Conductivity of a Partially Ionized Drifting Plasma

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Abstract

The conductivity of a partially ionized drifting plasma is discussed on the basis of the macroscopic transport equation. The resulting form of the conductivity differs from the ordinary one in that the perturbation of temperature leading to that of ionization plays a substantial role in the present case.

Our theory predicts that effective dielectric permittivity deduced from the imaginary part of the conductivity may exceed the unity.

I. Introduction

The present work is devoted to an account of the influence of a temperature perturbation on the conductivity of a partially ionized drifting plasma.

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The conductivity of gaseous plasma has been considered in the view of the response of the electron velocity to the applied a.c. electric field. The conductivity formula was derived starting from the equation of motion with allowance for the effect of collision. The complex conductivity thus obtained is expressed as

\[ \sigma = \frac{n e^2}{m} \frac{\nu_c}{\nu_c^2 + \omega^2} - i \frac{n e^2}{m} \frac{\omega}{\nu_c^2 + \omega^2} \]  

(1)

where \( n \) is the plasma density, \( m \) the mass of the electron, \( e \) the electronic charge, \( \nu_c \) the collision frequency, \( \omega \) the angular frequency of the applied a.c. field.

This formula has been accepted in microwave diagnostic technique as a fundamental relation describing the property of plasma.

However, this simplified formula must be modified if a perturbation of the electron temperature is considered. It causes a change of the ionization frequency resulting in a corresponding change of losses of the energy and the density of the electron. Therefore, the current response or the conductivity must be evaluated by considering the temperature perturbation in addition to these of the density and the velocity.

As the basic equations determining the relation
among these quantities the macroscopic transport equations representing the continuity, the conservation of the momentum and the energy are used.

The plasma concerned here has a drift due to a steady electric field compensating for the loss of the energy by elastic and inelastic collisions.

With the conductivity formula presented by us, the anomalous plasma permittivity will be discussed.

II. The plasma Conductivity

We start with one dimensional transport equation for the electron regarding that the ion has a negligible effect on the current response. It is also assumed that all the collision processes occur between the electron and the neutral molecule.

With these assumptions, the transport equations are written by\(^{(2)(3)}\),

\[
\frac{\partial}{\partial t} n + \frac{\partial}{\partial x} (n v) = n \nu_i - W \tag{1}
\]

\[
\frac{\partial}{\partial t} (n v) + \frac{\partial}{\partial x} \left\{ n \left( v^2 + kT/m \right) \right\} + eE_n/m = -\nu_e n v \tag{2}
\]

\[
\frac{\partial}{\partial t} \left\{ n \left( v^2 + kT/m \right) \right\} + \frac{\partial}{\partial x} \left\{ n \left( v^3 + 3v kT/m \right) \right\}
+ 2eE_n v/m = -2\nu_e n kT/M - \nu_i eV_i n/m \tag{3}
\]
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where \( v \) is the directed velocity of the electron, \( \nu_i \) the ionization frequency, \( W \) the loss in the number density \( n \), \( k \) the Boltzmann constant, \( T \) the electron temperature, \( E \) the electric field, \( M \) the mass of the neutral molecule, and \( V_i \) the ionization potential of the gas used.

The conduction current density \( J \) is given by

\[
J = -nev
\]

The smallness of the a.c. components allows us to linearize the above basic equations. The ionization frequency strongly depends upon the electron temperature, as seen from the following expression:

\[
\nu_i = A \left( \frac{kT_e}{e} \right)^{1/2} e \mu ( -eV_i / kT )
\]

where \( A \) is a constant. The collision frequency \( \nu_c \) is also a function of the electron temperature, i.e., it is proportional to the root of \( T \).

To proceed the analysis, we make use of the d.c. equations as follows:

\[
\frac{ne}{\nu_i} = W
\]

\[
eE/m = -\nu_c \nu_o
\]

\[
\nu_o^2 = kT_e/M + \nu_i eV_i / 2m \nu_c
\]

which are obtained from eqs. (1) to (3).
The linearized a.c. equations can be solved for vanishing space derivatives of all quantities appearing in the equations. The conductivity for this case is easily found taking the ratio of the a.c. current to the a.c. electric field. The conductivity thus obtained is ultimately given by

\[
\sigma = \frac{\omega_p^2}{4 \pi} \left\{ \frac{kT_o}{m} (Q - R) \omega^2 + \left( \frac{PR - SQ}{m} \right) \right\} + \frac{i \omega_p^2}{4 \pi \omega} \left\{ -PS + \left( \frac{kT_o}{m} P + \frac{kT_o}{m} S - RQ \right) \right\}
\]

where

\[
p = \nu_0 \left\{ \left( \frac{kT_o}{M} + \frac{\nu_{io} e V_i}{2m \nu_c} \right) \left( \nu_{co} + 3 \nu_{io} \frac{e V_i}{kT_o} \right) + \frac{e V_i \nu_{io}}{kT_o} \left( \frac{1}{m} + \frac{1}{m} \right) + \frac{3kT_o}{M} \nu_{co} \right\}
\]

\[
Q = -\left( \frac{kT_o}{M} + \frac{\nu_{io} e V_i}{2m \nu_c} \right) \left( \nu_{co} + \nu_{io} \frac{e V_i}{kT_o} \right) + \nu_{io} \left( \frac{e V_i}{m} + \frac{e V_i}{kT_o} \right) + \frac{3kT_o}{M} \nu_{co}
\]

\[
R = -\left( \frac{kT_o}{M} + \frac{\nu_{io} e V_i}{2m \nu_c} \right) \left( \nu_{co} + \nu_{io} \frac{kT_i}{kT_o} \right) + \frac{3 \nu_{co} kT_o}{M} + \frac{e V_i}{m} \left( \frac{e V_i}{kT_o} \right) + \frac{3K_T o}{M} \nu_{io}
\]

\[
S = 4 \left( \frac{kT_o}{M} + \frac{\nu_{io} e V_i}{2m \nu_c} \right) \nu_{io} \nu_{co} \frac{e V_i}{kT_o}
\]

\[
\omega_p^2 = \frac{4 \pi e^2 n_o}{m}
\]
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$\nu_{io} = 3 \times 10^4$/sec., $\nu_{co} = 2 \times 10^5$/sec., and $T = 3 \times 10^4$ °K, which are typical for He plasma of 1 mm Hg, while in the latter case we take $\nu_{io} = 3 \times 10^6$/sec. and $\nu_{co} = 2 \times 10^7$/sec., but leaving the remaining values unchanged.

Fig. 1 shows the plot of the real part of the conductivity $\sigma_r$ as a function of the frequency for the former case with different plasma density. In fig. 2 these for the latter case are shown. In both cases, the plot of $\sigma_r$ basing on eq. (1) are shown for comparison.
Fig. 1 (b)

$\eta_0 = 10^9 / \text{cm}^3$

$\sigma_T (\mathcal{U}) 10^3$

$10^{-2}$

$10^4$

$10^5$

$10^6$

$10^7$

$10^8$

$10^9$

$f (\text{c/sec.})$

Eq. (1)

Fig. 1 (c)

$\eta_0 = 10^7 / \text{cm}^3$

$\sigma_T (\mathcal{U}) 10^3$

$10^4$

$10^5$

$10^6$

$10^7$

$10^8$

$10^9$

$f (\text{c/sec.})$

Eq. (1)
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Fig. 2 (a)

Fig. 2 (b)
Concerning the imaginary part of the conductivity, it is pointed out that it may possess a positive value if the frequency lies between \( f_1 \) and \( f_2 \) given by

\[
\omega_{1,2}^2 = \frac{1}{2} \left( \frac{m}{kT} \right)^2 \left[ \left( \frac{kT_m}{m} \right)(P+S) - RQ \right] \pm \sqrt{\left( \frac{kT_m}{m}(P+S) - RQ \right)^2 - 4 \left( \frac{kT_m}{m} \right)^2 PS}
\]

From figs. 3 and 4 in which \( \sigma_i \) for two cases are plotted, it is evident that for both cases there appears a frequency range having a positive \( \sigma_i \). For example, the numerical values of \( f_1 \) and \( f_2 \) for the former case are found to be around \( 6 \times 10^6 \) c/sec.
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Fig. 3 (a)

Fig. 3 (b)
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Fig. 3 (c)

\[ \eta_0 = 10^7 / \text{cm}^2 \]

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Fig. 4 (a)

\[ \eta_0 = 10^7 / \text{cm}^2 \]

\[ \eta_0 = 10^7 / \text{cm}^2 \]
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Fig. 4 (b)

\[ \sigma_c(U) \]

\[ n_e = 10^9 / \text{cm}^3 \]

\[ f (\text{c/sec.}) \]

(c)

\[ \sigma_c(U) \]

\[ n_e = 10^7 / \text{cm}^3 \]

\[ f (\text{c/sec.}) \]
As a consequence, the effective permittivity defined by \( \varepsilon = 1 + 4 \pi \sigma f / \omega \) is given by

\[
\varepsilon = 1 - \frac{\omega_p^2}{\omega^2} \left[ \frac{PS - \left\{ \frac{kT_0}{m} (p + s) - RQ \right\} \omega^2 + \left( \frac{kT_0}{m} \right)^2 \omega^4}{\left\{ p - \left( \frac{kT_0}{m} \right) \omega^2 \right\}^2 + \omega^2 Q^2} \right]
\]  

(12)

which is unlike the ordinary form, \( \varepsilon = 1 - \frac{\omega^2}{\nu_e^2 + \omega^2} \). In other words, the permittivity or dielectric constant may exceed unity under a certain condition.

In conclusion, it is noted that the dynamic response of a partially ionized plasma is strongly affected by the ionizing collision in such a way that a positive \( \sigma_i \) appear.

References


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Figure caption

Fig. 1. The plot of $\sigma_r$ as a function of frequency.
Fig. 2. The plot of $\sigma_r$ as a function of frequency.
Fig. 3. The plot of $\sigma_i$ as a function of frequency.
Fig. 4. The plot of $\sigma_i$ as a function of frequency.