The electron density and the electron collision frequency in a decaying plasma are deduced from the simultaneous recording of two microwave detector signals. The plasma is contained in a small diameter, cylindrical glass tube placed across a waveguide, on which the two detectors are located at some known positions. This method is applied to the decay of an usual pulsed DC discharge, and electron densities between $5 \times 10^{11}$ and $2 \times 10^{13}$ cm$^{-3}$ are measured by a 8300 MC. wave. It is also found that the electron temperature, except in its very early period, decreases more slowly than expected on the basis of the simple assumption of energy equipartition between electrons and ions.

1. Introduction

Usual microwave techniques for the investigation of plasma impedance are based on the shift of the minimum amplitude position of a standing wave detector and on the change of the standing wave ratio. However, when the plasma is produced by a non-recurrent (non repeated or one shot) discharge, these standard techniques become inadequate. Since also the initial conditions of such transient discharges are not always reproducible as exactly as in the case of a weak recurrent discharge, a modified
method of microwave measurement has to be developed and is described in this article. This method can yield the instantaneous value of both the electron density and collision frequency in a very small volume of a time varying plasma. These parameters are calculated from the traces of two detectors, displayed simultaneously on an oscilloscope. For analysis these traces are photographically recorded.

The relative arrangement between the waveguide circuitry and the plasma, described in this article, allows the measurement of an electron density corresponding to a plasma frequency \( \omega_p \) higher than the wave frequency \( \omega \) \((\omega_p > \omega)\).

In order to confirm the applicability and the validity of this impedance measurement, it is applied to a typical recurrent decaying plasma. The electron temperature in this plasma is obtained from the theoretical temperature dependence of the electron-ion collision frequency, since the electron-atom collision frequency may be neglected in this experiment. Since the two detector signals are photographically recorded, this technique is especially adapted to plasmas produced by non repeated discharges.

2. Description of the equipment

The equipment consists in a glass tube, 0.5 cm. in diameter, 40 cm. in length, containing the time varying plasma to be investigated, and in an X-band waveguide circuitry \((8200-12000 \text{ MC})\). The glass tube is placed across that waveguide through a hole bored in its wide side wall. The electric field in the waveguide is therefore parallel to the tube axis \((\text{or plasma post axis})\).

At one end of the waveguide circuit, a cw, low power X-band klystron, produces a microwave at the desired frequency. This wave is reflected at the other end by a movable short and builds up as a standing wave as shown in Fig. 1. The two microwave probe detectors are located on the waveguide, detector 1 on the klystron side, detector 2 on the short side of the tube. In the absence of plasma, detector 1 is at a minimum amplitude position, whereas detector 2 is in the vicinity of a maximum amplitude position. The equivalent circuit of the waveguide and plasma post, the standing wave pattern in the waveguide, as well as the respective position of the probe detectors, are
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The position of the short is adjusted, in the absence of plasma, so as to have a maximum of the standing wave at the tube axis. If the tube material had no effect on the standing wave pattern, the distance between the tube axis and the movable short would be an odd multiple of a quarter waveguide wavelength. The tube wall, however, is made of glass, the equivalent of which is capacitive. In order to have, in the absence of plasma in the tube, the equivalence of an open circuit in the tube axis (theoretically infinite impedance), as viewed in the waveguide from the right hand side of the tube, the left hand side has to be made inductive by an amount equal in absolute value to the tube wall capacity. This is achieved by shifting the movable short over a distance \( \Delta \) toward the tube, as seen in Fig. 1. For our experiment, this distance corresponds to an electrical angle of 25°. This is not negligibly small.

The following procedure is used to find the correct position of the short. For some arbitrary position of the short, and in the absence of plasma, the probe of detector 1 is shifted a quarter wavelength from its minimum amplitude position. Thus, both probes 1 and 2 are in a maximum; by adjusting the gain of the corresponding amplifiers, the amplitudes of the probe signals, on an oscilloscope, given by the magnitude of the microwave cut-off pulse, is made equal. Then a discharge is produced in the tube by a repeated DC pulse of short duration and the effect of the plasma decay on the standing wave pattern is observed by the probes 1 (in its maximum position) and 2.

It should be noted that, for the arbitrary time of observation during the plasma decay, shown in Fig. 1, the standing wave pattern at the right side of the tube is shifted and attenuated with respect to its shape without plasma, whereas the one at the left side of the tube only decreases in amplitude but suffers no shifting. Therefore, during the plasma decay, the amplitude change \( h_1 \) of probe signal 2 is always proportional to the amplitude change \( h_2 \) on the tube axis. However, the amplitude change of probe signal 1, in its maximum position, is equal to \( h_2 \) only if the standing wave pattern at the right

* The principle of the measurement is a modification of the method previously published (J. of Phys. Soc. in Japan 16, 95 (1961)).
side of the tube has, without plasma, a maximum on the tube axis. Thus, by following procedure for each new position of the movable short, its correct position is experimentally approached and finally attained so as to the two signal amplitudes \( h_2 \) and \( h'_2 \) of probe 1 (in its maximum position) and 2 displaying the same cut-off amplitude, coincide exactly at any time during the plasma decay, except perhaps at its very beginning. This deviation of the two curves there, may be explained by the disturbing effect on the waveguide electric field distribution, of the high density plasma placed across it. The accuracy of this method leads to a maximum mismatching in the probe position smaller than 5 miles or 1 electrical degree.

3. Principle of the Measurement

The principle of the measurement is to relate the time varying magnitude of the two probe detector signals, \( h_1 \) and \( h_2 \), affected by the change of the plasma post impedance, directly to the time varying electron density \( n \) and collision frequency \( \nu \) in that portion of the plasma. The normalized equivalent impedance of this plasma post is a function of \( n \) and \( \nu \). On the other hand, the amplitudes \( h_1 \) and \( h'_2 \) of the signals from the two probe detectors 1 and 2, shown in Fig. 1, are also functions of the real and imaginary components of this impedance. Therefore, both \( n \) and \( \nu \) in the plasma post can be known, at any instant, from the signals amplitude \( h_1 \) and \( h'_2 \).

An advantage of this tube arrangement is that electron densities somewhat higher than the critical value can be measured, provided that their distribution along the axis of the tube is uniform. The amplitudes \( h_1 \) and \( h'_2 \) of both probe detectors 1 and 2, simultaneously displayed on the oscilloscope, can be calculated in a function of the post impedance, provided the crystals used in the probes have a square law characteristic.

\[
\begin{align*}
    h_1 &= \frac{1}{4} \left| 1 - \frac{z - 1}{z + 1} \right|^2 \\
    h'_2 &= 1 - \frac{1}{4} \left| 1 + \frac{z - 1}{z + 1} \right|^2
\end{align*}
\]
where the amplitudes $h_1$ and $h_2$ are normalized by dividing the maximum amplitude of the microwave cut-off pulse $h_m$ to have a maximum value of unity for $z = 0$. The terms $z$ and $(z-1)/(z+1)$ are respectively the normalized impedance of the plasma and its reflection coefficient.

Using $z = r + jx$, $r$ and $x$ are solved from Eq. (1)

$$x = \left( \frac{1}{h_1} - \frac{(h_1 + h_2)^2}{2h_1} \right)^{1/2},$$

$$r = \frac{h_2 - h_1}{2h_1}.$$  \hfill (2)

The equivalent normalized impedance of a plasma post is given by a formula indicated by Markuvtz. \(^{(1)}\) The complex dielectric constant $\varepsilon$ in this formula is, as well-known, related to the electron density $n$ and the collision frequency $\nu$ by

$$\varepsilon = 1 - \frac{n\varepsilon^2}{m_0^2 \omega} \frac{1}{\omega + j\nu}.$$  \hfill (3)

When Eq. (3) is put into the formula for the plasma impedance, the real and imaginary parts of $z$ can be separated, and finally $n$ and $\nu$ are deduced, as

$$n = \frac{A}{x-B}$$

$$\nu/\omega = \frac{r}{x-B}, \quad \text{or} \quad \nu/n = \frac{\omega r}{A}$$

where

$$A = \frac{m_0 \varepsilon^2 \omega^2 \left( \frac{a}{2\lambda_g} \right)}{\varepsilon^2} \left\{ 0.50 + 2 \left( \frac{\lambda}{\pi d} \right)^2 \right\}$$

$$B = \frac{a}{2\lambda_g} \left( S_0 + 0.25 \right)^{\star}$$

The constants $A$ and $B$ depend on the diameter of the plasma post $d$, the waveguide width $a$, and the free space and waveguide wavelength $\lambda$, $\lambda_g$. When $n$ is small, or $x$ is very large compared to $B$,

$$n = \frac{A}{x}, \quad \nu/\omega = \frac{r}{x}.$$  \hfill (4)

Thus Eqs. (2) and (4) relate $n$ and $\nu$ to the observed probe amplitudes $h_1$ and $h_2$.

In this method, Eq. (4) does not imply the condition $\nu \ll \omega$. However, there are some restrictions in

\[^{\star}\text{So is defined in reference (1).}\]
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the plasma impedance formula. They are:

\[
\sqrt{|\varepsilon|} < 4
\]

\[
d/\lambda < 0.15,
\]

\[
2a > \lambda > \frac{2}{3}a,
\]  

(5)

The relation \(n\) versus \(h_l\), for X-band frequency and the conditions of our equipment, is shown in Fig 2, for different parameters of \(\nu/n\). Electron densities higher than the critical value can be measured, provided the electron distribution along the axis of the tube is uniform, because then the electric field is almost perpendicular to the gradient of the electron density and the ac space charge does not have a disturbing effect on the probing microwave field. However, the restriction on \(\varepsilon\), limits the maximum electron density to \(2 \times 10^{13} \text{ cm}^{-3}\).

4. Experimental Results and Discussions

The method described in this article was used to measure the electron density, the collision frequency and the temperature in the decay of a plasma produced by a recurrent pulsed DC discharge in Argon or Neon at low pressures. By means of a pulse divider, the repetition frequency of these discharges is made half of that of the oscilloscope triggering. Therefore, four oscilloscope traces correspond to the change in the amplitudes \(h_1\) and \(h_2\) of the signals detected by the probes 1 and 2, both in the presence and in the absence of the decaying plasma. A typical exposure is shown in Fig. 3 (A), where the upper traces (with positive downward deflection) represent the signals picked up by probe 1 in its minimum position and the lower traces (positive upward deflection) those of the probe 2 in its maximum position; the horizontal traces correspond to probe signals 1 and 2 in the absence of plasma. The maximum amplitude of the microwave cut-off pulse (indicated by \(h_m\) in trace of probe 2, but not visible in trace of probe 1, because the latter is now in a minimum amplitude position) must obviously be made equal for both
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Fig. 3 (A)
traces in order to use the formulas derived in section 3. The time axis origin ($t = 0$) coincide in all the figures with the discharge triggering pulse.

Although the origin of the irregularity superimposed on the signals $h_1$ and $h_2$ in Fig. 3 (A), during the plasma decay, are not clearly explained at the present time, these traces were replotted in Fig. 3 (B) as smoothed curves,
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and the electron density and collision frequency are calculated from them with the help of Eqs. (2) and (4).

Figs. 4 and 5 show, in function of time, respectively the decay of the electron density $n$ and the change in the ratio of the electron ion collision frequency $\nu_{ei}$ to the electron density $n$, during the decay of the plasma located in the waveguide post.

In the ranges of pressure, density and temperature of our experiments, the electron-molecule collision frequency $\nu_{en}$ can usually be neglected with respect to the electron-ion collision frequency $\nu_{ei}$; if not, $\nu_{en}$ is taken into account. The relative importance of these parameters is
checked in Fig. 6, which plots the theoretical values of $\nu_{ei}/n_1$, (3) and experimental values of $\nu_{en}/p$, (5), (6) for Neon and Argon, where $p$ is the pressure in mm. Hg., in function of the electron temperature $T_e$. Since $\nu_{ei}$ is proportional to $n$ and $T_e^{-3/2}$, it is expected that, in the late period of the plasma decay, as the electron temperature approaches $300^\circ$K, the ratio $\nu_{ei}/n$ should reach a constant value. Fig. 5 shows that the experimental curve approaches the value $1.2 \times 10^{-2}$ cm$^3$/sec., which is compared to the theoretical and experimental values of the ratio $\nu_{ei}/n$ at room temperature derived by other authors and listed in Table I.

From the theoretical temperature dependence of the electron-ion collision frequency $\nu_{ei}$, the electron temperature in the decaying plasma is calculated from the measured $\nu_{ei}$ and plotted in Fig. 5. This curve shows, that the electron temperature takes more than 100 $\mu$ sec to fall down to $300^\circ$K. This time is much longer than expected, according to the general equation for the time rate of change of the electron temperature as follows.
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Fig. 6
<table>
<thead>
<tr>
<th>Electron Density</th>
<th>Theory</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Spitzer(8)</td>
<td>Ginsburg(3)</td>
</tr>
<tr>
<td></td>
<td>Anderson(4)</td>
<td>Dougal(7)</td>
</tr>
<tr>
<td>$1 \times 10^{11}$</td>
<td>$3.16 \times 10^{-3}$</td>
<td>$3.75 \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>$3.0 \times 10^{-3}$</td>
<td>$0.98 \times 10^{-2}$</td>
</tr>
<tr>
<td>$1 \times 10^{12}$</td>
<td>$2.36 \times 10^{-3}$</td>
<td>$2.88 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table I

Theoretical and experimental values of $\nu_e^{\hat{}} / n$ at room temperature (cm$^3$/sec).

I Phototube datas of luminous intensity

II microwave interaction

(7),(8) $\nu_e^{\hat{}}$ deduced from $\tau_e^{\hat{}}$. 
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\[
- \frac{dT_e}{dt} = \frac{1}{\tau_{ei}} (T_e - T_i) + \frac{1}{\tau_{en}} (T_e - T_g)
\]

where \( T_e, T_i \) and \( T_g \) are the electron, ion and gas temperature respectively, and \( \tau_{ei}, \tau_{en} \) are the characteristic times for equipartition of the temperature between electrons and ions, and electrons and atoms respectively. \( \tau_{en} \) is defined as \( \tau_{en} = \frac{1}{\nu_{en}} \frac{M}{2m} \), where \( M \) and \( m \) are the atom and electron mass respectively; similarly \( \tau_{ei} \) may be defined as \( \tau_{ei} = \frac{1}{\nu_{ei}} \cdot \frac{M_i}{2m} \) where \( M_i \) is the ion mass.

The theoretical values of \( \tau_{ei} \), derived in different ways, are summarized by Dougal and Goldstein. For Argon at room temperature and for the electron densities met in our experiments \((10^{11} - 10^{13} \text{ cm}^{-3})\), \( \tau_{ei} \) (which is much shorter than \( \tau_{en} \)) is of the order of a few \( \mu \text{sec} \). One reason which can possibly explain the long electron temperature decay time measured would be the presence of metastable atoms, heating the electron gas by the intermediary of the ion gas.

It was usually assumed that the maximum light intensity originating from the afterglow close to the beginning of a pulsed discharge corresponded to the time at which the electron temperature reached 300°k. The photomultiplier data of our experiments, especially in Neon afterglow, contradict this assumption, since the light intensity maximum appears in less than 20 \( \mu \text{sec} \), whereas the electron temperature reaches 300°k after more than 100 \( \mu \text{sec} \).

In order to check the error in the measurement of \( n \) and \( \nu \) due to the microwave energy lost by radiation through the waveguide hole or along the plasma post, the plasma was replaced by a metal slab of the same diameter. In that case, \( n \rightarrow \infty \); it roughly follows from Eq. (4), that \( x = B \) and \( r \rightarrow 0 \). Experimentally, \( r \) was found to be 0.1; that corresponds to \( \nu_{ei}/n = 1.5 \times 10^{-3} \text{ cm}^3/\text{sec} \).

Since the highest energy losses are expected at high electron densities, this corresponding value of \( \nu_{ei}/n \) is thought to be its maximum possible error. It is also probable that the plasma diameter is slightly smaller than the internal tube diameter, owing to the thin ion sheet found usually close to the walls. This and the possible non-uniformity of the electron distribution, due to the proximity of the metallic waveguide, contribute to the measurement error. If the equivalent plasma diameter could be known, then the constant \( A \) in Eqs. (4) and (4') would increase and thus explain in part the too high values of \( \nu_{ei}/n \) found in these experiments. This correction, however,
does not affect the electron temperature $T_e$, calculated, since only the $-3/2$ power of the temperature ratio is used $\left(\frac{\nu_e}{n} \sim \frac{(T_e/300)^{-3}}{2}\right)$.

This method was applied to the measurement of the plasma parameters $n$ and $\nu$ in a decaying plasma disturbed by the propagation of non-ionizing shock waves. It will be published later.

ACKNOWLEDGEMENT

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Figures

Fig. 1 Equivalent waveguide circuit and microwave standing wave pattern. Solid line: without plasma but with glass. Dashed line: with plasma and with glass. Dotted line: ideal case, without plasma and without glass.

Fig. 2 Electron density n versus normalized microwave signal amplitude $\frac{h^1}{h^m}$ of detector 1, for different values of the parameter $\nu/n$, where $a = 33$ mm and $d = 5$ mm are used.

Fig. 3 (A) Microwave signal amplitudes $h^1$ and $h^2$ from detectors 1 and 2 versus time, for a decaying plasma in Argon ($p = 8.3$ mm. Hg; 20 $\mu$ sec/ division; microwave cut-off pulse amplitude $h^m = 5$ divisions). Time starts with the discharge trigger pulse. (B) Same as in Fig. 3(A); curves smoothened.

Fig. 4 Electron density n versus time, calculated from Fig. 3(B).

Fig. 5 Ratio of the electron-ion collision frequency $\nu_{ei}$ to the electron density $n$, and electron temperature $T_e$ versus time, calculated from Fig. 3(B).

Fig. 6 Ratio of the electron-ion collision frequency to the electron density $\frac{\nu_{ei}}{n}$, and ratio of the electron-atom collision frequency to the gas pressure $\nu_{en}/p$ for Neon and Argon, versus electron temperature $T_e$. (p in mm. Hg, and n in cm$^{-3}$). $\nu_{ei}$ is deduced from Ginsburg's results$^{(3)}$ and $\nu_{en}$ calculated from the collision cross-section or collision probability determined in previously published papers.$^{(5),(6)}$