Ohmic Ignition of Neo-Alcator Tokamak with Adiabatic Compression

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Abstract

Ohmic ignition condition on axis of the DT tokamak plasma heated by minor radius and major radius adiabatic compression is studied assuming parabolic profiles for plasma parameters, elliptic plasma cross section, and Neo-Alcator confinement scaling. It is noticeable that magnetic compression reduces the necessary total plasma current for Ohmic ignition device. Typically in compact ignition tokamak of the minor radius of 0.47m, major radius of 1.5m and on-axis toroidal field of 20T, the plasma current of 6.8MA is sufficient for compression plasma, while that of 11.7MA is for no compression plasma. Another example with larger major radius is also described. In such a device the large flux swing of Ohmic transformer is available for long burn. Application of magnetic compression saves the flux swing and thereby extends the burn time.

Keywords:
adiabatic compression, Ohmic ignition, Neo-Alcator, saddle point, plasma current, tokamak,

1. Introduction

The tokamak Ohmic ignition device is essentially compact\(^{1-3}\), because its size is restricted by the upper bound of the available toroidal field intensity and the lower bound of the safety factor margin at the plasma surface, and is also simple because additional heating facilities are not necessary. Advantages of compactness and simple structure of the device will open the capability of realizing low cost and short construction time. Disadvantage of such device is that it requires too high toroidal field and large plasma current for achieving Ohmic ignition. Adoption of the reversed field pinch configuration\(^{4,5}\) is another possibility of Ohmic ignition, but the credible confinement data base is less sufficient and its collection is left for the future investigation.

Here we consider the start-up scenario of the Ohmic ignition tokamak on plasma axis with adiabatic magnetic compression of the plasma, which was first proposed in Reference 6. Effectiveness of combined compression has been discussed with some confinement scalings before the establishment of Alcator scaling\(^{7}\). Tokamak ignition with adiabatic compression has been discussed in ZEPHYR project\(^{8}\) and in Kurchatov Institute\(^{9}\) using Alcator scaling, where major-radius compression has been anticipated, and also for the case of traversing the ultra-low-q and low-q discharge in the course of tokamak start-up\(^{10}\). In the latter case, however, reliable confinement scaling is not yet available and thereby the introduction of uncertain assumption was necessary for the calculation. In this paper we focus on the normal tokamak mode of operation and use the more reliable Neo-Alcator scaling, in addition to brief
discussion on L-mode scaling\(^{(1)}\).

Adiabatic compression process of the tokamak plasma has been studied experimentally\(^{(12,13)}\), and improvement of plasma parameters has been observed. But for energy confinement time, both the improvement and degradation have been observed depending on the device.

2. Adiabatic compression

Consider the D-T plasma with the major radius \( R \) and the minor radius \( a \), and assume that density and temperature are equal for ions and electrons. At first, we assume circular cross-section plasma, and later we extend the calculation to the case of non-circular cross-section plasma. Let us assume profiles of the form

\[
x(r) = x_c \left[ 1 - \left( \frac{r}{a} \right)^2 \right]^\alpha, \tag{1}
\]

for the density \( n \), and the current density \( j \), and in order to meet Ohm's law \( j \propto T^{3/2} \), we take the form

\[
T(r) = T_c \left[ 1 - \left( \frac{r}{a} \right)^2 \right]^{2\alpha/3}, \tag{2}
\]

for the plasma temperature. Parameters with subscript \( c \) are values on the plasma minor axis. The peaks of the density and the current profiles are given by

\[
n_c = \frac{N}{2\pi^2 a^2 R} (\alpha + 1), \tag{3}
\]

and

\[
j_c = \frac{I_p}{\pi a^2} (\alpha + 1), \tag{4}
\]

respectively, where \( N \) is the total number of particles and \( I_p \) is the total plasma current in MA. The safety factor at the plasma surface is given by

\[
q = \frac{5a^2 R}{R I_p}, \tag{5}
\]

and that on the axis \( q_c \) is related to \( q \) by

\[
q = (\alpha + 1) q_c. \tag{6}
\]

Let us consider the compression scheme shown in Fig. 1, that is, we apply the minor radius compression first and the major radius compression next. Compression ratios of the minor radius and the major radius are given by

\[
C_a = \frac{a_0}{a_1}, \tag{7}
\]

and

\[
C_b = \frac{R_1}{R_2} = \left( \frac{a_1}{a_0} \right)^2, \tag{8}
\]

respectively, where values with subscripts 0 are those before compression, 1 after the minor radius compression, and 2 after the major radius compression. Geometrical restriction yields the relation

\[
C_e = \left[ \frac{\frac{a_0}{C_a} + \sqrt{\left( \frac{a_0}{C_a} \right)^2 + 4R_0(R_0-a_0)}}{2R_0} \right]^2, \tag{9}
\]
Assume that the toroidal flux in the minor radius is kept constant during compression, then toroidal fields after compression are given by

\[ B_1 = C_s^2 B_0 \]  
and

\[ B_2 = C_R B_1 \]  

Also from Eqs. (3), (4), (7) and (8), peak values of the density and the current density after compression are related to those before compression as follows:

\[ n_1 = C_s^2 n_0, \]  
\[ n_2 = C_R^2 n_1, \]  
\[ j_1 = C_s^2 j_0, \]  
and

\[ j_2 = C_R^2 j_1. \]  

Similarly for the plasma temperature, from the adiabatic relation \( T \propto n^{2/3} \), we obtain

\[ T_1 = C_s^{4/3} T_0, \]  
and

\[ T_2 = C_R^{4/3} T_1. \]  

These relations are similar to those derived in Reference 6.

In case that the plasma cross section is elliptic with the elongation ratio \( \kappa \), \( q \) is replaced by the cylindrical equivalent safety factor

\[ q^* = q \frac{1 + \kappa^2}{2}, \]
and the current density on axis is given by

\[ j_c = \frac{5B_1}{\pi q_\ast R} \frac{1 + \kappa^2}{2\kappa}, \]

(19)

where \( q_\ast \) is the safety factor on axis\(^{14}\). The toroidal field on the plasma axis after minor radius compression and major radius compression is obtained by inserting \( j_2 \) into Eq. (19) and given by

\[ B_2 = \frac{2\pi q_\ast j_1 R_0 \kappa}{5(1 + \kappa^2)}. \]

(20)

Neo-Alcator energy confinement time after compression is represented by

\[ \tau_\varepsilon = 0.07 a R^2 q_\ast \bar{n} \]

(21)

with average density

\[ \bar{n} = \frac{n_e}{\alpha + 1}. \]

(22)

In order to satisfy adiabatic condition, compression time of the magnetic field \( B(dB/dt)^{-1} \) is taken to be shorter than the energy confinement time.

3. Ohmic ignition condition

We seek for the Ohmic ignition condition with moderate toroidal field intensity and reasonable device size. Let us assume a D-T plasma with 50-50 D-T mixture, and consider the power balance equation on axis after minor radius and major radius compression as

\[ P = P_j + P_\alpha + P_L + P_B \]

\[ = C_j j_x T_{\alpha}^{1.5} + C_\alpha n_e^2 T_x^{3.5} - C_L \frac{n_e T_x}{\tau_\varepsilon} - C_B n_e^2 T_x^{2.5}, \]

(23)

where \( P_j \), \( P_\alpha \), \( P_L \), and \( P_B \) are power densities of Ohmic heating, alpha particle heating, transport loss, and Bremsstrahlung loss on plasma axis in MW/m\(^3\), respectively. Synchrotron radiation is neglected here, assuming the total reflection from inner wall of the vessel. Unit of plasma density is in \( 10^{20} \text{m}^{-3} \) and that of temperature in keV. Coefficients \( C \)'s of Eq. (23) are given by

\[ C_j = 2.97 \times 10^{-2} (0.65 Z_{\text{eff}} + 0.35) \]

\[ C_\alpha = 6.25 \times 10^{-5} \left( \frac{Z - Z_{\text{eff}}}{Z - 1} \right)^2 \]

\[ C_L = 4.81 \times 10^{-2} \]

and

\[ C_B = 4.93 \times 10^{-3} Z_{\text{eff}}. \]

where \( Z_{\text{eff}} \) is effective charge number of the plasma and \( Z \) is charge number of impurity species. Coefficient \( C_j \) contains the factor arising from the Spitzer resistivity only, and the neoclassical enhancement factor is not taken into account.

In the second term of the right hand side of Eq. (23), we adopted the fusion reaction rate coefficient which is proportional to the powers of temperature dependence 3.5 according to Reference 13. This choice of the temperature dependence makes it possible to express the density and the temperature in analytical form at the saddle point of the power density plane shown below.\(^{10}\).
We prescribe the ignition condition in such a way that the power density on plasma axis is positive at the saddle point so as that the thermal runaway caused by the alpha particle heating takes place. Figure 2 shows an example of 3D expression of the dependence of power density on temperature and density. The saddle point values are calculated from $\frac{\partial P}{\partial T} = \frac{\partial P}{\partial n} = 0$, and the marginal condition for ignition is given by $P=0$ at the saddle point. From these relations, we obtain the temperature, the density, and the current density at the saddle point as follows:

![Figure 2 Example of 3D plot of power density vs. plasma temperature and density. Saddle point is located at $T=5.2\text{keV}$ and $n=2.1\times10^{21}\text{m}^{-3}$.](image)

\begin{align}
T_s &= \sqrt[3]{\frac{C_B}{C_F}}, \\
n_s &= T_s^{1/4} \sqrt[6]{\frac{5C_b}{6C_0k}}, \\
j_s &= T_s^{5/4} \sqrt[8]{\frac{C_b}{C_Fk}},
\end{align}

where $K = \tau_{E_b}/n_s$ and $\tau_{E_b}$ is the global energy confinement time at the saddle point. These are marginal values requisite for ignition. Additionally, multiples of energy confinement time with density and current density become constant at the saddle point:

\begin{align}
n_s \tau_{E_b} &= \frac{4.01 \times 10^2}{C_0} \sqrt[6]{\frac{C_B}{C_F}}, \\
j_s \tau_{E_b} &= \frac{4.39 \times 10^2}{\sqrt{C_0C_F}}.
\end{align}

Parameters attained after compression and capable to achieve ignition are obtained by putting $T_2 = T_s$, $n_2 = n_s$, $j_2 = j_s$, and

\begin{equation}
K = \frac{\tau_{E_2}}{n_2} = \frac{0.07\,\text{mag} \times R_2^2}{\alpha + 1},
\end{equation}
Then the other parameters can be calculated from Eqs. (9) (21) for given minor radius compression ratio \( C_a \). Average beta after compression is given by

\[
\tilde{\beta}_a = \frac{2n_b T_b}{B_0^2 / 2 \mu_0 (5 \alpha / 3 + 1)},
\]

where \( \mu_0 \) is the vacuum permeability.

Examples of parameter sets with \( Z_{\text{eff}} \) of 1.5 and \( Z \) of 8 are listed in Table I. The parameter set No. 1 is the result of compression-free case, and is close to that of the Ohmic ignition tokamak IGNITEX\(^3\). For parameter set No. 2, the device parameters and the maximum toroidal field at the device center are adjusted to take those of IGNITEX, but the magnetic compression is applied in this case. Evolution of current density at each stage of compression is shown in Fig. 3. It should be noticed that the peak of current density increases drastically by compression, and this fact enables us to achieve Ohmic ignition with

<table>
<thead>
<tr>
<th>Table I Examples of Parameter Set of Neo-Alcator Tokamak</th>
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<tbody>
<tr>
<td>No.</td>
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<td>compression ratio ( C_a )</td>
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<td>compression ratio ( C_m )</td>
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<tr>
<td>elongation ratio ( \kappa )</td>
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<td>index of profile ( \alpha )</td>
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<td>minor radius (m)</td>
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<td>total current (MA)</td>
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<td>safety factor</td>
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<td>current density (MA/m(^2))</td>
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<td>density ( (\times 10^{20} \text{m}^{-3}) )</td>
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<td>temperature (keV)</td>
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<td>confinement time (s)</td>
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<td>average toroidal beta (%)</td>
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<td>magnetic energy (GJ)</td>
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<td>compression power (GW)</td>
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lower plasma current compared with compression–free tokamaks\textsuperscript{2,3}). Lower plasma current can be generated with smaller flux swing of the current transformer, and this fact makes the design of device easier especially in the case of compact device. It is required to increase the toroidal field from 11T to 20T within the energy confinement time 0.4s. Then the magnetic energy in the plasma volume after the compression is about 1.7GJ and the rate of magnetic energy increase corresponds to the power of 3.0GW. Those values are technically amenable. Small safety factor on plasma axis after the major radius compression might cause the excitation of resistive MHD modes and change the plasma current profile. For certain profiles, however, $q_c < 1$ regime can survive\textsuperscript{16,17).}

The parameter set No. 3 is an example of the case of larger major radius device without compression, and the set No. 4 is that with compression. Again the initial minor and major radii and the toroidal field at the device center are taken to be equal for both cases. With larger major radius device, longer discharge time is available for the sake of larger amount of flux swing in the current transformer. The magnetic compression improves this feature much more, because it reduces the total plasma current necessary for ignition. However, as is seen in the table, this improvement is possible at the expense of safety factor reduction. Larger major radius device can satisfy the ignition condition with smaller toroidal field, but in stead of that technological merit, larger total magnetic energy is necessary.

Density limit of Murakami scaling is satisfied for all cases in Table I.

4. Discussion and conclusion

The scheme discussed above requires fast rising high toroidal field and supporting power supply system. We will be able to realize such a system by using, for example, single turn toroidal coil divided into several parts in toroidal direction. Each part is energized by independent power supply which consists of a capacitor bank crowbarred by the homopolar generator. Feasibility of single turn toroidal coils with homopolar generators has been demonstrated in the design of very high field tokamak IGNITEX\textsuperscript{3).}

The necessary toroidal field intensity for Ohmic ignition cannot be reduced by the magnetic compression as shown in the previous section. Low–$q$ mode of operation can assist the reduction of toroidal field, and once the ignition takes place, as long as the sufficient alpha particle heating power is maintained, we will be able to increase $q$ value by decreasing the total plasma current or by increasing the minor radius of plasma through free expansion. Applicability of Neo–Alcator confinement scaling at low–$q$ regime ($q < 2$) is, however, not studied intensively yet.
In order to guarantee the attainment of Ohmic ignition, it is necessary to keep the current density on axis larger than \( j_s \) given by Eq. (26) until the onset of alpha particle heating. The current density on axis decreases as the plasma current channel expands or as the current density profile near the axis flattens because of the occurrence of sawtooth oscillations. In the present scheme, the required total plasma current is small and the increase of the plasma current so as to maintain the sufficient amount of current density for ignition is technically admissible.

Once the thermal runaway takes place at the plasma center, the plasma column is allowed to expand and back to its initial geometry before compression by using the external position control system. Hereafter, application of divertor action and/or plasma current drive might be possible.

An alternative way to achieve Ohmic ignition is to generate the plasma with the parameters equivalent to those of the compressed plasma (such as those listed in No. 2 or 4 of Table I) without compression. With the magnetic compression, however, flexibility of tokamak operation increases, \( q \) margin increases as discussed above, and improvement of transport by the separation between plasma surface and the first wall is anticipated\(^{18}\).

It would be not so obvious that L-mode confinement scaling\(^{19}\) should be applied for these high-field high-density tokamak plasmas. where ohmic heating power is dominant. We have, however, examined an ignition condition for L-mode scaling, if alpha-particle heating power is incorporated in an auxiliary heating power of L-mode scaling. Here we have adopted ITER-89 power law, and found that plasmas never ignite for L-mode condition. If H-mode is realized, an ignition condition is achieved with a small amount of auxiliary heating power (\( P_{aux} < 10\text{MW} \)). Especially we could expect H-mode for compressed plasmas, because plasma–wall interaction will be quite reduced, as clearly demonstrated in TUMAN-3 compression experiments\(^{18}\). Ohmic ignition with no auxiliary heating is also expected, if the density profile is more peaked in addition to the H-mode, demonstrated as PEP–H mode in JET\(^{20}\).

As for reactor-relevant plasmas, the role of alpha-particle heating for the plasma confinement is one of the key issues to achieve an ignition condition. In high-field tokamak the ohmic heating is directly replaced with alpha-particle heating by no way of high power auxiliary heating, that will give a very good opportunity to examine whether the alpha-particle heating triggers the confinement degradation, as observed in all of auxiliary heating experiments, or not.

In conclusion, magnetic compression makes the design of tokamak Ohmic ignition scheme easier, because of the reduction of required plasma current. Although the device parameters are not optimized here, examples of parameter set are within the limit of technical capability. The high-field tokamak device is suitable to study the confinement characteristics of alpha-particle heating, because an auxiliary heating power is very small or none. In compressed plasmas, in addition, H-mode is expected, even if plasmas would fall into the L-mode regime.

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