Ringless Quadrupole Configuration with Spiral Windings

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Toroidal configurations with a quadrupole-like separatrix are discussed. The configurations are produced by superposing a quadrupole field and a pair of radial transform fields on a toroidal field. They may have magnetic shears close to unity. The radial transform fields are produced by planar spiral coils which may be fabricated much more accurately than toroidal-helical coils and also leave the side of the torus accessible.

I. INTRODUCTION

Experiments with toroidal configurations with internal conductors have shown the improved confinement of plasmas and the effectiveness of magnetic well and magnetic shear in suppressing plasma instabilities. Since the disadvantage of having internal conductors for a fusion reactor is obvious, the search for configurations with the similar properties but without internal conductors has been pursued.

Configurations with a poloidal multipole-like separatrix have large magnetic shears when toroidal magnetic fields are superposed. Without the internal conductors, it is necessary to either have plasma currents or give up the axial symmetry. The first type has been discussed by the author.¹ Examples of the second type were discussed first by H. Furth.² In these examples, two sets of helical windings are used.

In this report, much simpler configurations with similar properties are discussed.

II. CONFIGURATION

The configuration may be produced by the following set of magnetic fields; a toroidal field, an axially symmetric quadrupole field, and a magnetic field produced by a pair of spiral windings. One of the spiral windings is placed on a plane parallel to the median plane and the other on a plane on the other side of the median plane with the same distance. The senses of the spiral are opposite to each other, i.e., one clockwise and the other counter-clockwise.

The spiral fields give radial transforms to the flux lines and are used to close the flux surface without inclusion of conductors. By and large, the separatrix is shaped by the quadrupole field except near the spiral windings. The overall configuration of the flux surfaces is similar to that of a quadrupole.

III. STRAIGHT CONFIGURATION

In this section, the flux surface configurations are calculated analytically for a straight system.

The magnetic potential given by

$$\chi = B_0 z + a x y + (b/\ell) e^{\ell' y} \sin(kz + \ell x) + (b/\ell) e^{-\ell' y} \sin(kz - \ell x) \quad (1)$$

describes the magnetic field configuration, where B_0 is the axial field, a is the strength of the quadrupole, the last two terms are transform fields, and $\ell'^2 = k^2 + \ell^2$.

The trajectory of a flux line is given by

$$\left\{ \begin{array}{l} \frac{dx}{dz} = \frac{\partial \chi / \partial x}{\partial \chi / \partial z} \\ \frac{dy}{dz} = \frac{\partial \chi / \partial y}{\partial \chi / \partial z} \end{array} \right. \quad (2)$$

The trajectories are the superposition of the slowly varying part and the periodic part with respect to kz . We put

$$\left\{ \begin{array}{l} x = \bar{x} + \tilde{x} \\ y = \bar{y} + \tilde{y} \end{array} \right. \quad (3)$$

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and also assume $k^2 \ll \ell^2$, $\ell' \sim \ell$ and $e^{\pm \ell y} \sim 0$ for $y \lesssim 0$. Then we have

$$\left\{ \begin{array}{l} \tilde{x} = \frac{(k-a/B_0)(b/B_0)}{k^2+(a/B_0)^2} \left[e^{\ell \tilde{y}} \sin(kz+\ell \tilde{x}) - e^{-\ell \tilde{y}} \sin(kz-\ell \tilde{x}) \right] \\ \tilde{y} = \frac{-(k+a/B_0)(b/B_0)}{k^2+(a/B_0)^2} \left[e^{\ell \tilde{y}} \cos(kz+\ell \tilde{x}) - e^{-\ell \tilde{y}} \cos(kz-\ell \tilde{x}) \right] \end{array} \right. \quad (4)$$

By using these solutions, the slowly moving parts are given by

$$\left\{ \begin{array}{l} \frac{d\tilde{x}}{dz} = \frac{\partial \tilde{\psi}}{\partial \tilde{y}} \\ \frac{d\tilde{y}}{dz} = -\frac{\partial \tilde{\psi}}{\partial \tilde{x}} \end{array} \right. \quad (5)$$

where

$$\tilde{\psi} = \frac{a}{2B_0} (\tilde{x}^2 - \tilde{y}^2) - \frac{2kb^2/B_0^2}{k^2+(a/B_0)^2} \operatorname{ch} \ell \tilde{y} \quad (6)$$

The surfaces with $\tilde{\psi} = \text{const}$ are the flux surfaces. In order that the magnetic axis $\tilde{x} = \tilde{y} = 0$ to be hyperbolic, the condition

$$\left| \frac{a}{2B_0} \right| > \frac{k\ell^2 b^2/B_0^2}{k^2+(a/B_0)^2} \quad (7)$$

has to be satisfied. The other magnetic axes are located at $\bar{x} = 0$ $\bar{y} = Y$, where Y satisfies

$$(a/B_0)Y + \frac{2k\ell b^2/B_0^2}{k^2 + (a/B_0)^2} \text{sh } \ell Y = 0 \quad (8)$$

which requires $ak/B_0 < 0$. For these axes to be elliptic, the condition

$$|a/2B_0| < \left| \frac{\ell^2 k b^2/B_0^2}{k^2 + (a/B_0)^2} \text{ch } \ell Y \right| \quad (9)$$

is necessary. The rotational transform angles ι around these elliptic axes are given by

$$\iota = L \left\{ (-a/B_0) \left(+ (a/B_0) + \frac{2k\ell^2 b^2/B_0^2}{k^2 + (a/B_0)^2} \text{ch } \ell Y \right) \right\}^{\frac{1}{2}} \quad (10)$$

when L is the length of the system.

If the conditions (6) and (9) are satisfied, the flux surfaces form a configuration like a quadrupole. The quadrupole separatrix is given by

$$\frac{a}{2B_0} (\bar{x}^2 - \bar{y}^2) - \frac{2kb^2/B_0^2}{k^2 + (a/B_0)^2} (\text{ch } \ell \bar{y} - 1) = 0 \quad (11)$$

There are the other types of separatrices which separate the flux lines encircling the transform windings from the ones encircling the quadrupole configuration. Their vertical positions are given approximately by

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$$\frac{2\ell k b^2/B_o^2}{k^2 + (a/B_o)^2} \operatorname{sh} \ell y + \frac{a}{B_o} y = \frac{k}{\ell} \quad (12)$$

The useful volume is limited to inside of these separatrices.

IV. TOROIDAL CONFIGURATION

In toroidal configurations, the magnetic potential is given in cylindrical coordinates by

$$\chi = B_0 r_0 \theta + \alpha z \ln (r/r_0) + \chi_+ + \chi_- \quad . \quad (13)$$

The first term represents the toroidal field, the second the quadrupole, and $\chi_{\pm}$ are the fields produced by the spiral windings. They are given by

$$\chi_{\pm} = b_{\pm} e^{\pm \ell z} \left\{ J_m(\ell r) \sin m\theta \pm N_m(\ell r) \cos m\theta \right\} \quad (14)$$

or equivalently,

$$\begin{aligned} &= b_{\pm} \frac{e^{\pm \ell z}}{\sqrt{(\pi/2) \ell r}} \left\{ P_m(\ell r) \sin(m\theta \pm \ell r - (m + \frac{1}{2}) \pi/2) \right. \\ &\quad \left. \pm Q_m(\ell r) \cos(m\theta \pm \ell r - (m + \frac{1}{2}) \pi/2) \right\} \end{aligned} \quad (15)$$

where J_m and N_m are the first and the second kind of Bessel functions,

P_m and Q_m are the first and the second kind of Legendre functions.

For large values of ℓr , $\chi_{\pm}$ are given approximately by

$$\chi_{\pm} \approx b_{\pm} \frac{e^{\pm \ell z}}{\sqrt{(\pi/2) \ell r}} \sin(m\theta \pm \ell r - (m + \frac{1}{2}) \pi/2) \quad . \quad (16)$$

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The trajectories of flux lines may be calculated approximately by using the same technique as in the previous section. The only difference is the slight distortion of the flux surfaces due to the toroidal effect.

To calculate the trajectory more accurately and test the effects of error fields, digital computations are necessary.

V. CHARACTERISTICS OF THE CONFIGURATION

A larger magnetic shear is possible in this configuration compared to "high shear stellerator." The transform angle decreases away from the elliptic magnetic axes toward the separatrix until it vanishes. It increases again outside the separatrix and reaches the maximum at the flux surface ψ_c . The magnetic shear conversely decreases from zero to $-\infty$ going from the magnetic axes to the separatrix and $+\infty$ to zero from the separatrix to the ψ_c surface. By choosing a small aspect ratio defined by the ratio of the toroidal circumference to the poloidal circumference, the average shear of approximately unity may be obtained.

The coil system is much simpler than that of a toroidal helical system. The quadrupole coils consist of four rings, two carrying positive and two negative current and one pair on the top and the other at the bottom of the torus. The transform coils are a pair of spiral coils one on top and the other at the bottom. They leave the side of the torus entirely accessible to injection of plasma, diagnostics, vacuum port, etc.

The accurate fabrication and the installation of these coils are straightforward, since all the coils are two-dimensional. Extensive digital computations are in order before an optimum design of the configuration with a combination of the largest magnetic shear and the least sensitivity to the error fields is obtained.

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REFERENCES

1. T. Ohkawa, Kakuyugo-Kenkyu 20, No. 6, 557 (1968).
2. H. P. Furth, J. L. Johnson, and G. V. Sheffield, Bull. Am. Phys. Soc. 13, 287 (1968), paper 3D-10.