J-Mirror, Magnetic Bottle Using the Longitudinal Invariant of Motion

J-Mirror, Magnetic Bottle Using the Longitudinal Invariant of Motion

T. Ohkawa

Gulf General Atomic Incorporated
John Jay Hopkins Laboratory for Pure and Applied Science
San Diego, California 92112

A magnetic bottle may be obtained by having a maximum of longitudinal invariant (J) in both the radial and the axial direction. The "loss disc" in this case is the region in the velocity space corresponding to small parallel velocities or small values of the invariant. Since the excursion of the orbit with a small J is confined to a small volume, the collisions only in this volume will lead to the loss of the particle. Consequently, the confinement time limited by collisions is much longer than that of the μ -mirror configuration. On the other hand, the constancy of J may be more readily destroyed by nonadiabaticity or low frequency electric field.

INTRODUCTION

The magnetic mirror device utilizes the constancy of the magnetic moment (μ) to confine the particle orbits. A magnetic bottle may similarly be obtained by using the constancy of the longitudinal invariant J defined by $J = m \oint v_{\parallel} ds$. If a magnetic configuration is designed such that J of a typical orbit decreases away from the confining region, (maximum J configuration) the particle orbits with J larger than a critical value are confined. In the following the advantages and the disadvantages of these configurations are compared with the ordinary mirror configuration.

MAXIMUM J CONFIGURATION

We shall take a linear quadrupole configuration as an example. The magnetic moment μ and the longitudinal invariant J are assumed to be constant of motion. This may be a poor approximation near the separatrix.

We shall use customary ψ (flux function) and χ (magnetic potential) coordinates. The longitudinal invariant is defined by

$$J = m \oint \frac{\dot{\chi}}{B^2} d\chi \quad (1)$$

It is equal to the area enclosed by the trajectory in the phase space for the trapped particles or by the trajectory and the periodic boundary of χ for the untrapped particles.

J-Mirror, Magnetic Bottle Using the Longitudinal Invariant of Motion

Since μ is a constant of motion, Eq. (1) may be written as

$$J(v, \psi, \mu) = \int \sqrt{mv^2 - 2\mu B(\psi, x)} (dx/B) \quad . \quad (2)$$

For a given v and ψ , the limiting values of J are:

- (a) particles without parallel velocity (maximum μ), $J = 0$ and
- (b) particles without perpendicular velocity ($\mu = 0$),

$$J = mv \oint (dx/B) = mv \oint ds \quad . \quad (3)$$

Therefore, the necessary condition for a maximum J configuration is that $\oint ds$ has a maximum. The particles with $J = 0$ are in the "loss disc", because $J = 0$ orbits cannot be contained by maximum J configuration.

For an intermediate value of J , Eq. (2) is differentiated with respect ψ ,

$$\frac{\partial J}{\partial \psi} = - \oint (mv^2 - 2\mu B)^{-\frac{1}{2}} (mv^2 - \mu B) B^{-2} (\partial B / \partial \psi) dx \quad . \quad (4)$$

It may be rewritten as

$$\frac{\partial J}{\partial \psi} = -m \oint (v_{\parallel}^2 + v_{\perp}^2/2) (1/BR_c) (ds/v_{\parallel}) = - \left(\frac{e}{c} \right) \oint v_g (ds/v_{\parallel}) \quad (5)$$

where R_c^{-1} is the curvature of the line and v_g is the drift velocity.

Equation (5) shows that the condition for a maximum J configuration is to have all particles drift, averaged over the orbits, in the good direction

For the particles which have the turning point at the maximum in x direction of the magnetic field strength, the integral is determined by the local value of the integrand. Therefore a maximum J configuration has to

Ohkawa

have the minimum B in ψ direction locally at the maximum in the χ direction. This has been suggested first by M. Rosenbluth⁽¹⁾ to stabilize the residual flute instability.

The maximum J region is defined by

$$\oint (mv^2 - 2\mu B)^{-\frac{1}{2}} (mv^2 - \mu B) B^{-2} (\partial B / \partial \psi) d\chi > 0 \quad (6)$$

for all values of μ between 0 and the maximum value given by $(mv^2 / 2B_m)$ where B_m is the magnetic field strength at the minimum on a flux line.

J may be made to have maximum in the axial direction by bringing current carrying rods of a linear multipole closer at the both ends. The magnetic field configuration is compressed at both the ends. The line length is reduced proportionally with the distance between the axis and the rods. The magnetic field increases proportionally. J for a given v , μ and ψ decrease towards the ends of the linear multipole. The maximum value of J at the end is given by

$$J^* = mv \oint_{\text{on the separatrix}} ds \quad (7)$$

The orbits having $J > J^*$ are confined in the J -mirror. The orbits drift in the axial direction as it approaches towards the separatrix. After it crosses the separatrix, it reverses the direction of the axial drift and moves away from the separatrix. The orbits are furthest from the separatrix at the center of the J -mirror.

The orbits with $J < J^*$ escape from the J -mirror. However, they are confined in a region near the minimum of the magnetic field on a flux line. This is the important property making the collisions between particles outside of this region irrelevant to the confinement time.

J-Mirror, Magnetic Bottle Using the Longitudinal Invariant of Motion

LOSS DUE TO COLLISIONS

If we denote the volume of the region containing the orbits with $J < J^*$ V_1 and the total volume V_0 , the confinement time τ is given by

$$\tau = (V_0/V_1) \tau_c \quad (8)$$

where τ_c is the collision time as in the ordinary μ -mirror. The ratio V_0/V_1 may be $5 \sim 10$. The improvement of this order over the μ -mirror is significant, because the μ -mirror loss-rate is barely tolerable.

STABILITY

The plasma will be stable against the low-frequency mode like residual flute mode, because all the particles are drifting in good direction. The source of the instability is the reduced density along the line of force near the minimum of the magnetic field, or the "loss-disc". There may be a mode near the bouncing frequency of ions filling the loss-disc.

NON-ADIABACITY

The largest problem is probably the non-adiabacity of the motion near the separatrix. The maximum J region is near the separatrix and the orbits

have to cross the separatrix before it reverses the axial direction. The further calculation, digital if necessary, is needed to test this point.

COLLISIONAL CASE

If collisions are frequent, the plasma will flow out from the ends. The flow velocity is given by E/B , if it is due to EXB drift, or by T/BR_c , if due to VB drift. In either case, the velocity v_f is given by

$$v_f = \frac{1}{\alpha} \frac{T}{rB} \quad (9)$$

where α is a numerical constant and it is assumed $E \propto T/r$ and $R_c \propto r$. Then the confinement time τ_e limited by the end loss is given by

$$\tau_e = \frac{\alpha B_2 r_2 L}{T} \left(\frac{r_1^2}{r_2^2} \right) \quad (10)$$

where r_1 and r_2 are the radii of the device and B_2 and B_1 are the magnetic field strength at the center and at the ends, and L is the length of the device. The confinement time due to radial loss τ_r may be given by

$$\tau_r = \alpha' B_1 r_1^2 / T \quad (11)$$

where a Bohm type diffusion loss is assumed.

J-Mirror, Magnetic Bottle Using the Longitudinal Invariant of Motion

For these two to be equal, we have

$$\alpha \frac{B_2}{B_1} \left(\frac{L}{r_2} \right) \approx \alpha' \quad (12)$$

Since α' larger than 1500 is preferred, the condition for a tolerable end loss is

$$\alpha \left(\frac{L}{r_1} \right) \left(\frac{r_1}{r_2} \right)^2 > 1500 \quad (13)$$

If we assume $(r_1/r_2) \approx 5$ and $(L/r_1) \approx 12$, an α larger than 5 is needed for the end loss to be smaller than the radial loss.

SUMMARY

The possibility of plasma confinement using longitudinal invariant in linear devices was studied. It has a longer confinement time limited by collisions than a μ -mirror. A further study is needed to test other loss processes.

REFERENCE

1. M. N. Rosenbluth, Phys. Fluids 11, 869 (1968).