ECH Current Drive by Asymmetric Heating  
Around the Median Plane  

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Abstract  

ECH current drive using asymmetric heating around the median plane of tokamak plasma is discussed. Efficiency comparable to Fisch's method is obtained. Improvement of efficiency is possible by the addition of electrostatic barrier potential.  

In toroidal confinement configurations, the magnetic field is necessarily nonuniform. In the application of ECH heating not only the perpendicular energy of electrons is increased but also the motion of the electrons in the direction parallel to the magnetic field is affected. In a previous report, (1) the trapping of the electrons due to the increase of the magnetic moment has been discussed. The preferential trapping leads to current drive which is in the opposite direction to the current drive due to the Fisch (2) mechanism.  

If there are density and temperature gradients in the direction parallel to the magnetic field, the current can be driven provided that they are appropriately distributed as discussed in another report. (3)  

We consider the ECH current drive method taking advantage of the effects in this note. The motion of electrons in the direction parallel to the magnetic field is given by  

\[ m \frac{\partial u}{\partial t} + n u \frac{\partial u}{\partial s} + \frac{l}{n} \frac{\partial p}{\partial s} + \frac{l}{nB} (p - p) \frac{\partial B}{\partial s} - e(E - \eta j) = 0 \]  

(1)  

where \( m \) is the electron mass, \( u \) is the parallel velocity, \( s \) is the distance along the flux lines, \( p \) and \( p \) are the perpendicular and parallel pressures, \( n \) is the density, \( B \) is the magnetic field strength, \( E \) is the electric field, \( \eta \) is the electrical resistivity and \( j \) is the current density.  

In a steady state current drive, the current density is given by  

\[ j = - \frac{l}{2\pi RN_\eta} \int_1^N \left[ \frac{\partial p}{\partial s} + \frac{l}{nB} (p - p) \frac{\partial B}{\partial s} \right] ds \]  

(2)  

where the flux line closes after \( N \) turns around the torus and \( R \) is the major radius. By using the temperatures \( T \) and \( T \) instead of the pressures, we obtain  

\[ j = - \frac{l}{2\pi RN_\eta} \int_1^N \left[ T \frac{\partial n}{\partial s} + (T - T) \frac{\partial B}{\partial s} \right] ds \]  

(3)  

The first term is the thermoelectric term. If there is a phase shift between the density and the temperature distributions as a function of \( s \), the term is finite. The second term is nonvanishing only if the anisotropy of the temperature is not a function of the magnetic
field strength only. The constancy of the magnetic moment tends to make the second term vanish because the distribution function is a function of the magnetic moment and the kinetic energy in the absence of forces other than the magnetic force.

There are two ways to obtain a finite contribution from the second term. The first method is to consider the fact that the bounce time of the electrons between the regions of \( \partial B / \partial s \geq 0 \) is finite. Hence some part of the anisotropy produced in one region will remain before it is bounce averaged. The second method is to separate two regions with an electrostatic potential barrier. Some electron orbits will be trapped partially electrostatically and the anisotropy of one region is retained independent of the other region. Or equivalently, the electrostatic potential barrier acts as a thermal barrier and slows down the heat transport and therefore the transport of the anisotropy between two regions.

As an example, we consider ECH heating localized above or below the median plane in a tokamak. We assume that the ECH heating will increase the perpendicular temperature of electrons locally. Simple zero-dimensional equations for the temperatures are given by

\[
\begin{align*}
\frac{dT_\perp}{dt} &= \frac{P}{nV} - \nu(T_\perp - T_\parallel) - \left( \frac{\omega_b}{\pi} \right)(T_\perp - T_\perp') \\
\frac{dT_\parallel}{dt} &= -\nu(T_\parallel - T_\perp) - \left( \frac{\omega_b}{\pi} \right)(T_\parallel - T_\parallel') \\
\frac{dT_\perp'}{dt} &= -\nu(T_\perp' - T_\parallel') - \left( \frac{\omega_b}{\pi} \right)(T_\perp' - T_\perp) \\
\frac{dT_\parallel'}{dt} &= -\nu(T_\parallel' - T_\perp) - \left( \frac{\omega_b}{\pi} \right)(T_\parallel' - T_\parallel)
\end{align*}
\]

(4)

where \( P \) is the heating power, \( V \) is the volume, \( \nu \) is the collision frequency, \( \omega_b \) is the bounce frequency and the primes denote the quantities of the region not heated. In a steady state, Eqs. (4) become

\[
\begin{align*}
T_\perp - T_\parallel &= \frac{P}{Vn} \frac{2\nu + \omega_b / \pi}{4\nu(\nu + \omega_b / \pi)} \\
T_\perp' - T_\parallel' &= \frac{P}{Vn} \frac{\omega_b / \pi}{4\nu(\nu + \omega_b / \pi)}
\end{align*}
\]

(5)

(6)

and

\[
(T_\perp - T_\parallel) - (T_\perp' - T_\parallel') = \frac{P}{Vn \cdot 2(\nu + \omega_b / \pi)}. \tag{7}
\]

By using Eq. (7) in Eq. (3), we obtain

\[
j = \frac{P_\varepsilon}{2e\eta Vn(\nu + \omega_b / \pi)Rq}
\]

(8)

where \( q \) is the safety factor and \( \varepsilon \) is the inverse aspect ratio. For cases of interest, the electrons are in the banana regime and \( \omega_b \gg \nu \). The current density becomes

\[
j \approx \frac{\pi P_\varepsilon}{2e\eta Vn\omega_b Rq}. \tag{9}
\]
The ECH heating acts both on the trapped electrons and the circulating electrons. The trapped electrons that have turning points in the resonance zone are heated most. We divide the power $P$ into the fractions absorbed by the trapped and circulating electrons $P_T$ and $P_C$. The bounce frequencies for the trapped electrons and the circulating electrons are $\sqrt{\varepsilon} v_e/R_q$ and $v_e/R_q$ where $v_e$ is the electron thermal velocity. We obtain

\begin{align}
  j &\approx \frac{\pi}{2e\eta Vnv_e} \left[ P_T + \varepsilon P_C \left( 1 - \sqrt{\varepsilon} \right)^{-1} \right] \tag{10} \\
  &= \frac{\pi P_e}{2e\eta Vnv_e} \tag{11}
\end{align}

and $I \geq \varepsilon \geq \varepsilon$

The value of $\varepsilon'$ depends on the detail of the ECH heating. The total current $I$ becomes

\begin{equation}
  I \approx \frac{\pi P e'}{2e\eta Vnv_e} \tag{12}
\end{equation}

where $S$ is the cross sectional area of the plasma.

The efficiency $Y$ is given by

\begin{equation}
  Y = \frac{I}{P} = \frac{e'}{2e\eta Rnv_e} \tag{13}
\end{equation}

By converting into the customary form, we obtain

\begin{equation}
  Y \left( \text{amps/watt} \right) = 5.2 \times 10^{-2} \varepsilon e' \frac{T(keV)}{R(m)n(20)} \tag{14}
\end{equation}

The above value is comparable to the value for the Fisch current drive.

It is noted that the driven current is independent of the safety factor $q$. The large safety factor lowers the bounce frequency. However, the force $T = \partial B/\partial s$ is also reduced. These effects compensate each other. It is also true within one bounce period. The barely trapped electrons have small bounce frequency, but they also receive a small force parallel to the magnetic flux lines. As a result, there is no significant enhancement due to the barely trapped electrons.

The efficiency can be improved by reducing the interaction of electrons in two regions. An electrostatic potential where the median plane is negative with respect to the rest of the plasma interferes with the electron communications between two regions. There are two ways to establish the potential. The first method is to use sloshing ions created either by beam injection or by ICRH heating. This is a well established technique in mirror devices. The turning points of sloshing ions have a higher plasma density and therefore a positive electrostatic potential. The magnitude of the potential $\phi$ is given by

\begin{equation}
  \phi = \frac{\Delta n T_e}{en} \tag{15}
\end{equation}

where $\Delta n$ is the incremental density due to the sloshing ions.

The second method is to apply an additional ECH on the median plane. The increased trapped electrons on the median plane will increase the density. The region will be charged negative to attract ions. The potential is given by

\begin{equation}
  \phi = - \frac{\Delta n T_i}{en} \tag{16}
\end{equation}
where $\Delta n'$ is the incremental density due to the additional ECH.

When the electrostatic potential barrier becomes higher than the parallel kinetic energy of electrons, the electron orbits in two regions are separated. The trapped electrons have smaller parallel energy than that of the circulating electrons. The condition for the separation is given by

\begin{equation}
|\phi| > \frac{e T_e}{\epsilon} \tag{17}
\end{equation}

or

\begin{equation}
\frac{\Delta n}{n} > \epsilon \tag{18}
\end{equation}

and

\begin{equation}
\frac{\Delta n'}{n} > \epsilon \frac{T_e}{T_i} \tag{19}
\end{equation}

The driven current density in these cases can be obtained by putting $\omega_B = 0$ in Eq. (8) for the trapped electrons and replacing $\nu$ with $\nu/\epsilon$. We obtain

\begin{equation}
Y = 3.8 \frac{\epsilon^2}{q} \frac{T(keV)^3}{R^2(m) n^2(20)} \tag{20}
\end{equation}

For ignited plasma where the temperature is greater than 10 keV, the current drive efficiency is excellent.

When the potential barrier is less than that given by Eq. (17), the current drive efficiency is in-between the values given by Eq. (14) and Eq. (20). A rough estimate of the current drive efficiency may be obtained by considering the behavior of the electrons trapped both magnetically and electrostatically. The average energy of the electrons is $(e\phi/\epsilon)$. The population is given by $n(e\phi/\epsilon)^{3/2}$. The force in the parallel direction per electron is proportional to $(e\phi/\epsilon) \frac{\partial (\ell n B)}{\partial s}$. The collision frequency of these electrons are increased by a factor of $(e\phi/\epsilon T)^{-3/2}$. All these factors together, the contribution of these electrons to the current drive, result in

\begin{equation}
Y = 3.8 \frac{\epsilon^2}{q} \frac{T(keV)^3}{R^2(m) n^2(20)} \left( \frac{e\phi}{\epsilon T} \right)^4 \tag{21}
\end{equation}

The above formula indicates that the effect of the electrostatic barrier is small until $(e\phi/(\epsilon T)) \sim 1$.

It has been proposed that the ECH heating asymmetric around the median plane will drive the current in tokamaks. The magnitude of the current is similar to the current driven by the Fisch mechanism. The current will be greatly increased if an electrostatic potential barrier is produced on the median plane.

REFERENCES