High Density and High Magnetic Field Approach to Fusion

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The high magnetic field pinches with liner implosion are discussed. Hydraulic compression of liquid liner appears attractive for a fusion reactor.

(1) INTRODUCTION

There seems an interesting regime for fusion between two extremes, i.e., low-density magnetic confinement and high-density inertial confinement approaches. The purpose of this note is to map out the area of interest and analyze basic problems in the hope that a promising approach may be discovered. The regime under discussion is centered around $n = 10^{19} \text{ cm}^{-3}$ and $B = 10^6 \text{ G}$. All shapes of magnetic bottles used in low density regimes are available and also various methods of producing a high magnetic field are available. The question is, which combination gives the best result.

(2) LAWSON CRITERION

The Lawson criterion $n\tau \geq 10^{14} \text{ cm}^{-3} \text{ sec}$ must be satisfied. Since there is a significant difference between the straight devices and the toroidal devices, we treat them separately.

(a) Straight Device (θ -Pinch)

The end loss time is assumed to be the dominant loss. Then the confinement time τ is given by

$$\tau \approx L/v_s, \quad (2-1)$$

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where L is the length of the device and v_s is the sound wave velocity.

For 10 keV temperature, $v_s = 10^8$ cm/sec and we have

$$L \geq \frac{10^{22}}{n(\text{cm}^{-3})} \text{ cm} \quad (2-2)$$

If the plasma is heated by compression, we have the following relations,

$$\left. \begin{aligned} n &= \alpha n_o \\ T &= \alpha^{\gamma-1} T_o \\ v_s &= \alpha^{(\gamma-1)/2} v_{so} \end{aligned} \right\} \quad (2-3)$$

where subscripts o denote the initial state, α is the compression ratio, and γ is the ratio of specific heat. The compression time τ_c must be shorter than the initial end loss time L/v_{so} . Therefore the ratio τ/τ_c is limited to

$$\tau/\tau_c \approx \frac{v_{so}}{v_s} \approx \alpha^{-(\gamma-1)/2} \quad (2-4)$$

For collisional compression, $\gamma = 5/3$, we have

$$\tau/\tau_c \approx \alpha^{-1/3} \quad (2-5)$$

Even for $\alpha = 10^3$, the compression time must be shorter than 10τ . For example, $L = 10$ m and $\tau = 10 \mu\text{sec}$, the compression with $\alpha = 10^3$ must be done in a period shorter than $100 \mu\text{sec}$. For a slow compression of $\tau_c = 1 \text{ msec}$, the length of the device must be 100 m.

The collision mean-free path λ scales as

$$\lambda = \alpha^{1/3} \lambda_o \quad (2-6)$$

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If the plasma is collisional in the final state, it is collisional during the compression. For $n = 10^{19} \text{ cm}^{-3}$ and $T = 10 \text{ keV}$, we have $\lambda = 50 \text{ cm}$. Therefore the plasma in a 10 m device is collisional at all times.

The adiabatic compression scales β as

$$\beta = \beta_0 \alpha^{\gamma-2} = \beta_0 \alpha^{-1/3} \quad . \quad (2-7)$$

For collisional compression β is reduced. This formula does not hold, if the plasma has a sharp boundary. That will be the case when the compression time is much shorter than the skin time. τ_s , which scales

$$\tau_s = \mu_0 \sigma r^2 = \alpha^{(3/2)\gamma-5/2} = \alpha^0 \quad , \quad (2-8)$$

where r is the plasma radius and σ is the conductivity of plasma. For $\sigma = 10^9 (\text{ohm m})^{-1}$ and $r = 1 \text{ cm}$, $\tau_s = 10^{-1} \text{ sec}$ which is much longer than the compression time. Therefore, β is likely to be near unity.

The minimum plasma radius is determined by heat conduction and skin effect considerations. The classical energy confinement time τ_E is given by

$$\tau_E = \frac{r^2}{D} \sqrt{\frac{m_e}{m_i}} \quad (2-9)$$

where D is the classical diffusion constant and is given by $D = \eta p / B^2$.

By using $D = \mu_0^{-1} \sigma^{-1} \beta$, we have

$$\tau_E = \mu_0 \sigma r^2 \beta^{-1} \sqrt{m_e / m_i} \quad .$$

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For $\beta = 1$ and $\sigma = 10^9 \text{ ohm}^{-1} \text{ m}^{-1}$, we have

$$\tau_E \approx 15 r^2 (\text{m})$$

and a radius of 1 mm is enough to give $\tau_E \sim 10^{-5}$ sec. It does not make sense to choose the plasma radius much smaller than the skin depth of the conducting fluid because the stored magnetic energy in the fluid becomes too large a fraction of the total energy.

$$r \geq \sqrt{\tau / \mu_0 \sigma_f} \quad (2-11)$$

where σ_f is the electric conductivity of the fluid. For $\sigma_f = 10^6 \text{ ohm}^{-1} \text{ m}^{-1}$ and $\tau = 10^{-4}$ sec, $r \sim 1$ cm.

For a 10 m device, a likely set of numbers is: $n = 10^{19} \text{ cm}^{-3}$, $T = 10 \text{ keV}$, $B = 2 \text{ MG}$, $\tau = 10^{-5}$ sec, and $\tau_c = 10^{-4}$ sec.

As long as the plasma column is straight, θ -pinch plasmas are known to be stable. However, the column may bend slightly due to the imperfections in the system and then become susceptible to an instability. The column is unstable against a ballooning mode if $\beta < r R n^2 / L^2$, where R is the radius of the curvature of the bend. By introducing Δ , the displacement of the column, the stability condition becomes $\Delta \leq \beta^{-1} r$. If the column stays straight within its radius, it is stable against the ballooning mode.

(b) Cusp

Past experiments have shown that the effective width of the line cusps is roughly $2 \rho_i$ (ion gyroradius in the external field). The confinement time τ is given by

$$\tau = \frac{V}{4\pi r_c \rho_i v_s} \quad , \quad (2-12)$$

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where r_c is the radius of the line cusp and V is the volume of plasma.

If we define the form factor f by $V = fr_c^3$, the confinement time is given by

$$\tau = \frac{fr_c^2}{4\pi} \frac{eB}{T} \quad (2-13)$$

The scaling is Bohm-like. For $B = 10^6$ G, $T = 10^4$ eV, $n = 10^{19}$ cm⁻³, and $f = 10^{-1}$, we obtain

$$r_c = 30 \text{ cm} \quad .$$

This value is reasonably small.

In the above calculation, the form factor is assumed to be independent of ρ_i . This is probably correct for sharp boundary cases. If the sheath thickness become large and comparable with the plasma size, the confinement time becomes much shorter than the value given in the above formula.

The compression of a cusp differs from the θ -pinch in scaling laws. They are

$$\left. \begin{aligned} n &= \alpha n_0 \\ T &= \alpha^{\gamma-1} T_0 \\ B &= \alpha^{2/3} B_0 \\ \tau &= \alpha^{-\gamma+1} \tau_0 \\ \beta &= \alpha^{\gamma-4/3} \beta_0 \end{aligned} \right\} \quad (2-14)$$

The initial magnetic field has to be larger than the θ -pinch case but the compression time can be a little longer. The advantage of the cusp is of

course the smaller size. On the other hand the cusp compression is less understood than the θ -pinch.

(c) Toroidal Geometry

A toroidal bottle does not have a leak along flux lines, but the plasma has to be in a toroidal equilibrium, which requires some form of poloidal magnetic field. The confinement time is determined by the instabilities or by the transport processes. Since Tokamaks have too small a β to be interesting, we consider a toroidal θ -pinch with a poloidal magnetic field. A high- β stellarator is probably too complicated a configuration for the liner implosion. A toroidal θ -pinch with toroidal plasma current is the simplest configuration, provided that the configuration is made mhd stable. For a torus of aspect ratio R/r , mhd equilibrium requires $\beta_p \leq R/r$. It means that the poloidal magnetic field strength B_p must be of the order of $B_p \approx \sqrt{r/R} B_t$ where B_t is the toroidal field strength. A high- β equilibrium requires a considerable poloidal field strength.

For a high density $n = 10^{19} \text{ cm}^{-3}$, the required confinement time is only $10 \text{ } \mu\text{sec}$ and it would not be hard to achieve, if the configuration is mhd stable. Therefore, the key to the toroidal compression is mhd equilibrium and stability.

(3) DYNAMICS OF LINER IMPLOSION

In this section we consider the dynamics of the liner implosion.⁽¹⁾

The simplest model is the implosion of a cylinder of an incompressible material in the vacuum. The cylinder has inner radius of r_1 , outer radius

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of r_2 and density ρ . At $t = 0$, a pressure p was applied around the cylinder. The following equations describe the motion:

$$\left\{ \begin{array}{l} \frac{d}{dt} \left[\left(\frac{dr_2^2}{dt} \right)^2 \ln \left(\frac{r_2}{r_1} \right) \right] = - \frac{4}{\rho} \frac{dr_2^2}{dt} p \\ \frac{d(r_2^2)}{dt} = \frac{d(r_1^2)}{dt} \end{array} \right. \quad (3-1)$$

$$(3-2)$$

To obtain an approximate solution, we neglect the time derivative of the logarithmic term. We obtain

$$\frac{d^2}{dt^2} (r_2^2) = - \frac{2p}{\rho} \left(\ln \frac{r_2}{r_1} \right)^{-1}, \quad (3-3)$$

and

$$r_2^2 = r_2^2(t=0) - \frac{2}{\rho \left(\ln \frac{r_2}{r_1} \right)} \iint_0^t p \, dt \, dt \quad (3-4)$$

$$r_1^2 = r_1^2(t=0) - \frac{2}{\rho \ln \left(\frac{r_2}{r_1} \right)} \iint_0^t p \, dt \, dt \quad (3-5)$$

Two simpler cases are: (1) the pressure is applied as an impulse and, (2) the pressure is constant during the compression. For the first case, we have

$$r_1^2 = r_1^2(t=0) - \frac{2t}{\rho \ln r_2/r_1} \int p \, dt \quad (3-6)$$

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The second case is described by

$$r_1^2 = r_1^2(t=0) - \frac{t^2 p}{\rho \ln r_2/r_1} \quad (3-7)$$

To minimize the pressure for a maximum compression, the pressure should last for most of the compression period. The compression time τ_c is given by

$$\tau_c = \left\{ \frac{[r_1^2(t=0) - r_1^2] \rho \ln r_2/r_1}{p} \right\}^{\frac{1}{2}} \quad (3-8)$$

For $p = 10^4$ atm, $\rho = 1\text{g/cm}^3$, and $r_1 = 10$ cm, the compression time is approximately 200 μsec .

With this high pressure, the assumption of incompressibility does not apply unless the transit time of the sound wave is much shorter than the compression time. The sound wave velocity is of the order of 10^5 cm/sec and the transit time is shorter but comparable. The inclusion of compressibility does not change the numbers very much.

The deceleration starts when the magnetic field inside the cylinder is compressed and its pressure becomes significant. From the flux conservation, we have

$$\frac{d}{dt} \left[\left(\frac{dr_1^2}{dt} \right)^2 \ln \left(\frac{r_2}{r_1} \right) \right] = \frac{4}{\rho} \frac{B_m^2 r_m^4}{2\mu_0} \frac{1}{r_1^4} \frac{dr_1^2}{dt} \quad , \quad (3-9)$$

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where the subscript m denotes the values is where the compression ceases.

By integrating over time, we obtain

$$\frac{dr_1^2}{dt} = \left(\frac{2B_m^2 r_m^2}{\mu_o \rho \ln \frac{r_2}{r_1}} \right)^{\frac{1}{2}} \left[1 - \frac{r_m^2}{r_1^2} \right]^{\frac{1}{2}} \quad (3-10)$$

and

$$\varphi + \frac{1}{2} \operatorname{sh} 2\varphi = \left(\frac{2B_m^2}{\mu_o \rho r_m \ln \frac{r_2}{r_1}} \right)^{\frac{1}{2}} (t_m - t) \quad (3-11)$$

where

$$r_1 = r_m \operatorname{ch} \varphi \quad . \quad (3-12)$$

(4) STABILITY OF IMPLODING LINER

When the imploding cylindrical liner is being decelerated, it becomes unstable against the Raleigh-Taylor instability. The growth rate G is given by⁽²⁾

$$G = \sqrt{kg} \quad (4-1)$$

where g is the deceleration and k is the wave number and is given by m/r . The deceleration is given by Eq. (3-10)

$$g = \frac{1}{r_m} \left(\frac{B_m^2}{\mu_o \rho \ln \frac{r_2}{r_1}} \right) \frac{r_m}{r_1} \left[\frac{2r_m^4}{r_1^4} - \frac{r_m^2}{r_1^2} \right] \approx \frac{B_m^2}{r_m \mu_o \rho \ln \frac{r_2}{r_1}} \quad . \quad (4-2)$$

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For $B_m = 10^6$ gauss, $r_m = 1$ cm and $\rho = 1$ g/cm³, we have

$$G = \sqrt{m} 8 \times 10^4 \text{ sec}^{-1} .$$

The growth time is comparable to the desired confinement time and dynamic dwelling time except for large m modes.

The instability may be stabilized by rotating the liner. If the liner is given angular velocity Ω_0 initially, the compression will increase the angular velocity. The radial distribution of the angular velocity depends on viscosity. The time constant for the viscose damping τ_v is long for the common fluid such as water or mercury. It is given by

$$\tau_v \approx \frac{\rho r^2}{\eta} \quad (4-3)$$

where η is the viscosity and 1.5 and 1.8 centipoise for mercury and water. For $r = 1$ cm the time constant is 800 sec and 60 sec, respectively. Therefore, the viscosity is negligible and the angular momentum of the fluid element is preserved. The angular velocity Ω is given by

$$\Omega = \Omega_0 r^2 (t = 0) / r^2 . \quad (4-4)$$

The centrifugal force at the interface is given by $\rho r_1^4 (t = 0) \Omega_0^2 r_1^{-3}$.

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With the centrifugal force, the growth rate becomes

$$G = \left[kg - 3k r_1^4 (t=0) r_1^{-3} \Omega_o^2 \right]^{\frac{1}{2}} \quad (4-5)$$

The stability condition is given by

$$\Omega_o > \frac{r_1^2}{r_1^2(t=0)} \sqrt{g/(3r_1)} \quad . \quad (4-6)$$

With $g = 10^7 \text{ msec}^{-2}$, $r_1(t=0)/r_1 = 30$, and $r_1 = 1 \text{ cm}$ we have

$$\Omega_o > 20 \text{ sec}^{-1} \approx 1200 \text{ r.p.m.}$$

This value is easy to obtain. If the angular velocity is too large, the compression will stop before the inside magnetic pressure builds up to the desired value. This sets an upper limit on Ω_o . The condition is given by

$$\frac{r_1^2}{r_1^2(t=0)} \sqrt{g/r_1} > \Omega_o > \frac{r_1^2}{r_1^2(t=0)} \sqrt{g/3r_1} \quad (4-7)$$

For the above example, we have

$$1500 \text{ r.p.m.} > \Omega_o > 1200 \text{ r.p.m.}$$

(5) LINER FOR CUSP

The nonuniform magnetic field in this case makes the instability worse. The growth rate is given by

$$G = \sqrt{k \left(g + \frac{B}{\rho \mu_0} \frac{\partial B}{\partial r} \right)} \approx \sqrt{kg(1 + \ln r_2/r_1)} \quad . \quad (5-1)$$

The stable window for Ω_0 is almost closed. However, it may not be necessary to stabilize the mode completely but to reduce the growth rate until it becomes comparable to the confinement time. Also, the line tying for this short device may help reduce the growth rate.

(6) TOROIDAL LINER

Because of the toroidal curvature, there is a negative hoop force in the toroidal vacuum region. That is, the major radius will decrease, when the minor radius is compressed. A poloidal magnetic field is necessary to maintain equilibrium for both the plasma and the liner. The necessary strength of the poloidal field is $\sqrt{r/R}$ of the toroidal field.

There is a possibility of applying a vertical field initially and let both the major radius and the minor radius shrink. The toroidal vacuum region shrinks in the major radius because of the negative hoop force and the plasma major radius shrinks because of the increasing vertical field.

It is difficult to stabilize the liner instability by rotation for the toroidal case. A poloidal field comparable to the toroidal field will stabilize it.

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The poloidal field can be produced either by an internal ring or by plasma current. There are two ways of producing toroidal current:

(A) gap, and (B) helical cut. In the first method the liner has an overlapping gap and the current is induced in a standard manner. As soon as the compression starts, the gap shorts and acts as a closed torus. In the second method, helical grooves are cut in the toroidal liner. The compression will induce a toroidal electric field; therefore, a toroidal current. Early during the compression, the grooves are smoothed out and the torus becomes axially symmetric.

It is hard to stabilize the plasma if at all possible, without the internal ring. The stabilization of the liner requires a poloidal field of about the same strength as the toroidal field. This makes the kink mode hard to stabilize.

With an internal ring, the plasma can be made sufficiently stable. The compression is done by the toroidal magnetic field and the force due to the poloidal field is directed outward making the configuration mhd stable. The use of a disposal ring in addition to the disposable liner is not much of a disadvantage. The ring however, is susceptible to the instability, if the magnetic pressure exceeds the stress limit of the ring.

The toroidal configurations are much more complicated than the linear configurations, notwithstanding their potential advantages. They require much more detailed studies before an adequate assessment can be made.

(7) MAGNETIC DRIVE

A simple way of compressing the liner is to use magnetic pressure.⁽¹⁾ A coil is wound around the liner. The minimum energy W_m required for a

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cylindrical liner is given by

$$W_m = 2 \frac{H_m^2}{2\mu_0} \pi r_m^2 \quad . \quad (7-1)$$

For $B_m = 2 \times 10^6$ gauss, $r_m = 1$ cm, and $L = 10$ m, we have

$$W_m = 120 \text{ MJ} \quad .$$

The magnetic field B_c to be produced by the coil is then given by

$$B_c = (\mu_0 W_m / L \pi r_c^2)^{\frac{1}{2}} \quad . \quad (7-2)$$

where r_c is the radius of the coil and 100% efficiency is assumed.

For $W_m = 10^8$ J, $L = 10$ m, $r_c = 0.3$ m, $B_c = 60$ kG. With a realistic value for efficiency, the field is probably in the range of 150 ~ 200 kG and the total energy involved is of the order of 10^9 J.

(8) HYDRAULIC DRIVE

Since the compression is done mechanically by a liner, it may be less expensive to use a mechanical drive rather than a magnetic drive. A hydraulic drive seems most attractive among the various mechanical approaches. It has an added advantage of eliminating the liner by using conducting liquid as a hydraulic fluid. The key question for this approach is whether the compression could be done fast enough.

Consider a cylindrical vacuum space of radius r_1 surrounded by a fluid to radius r_2 . The vacuum space is maintained either by having a liner or by rotating a fluid. Axially symmetric plungers give an impulse at radius r_2 . For the case where the impulse is slowly applied, that is,

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the wavelength of the acoustic wave corresponding to the period of the impulse is larger than r_2 , the calculations in Sec. 3 are applicable.

For an impulse of shorter periods, we have to take compressibility, thus acoustic wave, into account. The equation motions are given by

$$\rho \left(\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} \right) = - \frac{\partial p}{\partial r} \quad (8-1)$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial(\rho v r)}{\partial r} = 0 \quad (8-2)$$

We introduce the density potential ψ defined by

$$\left\{ \begin{array}{l} r\rho = \frac{\partial \psi}{\partial r} \end{array} \right. \quad (8-3)$$

$$\left\{ \begin{array}{l} r\rho v = - \frac{\partial \psi}{\partial t} \end{array} \right. \quad (8-4)$$

By going from Eulerian coordinates (r, t) to Lagrangian or material coordinates (ψ, t) , we obtain

$$\left\{ \begin{array}{l} v = \frac{\partial r}{\partial t} \end{array} \right. \quad (8-5)$$

$$\left\{ \begin{array}{l} \frac{\partial^2 r}{\partial t^2} = - r \frac{\partial p}{\partial \psi} \end{array} \right. \quad (8-6)$$

By using $dp/d\rho = v_s^2$ where v_s is the sound velocity, we obtain

$$\frac{\partial^2 r}{\partial t^2} = + v_s^2 \left(\frac{\partial r}{\partial \psi} \right)^{-2} \left(\frac{1}{r} \right) \frac{\partial^2 (r^2/2)}{\partial \psi^2} \quad (8-7)$$

$$= + v_s^2 \rho^2 r \frac{\partial^2 (r^2/2)}{\partial \psi^2} ,$$

or

$$\frac{\partial^2}{\partial t^2} \left(\frac{r^2}{2} \right) - \frac{1}{r^2} \left[\frac{\partial}{\partial t} \left(\frac{r^2}{2} \right) \right]^2 = v_s^2 \frac{\partial^2}{\partial \psi^2} \left(\frac{r^2}{2} \right) \quad (8-8)$$

For a small amplitude wave, we put

$$\psi \sim \frac{r_0^2}{2} \rho_0 \quad (8-9)$$

$$r^2 = r_0^2 + 2\xi \quad (8-10)$$

where ρ_0 is the undisturbed density and r_0 is the initial position of fluid elements. Then Eq. (8-8)

$$\frac{\partial^2 \xi}{\partial t^2} \approx v_s^2 \frac{\partial^2 \xi}{\partial r_0^2} \quad (8-11)$$

This is a standard acoustic wave equation. The solution is given by

$$\xi = \xi(v_s t - k r_0) \quad , \quad (8-12)$$

where k is the wave number.

For a large amplitude wave, the equation has to be solved numerically. The main effect of the finite amplitude is to steepen the wave front provided that the dissipation is negligible at higher harmonic frequencies. At the frequency range of interest, 10^4 sec^{-1} , the dissipation is negligible.

The acoustic velocities for typical fluids, water and mercury, are $1.5 \times 10^5 \text{ cm/sec}$. The wavelength corresponding to 10^4 sec^{-1} frequency is 90 cm, which is larger but comparable to a typical value for the radius.

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The impulse may be produced by the combination of a hammer and a piston. For a cylindrical geometry the piston must be shaped in such a way to produce an approximate axially symmetric impulse. The deceleration time of the hammer must be matched with the frequency of the wave generated. For a hammer with a mass M per unit area of fluid and initial velocity u , we have

$$\omega M u \approx p$$

$$\approx \rho_1 v_s^2 = \rho_0 v_s u \quad ,$$

where p is the pressure generated and ρ_1 is the density increment. By equating the deceleration time and the acoustic frequency, we obtain

$$M = \rho_0 / k \quad .$$

Since $kr_2 \gtrsim 1$, the mass of the hammer is comparable to the fluid mass. The velocity of the hammer is given by

$$u = p / (\rho_0 v_s) \quad .$$

For $p = 10^{10}$ dyne/cm² (corresponding to 500 kG magnetic pressure), $\rho_0 = 10$ gm⁻³ and $v_s = 1.5 \times 10^5$ cm/sec, we have $u = 6 \times 10^3$ cm/sec which is not a very high velocity.

(9) CURRENT DENSITY LIMITATION

The maximum magnetic field obtainable by liner or fluid compression may be limited by the maximum current density in the conductor. A rough estimate may be obtained by calculating the time τ_b it takes to change the phase of the conductor.

$$\tau_b \approx \frac{C(T_b - T_o)}{\eta j^2} \quad , \quad (9-1)$$

where η is the average resistivity, j is the current density, C is the heat capacity per volume, and T_b and T_o are the boiling or melting points and the initial temperature. For the copper liners, we have $C \approx 10^6$ cal/m³, $T_b \sim 1000^\circ$ C, $\eta \sim > \times 10^8$ ohm. With $\tau_b = 10^{-4}$ sec, the current density is given by

$$j \approx \left[\frac{C(T_b - T_o)}{\eta \tau_b} \right]^{\frac{1}{2}} \quad (9-2)$$

$$\approx \left(\frac{4 \times 10^6 \times 700}{7 \times 10^{-8} \times 10^{-4}} \right)^{\frac{1}{2}} = 2 \times 10^{10} \text{ amp/m}^2 \quad .$$

The maximum field and current density are related through

$$B_m = \mu_o j \delta \quad , \quad (9-3)$$

where δ is the skin depth and is given by $\delta = (\mu_o / \tau_b \eta)^{-\frac{1}{2}}$. Then Eq. (9-3) becomes

$$B_m = \mu_o \left[\frac{C(T_b - T_o)}{\eta \tau_b} \right]^{\frac{1}{2}} \left[\frac{\tau_b \eta}{\mu_o} \right]^{\frac{1}{2}} \quad (9-4)$$

$$= \left[C(T_b - T_o) \mu_o \right]^{\frac{1}{2}}$$

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For copper we have

$$B_m \approx 500 \text{ kG} \quad .$$

This value is pessimistic because copper is still a pretty good conductor after it is melted. Also, the high pressure may help in increasing the melting points.

For a fluid conductor, the evaporation rate limits the maximum magnetic field strength. The evaporation at the vacuum and fluid interface will move back the current carrying layer. If the displacement is small compared to the skin depth, then the evaporation is tolerable. The limit on the field is then given by

$$(B_m^2/\mu_0^2) < \frac{H\delta^2}{\eta\tau}$$

where H is the heat of evaporation per unit volume, δ is the skin depth, and τ is the duration. By using the formula for the skin depth, we obtain

$$B_m^2/\mu_0 < H \quad .$$

For a typical value of $H = 4 \times 10^4 \text{ J/cm}^3$, $B_m \sim 2 \times 10^6 \text{ G}$.

(10) FUSION ENGINEERING CONSIDERATION

The hydraulic drive without a liner has a great advantage as a reactor. If molten lithium is used as a working fluid, it can also double as a blanket. Also, the first wall problem will be absent. The absence of the solid liner also helps to increase the repetition rate. The rotation of fluid can be achieved by driving the fluid inductively similar to an induction motor. The vapor pressure of molten lithium is tolerable (1 mm Hg at 700°C) because of its low Z . Also, the centrifugal force due to the rotation tends to keep the vapor near the interface.

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