PLASMA POTENTIAL OF OHMICALLY HEATED TOKAMAK PLASMAS

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ABSTRACT

The presence of neutral atoms in the tokamak plasma results in the negative plasma potential. The experimental results in ohmically heated plasmas can be explained by this model.

Plasma potential of tokamak plasmas has been measured by using heavy ion beam techniques. The result from ISX-B⁽¹⁾ shows that the plasma is negatively charged and the potential depth is approximately equal to the plasma temperature. The ions are confined electrostatically and the electrons carry the diamagnetic current. In this note, it will be shown that the presence of neutral atoms in the plasma causes the plasma to charge up negatively and the potential depth is about the ion temperature.

We consider the equilibria for the ion and the electron fluids. The force balance equations are given by

$$-\frac{\partial p_i}{\partial r} + enE_r + env_\theta B_\phi - env_\phi B_\theta = 0 \quad (1)$$

$$-\frac{\partial p_i}{r\partial\theta} + enE_\theta - env_r B_\phi - nv_{i\theta} m_i v_\theta = 0 \quad (2)$$

$$env_r B_\theta - nv_{i\theta} m_i v_\phi = 0 \quad (3)$$

$$-\frac{\partial p_e}{\partial r} - enE_r - enu_\theta B_\phi + enu_\phi B_\theta = 0 \quad (4)$$

$$-\frac{\partial p_e}{r\partial\theta} - enE_\theta + enu_r B_\phi - nv_{e\theta} m_e u_\theta = 0 \quad (5)$$

$$-enu_r B_\theta - nv_{e\theta} m_e u_\phi = 0 \quad (6)$$

where p_i and p_e are the ion and the electron pressures, v and u are the ion and the electron velocities, ν_{io} and ν_{eo} are the ion and electron collision frequencies with neutral atoms, n is the plasma density, m_i and m_e are the ion and the electron masses, θ and ϕ are the poloidal and the toroidal angles, and the circular cross section is assumed. The collisions between the ion and the electrons are neglected because they do not lead to charge separation.

The diamagnetic currents are divergence free and we have

$$\frac{\partial(nv_\theta R)}{\partial\theta} = \frac{\partial(nu_\theta R)}{\partial\theta} = 0 \quad (7)$$

where R is the major radius.

By assuming that the inverse aspect ratio ϵ is much smaller than unity and also that the plasma is not rotating in the ϕ -direction as a whole in 0th-order of ϵ , we obtain

$$v_\phi = \frac{2\epsilon\cos\theta}{enB_\theta} \left(-\frac{\partial p_i}{\partial r} + enE_r \right) \quad (8)$$

$$u_\phi = \frac{-2\epsilon\cos\theta}{enB_\theta} \left(-\frac{\partial p_e}{\partial r} - enE_r \right) . \quad (9)$$

The radial velocities are calculated from Eqs. (3) and (6) and they are

$$v_r = \frac{2\epsilon\nu_{io}m_i\cos\theta}{e^2nB_\theta^2} \left(-\frac{\partial p_i}{\partial r} + enE_r \right) \quad (10)$$

$$u_r = \frac{2\epsilon\nu_{eo}m_e\cos\theta}{e^2nB_\theta^2} \left(-\frac{\partial p_e}{\partial r} - enE_r \right) . \quad (11)$$

The velocities averaged over a flux surface $\langle v_r \rangle$ and $\langle u_r \rangle$ are given by

$$\langle v_r \rangle = \frac{\nu_{io}m_i\epsilon^2}{e^2nB_\theta^2} \left(-\frac{\partial p_i}{\partial r} + enE_r \right) \quad (12)$$

$$\langle u_r \rangle = \frac{\nu_{eo}m_e\epsilon^2}{e^2nB_\theta^2} \left(-\frac{\partial p_e}{\partial r} - enE_r \right) . \quad (13)$$

Quasi-neutrality requires that these velocities be equal and we obtain

$$E_r = \frac{\nu_{io} m_i \frac{\partial p_i}{\partial r} - \nu_{eo} m_e \frac{\partial p_e}{\partial r}}{en(\nu_{io} m_i + \nu_{eo} m_e)} \quad (14)$$

It is reasonable to assume that $\nu_{io} m_i \gg \nu_{eo} m_e$, and the electrostatic potential Φ is given by

$$-\frac{\partial \Phi}{\partial r} \approx \frac{1}{en} \frac{\partial p_i}{\partial r} \quad (15)$$

The above equation shows that the potential is negative and the depth is approximately equal to the ion temperature.

Since the neutral atom density in the interior of the plasma is small, the potential may take a long time to develop. We estimate the time constant.

The Poisson equation combined with the radial velocities yields

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} \frac{\partial \Phi}{\partial t} = \frac{e}{\epsilon_o} \frac{\partial n}{\partial r} (<v_r> - <u_r>) \quad (16)$$

By replacing $\partial/\partial r$ with r^{-1} , the initial rate of increase for the potential becomes

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{e\Phi}{T_i} \right) &\approx - \frac{\nu_{io} m_i n \epsilon^2}{\epsilon_o B_\theta^2} \\ &= - \frac{\nu_{io} \mu_o m_i n c^2 q^2}{B_\phi^2} \end{aligned} \quad (17)$$

Where q is the safety factor. The rate is given by the collision frequency multiplied by the plasma dielectric constant and the square of the safety factor. The collision frequency is given by

$$\nu_{io} = \nu_i \sigma_{io} n_o \quad (18)$$

where ν_i is the ion thermal velocity, σ_{io} is the collision cross section and n_o is the density of the neutral atom.

By using $\sigma_{io} = 10^{-20} \text{ m}^2$, $\nu_i = 10^4 (T_i/e)^{1/2} \text{ m} \cdot \text{sec}^{-1}$ and $m_i = 1.7 \times 10^{-27} \text{ kg}$, we obtain

$$\frac{\partial}{\partial t} \left(\frac{e\Phi}{T_i} \right) = -1.8 \times 10^{-32} \frac{n_o n q^2}{B_\phi^2} \left(\frac{T_i}{e} \right)^{1/2} \quad (19)$$

A typical set of parameters ($n = 4 \times 10^{19} \text{ m}^{-3}$, $q = 3$, $T_i = 400 \text{ eV}$ and $B_\phi = 1T$) yields

$$\frac{\partial}{\partial t} \left(\frac{e\Phi}{T_i} \right) = -1.3 \times 10^{-10} n_o \quad (20)$$

A neutral atom density as low as 10^{15} m^{-3} still gives a time constant of $10 \mu\text{sec}$.

It has been shown that the presence of the neutral atoms can explain the observed plasma potential. Although there may be nonclassical processes leading to nonambipolar transport, the above calculation shows that the classical effects are sufficient to produce the negative plasma potential.

REFERENCE

- (1) M. Murakami, IAEA Paper No. CN-44/A-II-a (London, 1984).