to select the currents (1) so as to diminish the resultant torque applied to the column, namely

$$\int_0^a 2\pi r^2 \left(\sum_{i_z, i_z} \bar{f}_\theta \right) dr = 0 \dots\dots\dots (40)$$

5. Motion of a charged particle in the vacuum region

It was inferred that in such a highly gradiented field as is given by (19) to (21) the charged particles may be driven toward the direction where the field decreases its strength. If it should be the case, the charged particle which left the plasma region would be sent back toward the Z axis in the confining scheme shown in fig. 3, and the loss of particles may possibly be eliminated.

This inference seems rather unusual, because in the static inhomogeneous magnetic field, the charged particles drift toward the direction $\pm \vec{B} \times \vec{\nabla} B$ according to the sign of the charge and not in $-\vec{\nabla} B$ direction.

Since the field given in (19) to (21) are pretty complicated to handle, the field at the limit when the conductivity of the plasma column becomes infinite will be considered throughout the section. The field in such limit can be calculated as above by imposing the subconditions as shown below in place of (13) to (14).

$$\left. \begin{aligned} \vec{H}(\infty, \theta, z) &= 0 \\ \vec{H}_{II}(b, \theta, z) &= \vec{H}_{III}(b, \theta, z) \\ \vec{H}_{III}(c, \theta, z) &= \vec{H}_{IV}(c, \theta, z) \\ H_{rII}(a, \theta, z) &= 0 \quad H_{\theta II}(a, \theta, z) = j_z^* \end{aligned} \right\} \dots\dots\dots (41)$$

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As the results, the magnetic induction can be expressed as

$$B_r = B (r^{m-1} - \alpha^{2m} r^{-m-1}) \cos \xi \dots\dots\dots (42)$$

$$B_\theta = -B (r^{m-1} + \alpha^{2m} r^{-m-1}) \sin \xi \dots\dots\dots (43)$$

$$E_z = -\frac{\omega}{m} B (r^m - \alpha^{2m} r^{-m}) \cos \xi \dots\dots\dots (44)$$

where

$$B = \mu_0 Q$$

The equations which determine the particle motion in the above field can be derived immediately.

$$\frac{M}{\varphi_B} \ddot{z} = -\dot{r} (r^{m-1} + \alpha^{2m} r^{-m-1}) \sin \xi$$

$$- \left(\theta + \frac{\omega}{m} \right) (r^m - \alpha^{2m} r^{-m}) \cos \xi$$

$$\frac{M}{\varphi_B} (\ddot{r} - r \dot{\theta}^2) = \dot{z} (r^{m-1} + \alpha^{2m} r^{-m-1}) \sin \xi$$

$$\frac{M}{\varphi_B} (\ddot{\theta} + 2 \frac{\dot{r}}{r} \dot{\theta}) = \dot{z} (r^{m-2} - \alpha^{2m} r^{-m-2}) \cos \xi$$

where M and g indicate the mass and the charge of a particle respectively. When we introduce the non-dimensional variables,

$$T = \frac{gB a^{m-1}}{M} t, R = \frac{r}{a}, \theta = \theta, Z = \frac{z}{a}, Q = \omega \frac{M}{gBa^{m-1}},$$

the equations (45) to (47) are reduced to

$$\ddot{Z} = -\dot{R} (R^{m-1} + R^{-m-1}) \sin \xi \dots \dots \dots (45)$$

$$- \left(\dot{\theta} + \frac{Q}{m} \right) (R^m - R^{-m}) \cos \xi$$

$$\ddot{R} - R \dot{\theta}^2 = \dot{Z} (R^{m-1} + R^{-m-1}) \sin \xi \dots \dots \dots (46)$$

$$\ddot{\theta} + 2 \frac{\dot{R}}{R} \dot{\theta} = \dot{Z} (R^{m-2} - R^{-m-2}) \cos \xi \dots \dots \dots (47)$$

In order to get some qualitative understanding on the particle orbit in this region, the above equations were applied for computations through IBM 7090 digital computer. The Euler method was adopted for numerical evaluation. The orbit was pursued for a particle whose initial speed is 15, and each of the 6 cases where the initial velocities were assigned in $\pm R$, $\pm \theta$ and $\pm Z$ directions respectively was computed. The particle starts the point $R=2.5$ and $\theta=0$ at $T=0$. Q was chosen to be 400, while $\Delta T=0.0002$ was taken as the infinitesimal time duration for computation, which proved to be sufficiently small for the purpose.

Fig's 5 to 8 are to show the results of computation for the individual cases. In fig. 5, the θ component of the magnetic induction which the particle experiences is also shown. Looking at these results, it will become clear that any particle is reflected back toward the z axis. This effects may be visuali-

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fig. 5

particle orbit in the vacuum region (1)

fig. 6 particle orbit in the vacuum region (2)

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fig. 7

particle orbit in the vacuum region (3)

fig. 8
 particle orbit in the vacuum region (4)

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zed well, if we plot the orbit on the $R-\theta$ plane as shown
in fig. 9 for the case 3.

fig. 9 $R-\theta$ projection of the particle motion
for the case 3.

The results shown in fig. 6 shows the tendency as if the particle goes away from the column gradually, but the slight decrease of the initial speed θ_0 results the particle to come to touch the column sooner. More important seems the fact that particle of a certain speed seems to be unable to go beyond a circle of the radius which is determined actually by the current density I in (3).

To treat the problem more generally, some analytical efforts have been added on the equations (45) to (47). One approach for this is to find the particle orbit in the limit when the frequency of the field is high enough so that the particle experiences quite a number of cycles of the field variation before it moves an appreciable distance. Since the equation (45) can be integrated directly to take the form

$$\dot{Z} = -\frac{1}{m}(R^m - R^{-m}) \sin \xi + C \dots \dots \dots (48)$$

(46) and (47) can be written down as

$$\ddot{R} - R \dot{\theta}^2 = -\frac{1}{m} \cdot \frac{1}{R} (R^{2m} - R^{-2m}) \sin^2 \xi \\ + C (R^{m-1} + R^{-m-1}) \sin \xi \dots \dots \dots (49)$$

$$R \ddot{\theta} + 2 \dot{R} \dot{\theta} = -\frac{1}{m} \cdot \frac{1}{R} (R^m - R^{-m})^2 \sin \xi \cos \xi \\ + C (R^{m-1} - R^{-m-1}) \cos \xi \dots \dots \dots (50)$$

In the limit mentioned above, the oscillating terms such as $\sin \xi$, $\cos \xi$ or $\sin \xi \cos \xi$ may be averaged away in its effects on the particle motion, except that which includes $\sin^2 \xi$. Therefore, in these limiting cases, (48) to (50) can be simplified to

$$\dot{Z} = C \dots \dots \dots (51)$$

$$\ddot{R} - R \dot{\theta}^2 = -\frac{1}{2} \cdot \frac{1}{m} \cdot \frac{1}{R} (R^{2m} - R^{-2m}) \dots \dots \dots (52)$$

$$R \ddot{\theta} + 2 \dot{R} \dot{\theta} = 0 \dots \dots \dots (53)$$

The equation (53) can be integrated to become,

$$\dot{\theta} R^2 = D$$

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For comparison, the equations (51), (52) and (54) were computed by the digital computer under the same conditions as before. The results were plotted together on the fig's 5 to 8, which are recognized to be considerably close to those calculated before. The corresponding $R-\theta$ projection to the case shown in fig. 6 (case 3) is shown in fig. 10.

fig. 10 $R-\theta$ projection of the particle orbit approximately calculated for the case 3

As will be imagined in the equation (52), the similar calculation for the case when the charge of the particle is negative gave almost the same results.

6. Particle motion in a simple gradiented travelling field

Another approach into the problem how a charged particle is driven in the gradiented travelling field as is given in (19) to (21) is to treat the matter in the travelling field of more simple form. The most simple gradiented travelling field conceivable may be the one which is produced in the vacuum region by the one-dimensional sheet-like current distribution as given by (in cartesian coordinates)

$$i_z = I \sin \left(\omega t + \frac{2\pi}{\lambda} x \right) \quad \left[\frac{\text{A}}{\text{m}^2} \right] \dots\dots\dots (55)$$

$$\text{in } y=0, -\infty < z < \infty \quad -\infty < x < \infty$$

The electromagnetic field to be produced by the above current distribution can be derived as follows, neglecting the displacement current as before,

$$B_z = E_x = E_y = 0 \dots\dots\dots (56)$$

$$B_x(x, y, z) = -B e^{-\beta y} \sin(\omega t + \beta x) \dots\dots\dots (57)$$

$$B_y(x, y, z) = -B e^{-\beta y} \cos(\omega t + \beta x) \dots\dots\dots (58)$$

$$E_z(x, y, z) = -\frac{\omega}{\beta} B e^{-\beta y} \cos(\omega t + \beta x) \dots\dots\dots (59)$$

for $y > 0$

where

$$B = \frac{\mu_0 I}{2}, \quad \beta = \frac{2\pi}{\lambda}$$

The particle orbit in this field is determined by the following equations of motion.

$$M \ddot{x} = -g \dot{z} B_y = g B \dot{z} e^{-\beta y} \cos(\omega t + \beta x) \dots\dots\dots (60)$$

$$M \ddot{y} = g \dot{z} B_x = -g B \dot{z} e^{-\beta y} \sin(\omega t + \beta x) \dots\dots\dots (61)$$

$$\begin{aligned} M \ddot{z} &= g (\dot{x} B_y - \dot{y} B_x) + g E_z \\ &= -g B \left(\dot{x} + \frac{\omega}{\beta} \right) e^{-\beta y} \cos(\omega t + \beta x) + g B \dot{y} e^{-\beta y} \sin(\omega t + \beta x) \\ &\dots\dots\dots (62) \end{aligned}$$

Putting

$$w = x + \frac{\omega}{\beta} t$$

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and by introducing the non-dimensional variables

$$T = \frac{\omega_B}{M} t, \quad W = \beta w, \quad Y = \beta y, \quad Z = \beta z$$

the equations (60) to (62) are transformed into

$$\ddot{W} = \dot{Z} e^{-Y} \cos W \dots \dots \dots (63)$$

$$\ddot{Y} = -\dot{Z} e^{-Y} \sin W \dots \dots \dots (64)$$

$$\ddot{Z} = \dot{Y} e^{-Y} \sin W - \dot{W} e^{-Y} \cos W \dots \dots \dots (65)$$

(65) is integrated to become

$$\dot{Z} = -e^{-Y} \sin W + C$$

which is substituted into (63) and (64).

$$2\ddot{W} = -e^{-2Y} \sin 2W + C' e^{-Y} \cos W \dots \dots \dots (66)$$

$$2\ddot{Y} = e^{-2Y} (1 - \cos 2W) - C' e^{-Y} \sin W \dots \dots \dots (67)$$

In the limit when the frequency of the field variation is high enough, the above equations are reduced to

$$\dot{Z} = C \dots \dots \dots (68)$$

$$\dot{W} = D \dots \dots \dots (69)$$

$$\ddot{Y} = \frac{1}{2} e^{-2Y} \dots \dots \dots (70)$$

in the similar way as before. The solution of the equation (70) is

$$Y = \ln \frac{1}{\sqrt{2} \alpha} \cosh \{ \alpha (T + r) \} \dots \dots \dots (71)$$

for $Y > 0$

which indicates the fact that the particles are driven toward

$Y \rightarrow \infty$ as $T \rightarrow \pm \infty$. This is to confirm that the particles in these highly gradiented travelling field is reflected from the region where the field strength is higher to the region of lower field strength. And the plasma may be confined in the region $Y > k$ ($k > 0$) if the current density I is chosen to be high enough, since the sign of the particle charge is not involved in the above calculations. Though we restricted our discussion for $Y > 0$ in the above, it would be seen easily that the similar conclusion is deduced for $-\infty < Y < +\infty$. The orbit (71) has a minimum point when the particle starts with the negative velocity component, that is $\dot{Y}_0 < 0$. The minimum point in this case is expressed as

$$Y_{\min} = \ln \frac{1}{\sqrt{2} \alpha} \dots \dots \dots (72)$$

In addition, α and r in (71) are derived as

$$\alpha = \frac{1}{\sqrt{2} e^{Y_0}} \sqrt{1 + 2 (\dot{Y}_0 e^{Y_0})^2} \dots \dots \dots (73)$$

$$r = \frac{\sqrt{2} e^{Y_0}}{\sqrt{1 + 2 (\dot{Y}_0 e^{Y_0})^2}} \sinh^{-1} \{ \sqrt{2} (\dot{Y}_0 e^{Y_0}) \} \dots \dots \dots (74)$$

The general discussion on the equations (66) and (67) is thought to be of considerable complication. The case when $C' = 0$ in them seems somewhat intuitive. If we put

$$Y + iW = \ln \left\{ \frac{R}{\sqrt{2}} e^{i\theta} \right\} \dots \dots \dots (75)$$

($R > 0$)

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the equations can be rewritten as

$$R \ddot{R} - \dot{R}^2 = 1 - \cos 2\theta \dots\dots\dots (76)$$

$$R^2 \ddot{\theta} = -\sin 2\theta \dots\dots\dots (77)$$

The right side of the equation (76) is almost always positive, while the equation

$$R' \ddot{R}' - \dot{R}'^2 = a^2 \dots\dots\dots (78)$$

has the solution

$$R' = \cosh \{ a (T + \delta) \} \dots\dots\dots (79)$$

This is to suggest the likelihood for $R(>0)$ to inherently increase as the time elapses, which shows in turn $Y \rightarrow \infty$ as $T \rightarrow \pm \infty$. As before, we have avoided the complication by confining the discussion for $\gamma > 0$. This may be justified by selecting the field strength and the starting point of a charge properly.

In connection to this problem, the complex differential equation

$$\ddot{\vec{Y}} = e^{-2\vec{Y}} \dots\dots\dots (80)$$

shows a suggestive characteristics. The general solution of the above is easily found to be

$$\vec{Y} = \ln \frac{i}{A + iB} \sinh \{ (A + iB) (T + \Gamma + i\tau) \} \dots\dots\dots (81)$$

$A, B, \Gamma, \tau; \text{real}$

If we put

$$\vec{Y} = Y + iW = \ln \{ R'' e^{i\theta''} \}$$

the equations for R'' and θ'' has an analogous form as (76) and (77),

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$$R''R'' - \dot{R}''^2 = \cos 2\theta'' \dots\dots\dots(82)$$

$$R''^2 \ddot{\theta}'' = -\sin 2\theta'' \dots\dots\dots(83)$$

and has the solution

$$Y = \ln \frac{1}{2(A^2 + B^2)} \{ \cosh(2\{A(T+\Gamma) - B\tau\}) - \cos 2(\{B(T+\Gamma) + A\tau\}) \} \dots\dots\dots(84)$$

which shows, except for a limited cases of initial conditions, (eg, A=0) the same behavior as before that $Y \rightarrow \infty$ when $T \rightarrow \pm\infty$.

We have not discussed on the influence of superposing the other travelling field on the field so far been given consideration. Some trial calculations have shown that the effects reflecting the charged particles toward the region of lower field strength seems to be added together in this case.

7. Addition of static axial field.

The limiting case which was adopted in the previous section for approximate calculation of the particle orbit in the gradiented travelling field has shown to be quite satisfactory for the plasma confinement. However, for a particle of energy of the order of ~ 10 to ~ 100 KeV, the frequency of the field should be tremendously high in order such approximation be right. For example, the frequency of the travelling field should be of the order of ~ 10 to ~ 100 Mc/S for deuterons. Such would require the effects of displacement currents to be taken into account; but, most of all, it would lay a severe problem from the engineering standing point of view.

However, the fact that the particle motion can be determined without including the z component explicitly as seen from (49) and (50), for example, would show the usefulness of applying the axial static induction to solve the problem. The particles would be frozen around the induction, and consequently their motion in r and θ directions is braked. This would equivalently decrease the speed of the particles in $r = \theta$

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plane, and the field of considerably lower frequency would become sufficient to realize the particle motion which is similar to that obtained in the limiting case where the frequency was assumed extremely high.

8. Conclusions

The plasma confinement with the travelling magnetic field has been treated with the finite conductivity of the plasma being taken into account. As the result the need for adopting two travelling fields superimposed which travel in the opposite directions to each other and have different frequencies for cancelling the driving force of the fields in their travelling directions became obvious.

Some efforts have been added to understand the particle motion in the highly gradiented travelling field as is realized in these confining scheme. The numerical calculations and the approximate analytical investigation have shown that the particles are reflected from the region of higher field strength. This is thought to be due to the mirror-like effects which are inherent to these gradiented travelling field. By virtue of these effects the proposed scheme is expected to provide a completely stable confinement of plasma with little particle losses. It became obvious, in addition, that the superposition of the static magnetic induction in the z direction on the scheme is necessary from the engineering point of view.

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References;

- (1) S. M. Osovets: Confinement of a plasma by a travelling magnetic field. "Plasma Physics and the Problem of Controlled Thermonuclear Reactions" Pergamon Press vol. IV (1959)
- (2) H. Kondo, K. Toshioka: to be published