Theoretical Analysis of a Thermal Inertia Model for Soil Moisture Estimation and its Application to Remote Sensing

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Abstract: For soil moisture sensing, a thermal inertia model based on the heat balance and one-dimensional heat transfer of the earth's surface was constructed analytically by applying some basic relationships between the daily range of surface temperature and the heat balance terms formulated by Laiktman and Uchijima, and the solution of heat transfer derived by Ohga. This model was enlarged for airborne and satellite remote sensing of soil moisture. The procedure was verified using heat balance data obtained at a ground truth center in the Kujukuri plain.

In the heat transfer theory, the concept of thermal inertia explaining the heat conductance inherent to each material is well known. This thermal inertia model has been developed for soil moisture sensing from evaluation of previous thermal inertia models, and is more advantageous than other models because of its simplicity, the validity of its physical/mathematical solution and its incorporation of surface relative humidity in appropriate heat balance terms.

It is also practical, since it allows the use of two datasets from remote sensing twice a day. The daily range estimated from measurements made in the early morning and at noon can eliminate the noise of measurement error for the larger dynamic range of measurement values. From this evaluation of thermal inertia it was clarified that soil moisture remote sensing and its procedure based on heat balance and thermal inertia derived from physical and mathematical solutions is more accurate than that based on apparent thermal inertia, and results of simulation and iteration. Although some problems still remain with this procedure, it can be used as a basis for remote sensing of soil moisture.
1. INTRODUCTION

By remote sensing, wide areas of the earth's surface can be observed and information with a homogeneous quality can be obtained. Remote sensing technology has been applied for examining environmental pollution and for evaluating the development of land use and natural resources (i.e., crop production and desertification). Soil moisture is an important factor for agricultural production, together with other environmental parameters such as air temperature, surface albedo, evaporation and water balance.

Some empirical equations for soil moisture estimation have been developed using parameters such as weather elements, visible, near and thermal IR data, brightness temperature and microwave backscattering. Thermal parameters such as surface temperature, soil-air temperature difference and the rate of increase of surface temperature have been evaluated for soil moisture estimation1)-22).

Against this background, thermal inertia as an intrinsic parameter has been examined to determine soil moisture and evapotranspiration23)-49).

As Smith (1983)50) described, there have been two different approaches for determination of thermal inertia (TI). One is the iterative calculation method of Rosema and coworkers11),28),36),39),41),42) In this method, heat balance and heat transfer equations are solved repeatedly for various sets of albedo, surface roughness, surface humidity, thermal conductivity, and surface temperature. Also, surface relative humidity is sometimes assumed to be saturated. In this approach, the relationship between albedo, thermal inertia and soil moisture was examined, and some attempts were made to simplify the heat balance equation to reduce the considerable calculation time.

The other approach is the analytical procedure of Watson and other investigators23),25)-27),38) Watson tried to evaluate thermal inertia theoretically on the basis of a one-dimensional heat transfer model. However, Watson's theoretical examination still included some iterative solution methods for determination of some parameters.

In some previous approaches based on one-dimensional heat transfer and a heat balance model, some heat balance terms were excluded. For example, the latent heat flux was assumed to be zero because of the lack of latent heat flux in arid or semi-arid areas33),34). A model developed by Price (1980) excluded soil heat flux by time-averaging over a 24-h period41).

Price (1985)43) also evaluated apparent thermal inertia (ATI) assuming it to be a function of other variables such as net radiation, surface temperature, solar constant, atmospheric transmittance and ground heat flux. Thus, his ATI was not based on a physical and mathematical solution, but on an iteration method.

Recently, Ho (1987)21) proposed a soil thermal model related to soil heat flux using the first harmonic of the variation of the surface temperature curve. Some estimated parameters of heat balance terms used as a basis for his soil thermal model were derived from regression analyses.

In some previous studies, thermal inertia and albedo were handled simultaneously, because of failure of a mathematical solution for evaluation of thermal inertia based on one-dimensional heat transfer using heat balance models of the earth's surface. The previous models included many complex input parameters for atmospheric correction and other subjective parameters, and lacked essential parameters of heat balance. Sometimes, the relative humidity of the soil surface in the model was ignored or insufficiently adopted and was assumed to be saturated. As mentioned above, these previous models have some ambiguities. The purpose of the present study was to construct analytically a fundamental thermal inertia model for soil moisture sensing and to evaluate the model by applying it to airborne remote sensing data.
2. Thermal Inertia Modeling for Soil Moisture Estimation

(1) Thermal model

As described by Budyko (1956), the energy balance of the earth's surface is essentially formulated as follows:

\[ S = 1E + H + B \] ..........................(1)

where \( S \) is net radiation flux, \( 1E \), latent heat flux, \( H \), sensible heat flux, and \( B \), soil heat flux.

For one-dimensional heat transfer, \( 1E, H \) and \( B \) are given by Eqs. (2), (3) and (4), respectively (Oga, 1931; Laiktman, 1961; Uchijima, 1964, 1974; Monteith, 1973).

\[ 1E = -K_s \frac{\partial q}{\partial h} \bigg|_{h=0} \] ................................(2)

\[ H = -K \frac{\partial \theta}{\partial h} \bigg|_{h=0} \] ................................(3)

\[ B = -\frac{\lambda}{2} \frac{\partial \theta}{\partial z} \bigg|_{z=0} \] ................................(4)

Substituting Eqs. (2), (3), (4), and (5) into Eq. (1), we obtain:

\[ S = dS \cos \alpha \cos \theta \bigg|_{h=z=0} \]

\[ = dS \cos 2\pi \frac{L}{\tau} \bigg|_{h=z=0} \] ................................(5)

where \( K_s \) is the molecular diffusion coefficient for water vapor, \( K \), the coefficient of thermal conductivity of air, \( \lambda \), thermal conductivity of soil, \( q \), specific humidity, \( \theta \), temperature, \( h \), height and \( z \), depth, \( dS \), daily amplitude of net radiation, \( \omega \), angular velocity \( (\omega=2\pi/\tau) \), \( t \) and \( \tau \), time, and \( \tau \), cycle. The boundary condition of height \( h \) and depth \( z \) is zero. As for sign, all terms except net radiation customarily have minus signs in the heat balance equation.

Substituting Eqs. (2), (3), (4), and (5) into Eq. (1), we obtain:

\[ -K_s \frac{\partial q}{\partial h} - K \frac{\partial \theta}{\partial h} - \frac{\lambda}{2} \frac{\partial \theta}{\partial z} = dS \cos 2\pi \frac{L}{\tau} \] ..........................(6a)

\[ -\text{k}w \rho \frac{\partial q}{\partial h} - \text{k}a\text{Cp} \frac{\partial \theta}{\partial h} - \frac{\text{acr}}{\partial z} = dS \cos 2\pi \frac{L}{\tau} \] ..........................(6b)

where \( K_s \) is the molecular diffusion coefficient for water vapor, \( K \), coefficient of thermal conductivity of air, \( K_a \), thermal diffusivity of air \( (ka=K/\rho C_p, \text{cm}^2/\text{s}) \), \( \rho \), air density \( (\text{g/cm}^3) \), \( Cp \), specific heat of air \( (\text{cal/g}^\circ \text{C}) \), \( a \), thermal diffusivity of soil \( (a=\lambda/cr, \text{cm}^2/\text{s}) \), \( \lambda \), thermal conductivity \( (\text{cal/cm.s.}^\circ \text{C}) \), \( c \), specific heat of soil \( (\text{cal/g}^\circ \text{C}) \), \( r \), specific gravity of soil, \( l \), latent heat of vaporization \( (\text{cal/g}) \).

According to Laiktman and Uchijima, the specific humidity \( q \) at the earth's surface can be approximated by the saturated specific humidity of the soil surface at a surface temperature \( T_s \):

\[ q = \mu q(T_s) \] ..........................(7)

where \( \mu \) is surface relative humidity, \( q \), specific humidity, and \( q(T_s) \), saturated specific humidity at a surface temperature \( T_s \).

Because of the decreased evaporation from dry soil, the evaporation rate is overestimated without using the parameter \( \mu \). However, the surface relative humidity should be introduced for precise estimation of the evaporation from dry soil. This was described by Laiktman (1961) as "\( g \)" and subsequently defined by Uchijima (1964).

In this paper the parameter \( \mu \) is defined as:

\[ \mu = \frac{(1E/\rho D + e(T))}{e(Ts)} \] ..........................(8a)

because

\[ 1E = \rho D (\mu e(Ts) - e(T)) \] ..........................(8b)

where \( D \) is diffusion velocity \( (\text{cm/s}) \), \( T \), air temperature \( (\circ \text{C}) \), \( T_s \), surface temperature \( (\circ \text{C}) \), and \( e \), water vapor pressure \( (\text{mmHg}) \).

To simplify the analysis, the value of the diffusivity of water vapor in air, \( Kw \), is assumed to be equal to that of the thermal diffusivity of air, \( Ka \), and both can be expressed as \( K \). This assumption is the basis of the analysis of the heat balance method.

The specific humidity at the earth's surface can be expressed in terms of temperature (Laiktman, 1961; Uchijima, 1964).

\[ -\text{k}w \rho \frac{\partial q}{\partial h} = -k \rho \mu \Phi \frac{\partial \theta}{\partial h} \] ..........................(9)

where \( \Phi \) is the gradient of a line tangential to the
saturation vapor pressure-temperature curve, 
\( \Phi = \frac{dq(T_s)}{dT_s} \) (mmHg/°C).

Rearrangement of Eq. (6b) in the same way as that for the solution of Laiktman (1961)\(^{53}\), and substitution of Eq. (9) into Eq. (6b) gives:

\[-k_0C_p(1 + \frac{1}{C_p}) \frac{d\theta}{dh} - \lambda \frac{d\theta}{dz} = dS \cos 2\pi \tau_s \]  

We can rewrite Eq. (10a) into Eq. (10b), because 
\[-k = K/\rho C_p\]  

According to a brilliant solution for heat transfer by Oga (1931)\(^{52}\), heat flux into air, \(dQ_{\text{air}}\) and soil, \(dQ_{\text{soil}}\), which are the first and second terms of Eq. (10b), are given in the following equations:

\[dQ_{\text{air}} = -\sqrt{\lambda C_t} dS \sqrt{\frac{2\pi}{\tau_s}} \cos \left(2\pi \frac{\tau_s}{\tau_s} + \frac{\pi}{4}\right) dt \]  

\[dQ_{\text{soil}} = -\sqrt{\lambda C_t} dS \sqrt{\frac{2\pi}{\tau_s}} \cos \left(2\pi \frac{\tau_s}{\tau_s} + \frac{\pi}{4}\right) dt \]  

In these equations, a unit area is assumed to be 1cm\(^2\) and thus eliminated.

Each heat flux (Q) in a half cycle can be written as

\[Q_{\text{air}} = \int_0^{\tau_s} dQ_{\text{air}} = -\sqrt{\lambda C_t} dS \sqrt{\frac{2\pi}{\tau_s}} \cos \left(2\pi \frac{\tau_s}{\tau_s} + \frac{\pi}{4}\right) \]  

and

\[Q_{\text{soil}} = \int_0^{\tau_s} dQ_{\text{soil}} = -\sqrt{\lambda C_t} dS \sqrt{\frac{2\pi}{\tau_s}} \cos \left(2\pi \frac{\tau_s}{\tau_s} + \frac{\pi}{4}\right) \]  

where \(Q_{\text{air}}, Q_{\text{soil}}\) are heat flux into air and soil in a half cycle, respectively.

On the other hand, net radiation in a half cycle is obtained by integration of the absolute value of \(dS\) (cos \(2\pi \tau_s/\pi\)) on the right hand side of Eq (10b), giving \(dS \cdot (\tau_s/\pi)\).

Heat balance in a half cycle can be expressed by the following equation:

\[-Q_{\text{air}} - Q_{\text{soil}} = dS \frac{\tau_s}{\pi} \]  

Substitution of Eqs. (13) and (14) into Eq. (15) results in

\[-\sqrt{\lambda C_t} \int_0^{\tau_s} (1 + \frac{1}{C_p}) \frac{d\tau}{d\tau_s} \sqrt{\frac{2\pi}{\tau_s}} \cos \left(2\pi \frac{\tau_s}{\tau_s} + \frac{\pi}{4}\right) dt = dS \frac{\tau_s}{\pi} \]  

We can rewrite Eq. (16) as follows:

\[\sqrt{\lambda c_t} = \frac{dS}{dT_s \sqrt{2\pi}} - \left(\sqrt{\lambda} \mu C_p \left(1 + \frac{1}{C_p}\right) \right) \ldots (17)\]  

because \(K = k_0 \rho C_p\).

The units of the parameters are as follows:

\[\sqrt{\lambda c_t} \text{ cal/cm}^2 \text{ sec}^{-1/2} \text{C}, \ dS, \ \text{cal/cm}^2 \text{ sec}, \ dT_s, \ \text{C}, \ \tau_s \text{ sec}, \ k, \ \text{cm}^2/\text{sec}, \ \rho, \ 1.2 \times 10^{-3} \text{g/cm}^3, \ C_p, \ 0.24 \text{ cal/g} \text{C}, \ \mu, \ \text{absolute value}, \ I, \ 580 \text{ cal/g}, \ \Phi, \ \text{mmHg}/\text{C}.

The thermal inertia \(\sqrt{\lambda c_t}\) is the heat conductance capacity and has an inherent value for each material (Oga, 1931)\(^{52}\). Therefore, this controls heat diffusion in the soil. In the precise definition of thermal inertia, the second term on the right hand side of Eq (17) cannot be neglected. The parameter relating to the diffusion coefficient and the relative humidity at the soil surface is also calculated using heat balance terms.

(2) Soil physics

As described above, thermal inertia consists of thermal conductivity \(\lambda\), specific heat \(c\) and specific gravity \(r\). In soil, \(\lambda\) is a function of soil moisture and expressed as \(\lambda = f(WV)\). Multiplication of specific heat \(c\) by specific gravity \(r\) gives the volumetric heat capacity \(C_v\), which is also well known as a function of volume wetness \(WV\), and expressed as:\(^{57}^{58}\):

\[C_v = 0.46 f_m + 0.60 f_o + f_w \]  

\[C_v = 0.2 \nu + WV / 100 \]  

where \(C_v\) is volumetric heat capacity (cal/cm\(^3\)C), \(\nu\), bulk density (g/cm\(^3\)), and \(f_m, f_o, f_w\), the solid phase elements of soil such as mineral, organic matter and water.

Therefore, the parameter \(\sqrt{\lambda c_t}\) is a function of volume wetness (WV), and soil moisture can be estimated from a reversed function of the following form:

\[WV = f^{-1}(\sqrt{\lambda c_t}) \]  

Soil moisture influences the physical properties of soil such as thermal conductivity, thermal diffusivity, specific heat, heat capacity and evaporation. A linear relationship exists between evaporation and soil moisture below a critical value which
is about 60% of the field capacity (Uchijima, 1964). The daily range of Ts for wet soil is lower than that for dry soil because of the smaller heat capacity of the soil layer with only a small volume of water and the larger soil porosity. Therefore, thermal inertia at the soil surface based on the energy balance theory can be used to estimate soil moisture.

3. Thermal inertia from airborne MSS and ground truth data

(1) Determination of energy balance term

(a) Longwave radiation and effective radiation

The energy balance at the earth’s surface is also expressed as components of short and long wavelengths:

\[ (1 - \alpha) R + L = U + 1E + H + B \]  \quad (20)

Substitution of Eq. (1) into Eq. (20) yields the following Eq. (21):

\[ L = U + S - (1 - \alpha) R \]  \quad (21)

where S is net radiation, R global solar radiation, L downward longwave radiation from the sky, U upward longwave radiation from the soil surface, and \( \alpha \) albedo of the soil surface.

Meteorological elements were observed in and around the ground truth center. These were heat balance terms such as air temperature, relative humidity, wind direction, wind speed at levels of 0.5 and 1.5 m above the ground, global solar radiation, net radiation, albedo, air pressure at a level 1.5 m above the ground, surface temperature, and heat flow from a depth of 0.5 to 1.0 cm.

Longwave radiation from the earth’s surface (U) is calculated from Eq. (22):

\[ U = \delta \sigma (Ts + 273.15)^4 + (1 - \delta) L \]  \quad (22)

where Ts is surface temperature obtained using a hand-carried thermal IR radiometer and MSS, \( \delta \), emissivity of soil surface and \( \sigma \), the Stefan-Boltzmann constant. In this study, emissivity (\( \delta \)) was assumed to be 0.961. The second term on the right hand side of Eq. (22) can be disregarded because it is negligibly small when the emissivity \( \delta \) is larger than 0.95 (Uchijima, 1974). Therefore, the downward longwave radiation (L) at the ground truth center is obtained using Eq. (21). Then, the parameters U and L can be calculated easily for the ground truth center. Effective radiation (F) is calculated using Eq. (23):

\[ F = L - U \]  \quad (23)

An airborne sensor (MSS) facilitates the measurement of reflected spectral energy within a narrow wavelength and also the thermal IR radiation in each pixel. As described above, R and L are obtained using data from the ground truth center. Strictly speaking, the downward longwave radiation (L) varies according to location (pixel) because of the change in air temperature. However, the radiation can be applied in a narrow area such as the 512×512 pixels of airborne MSS data used in this study. Downward longwave radiation, thus obtained in a short time during MSS measurement, was substituted into Eq. (23).

(b) Albedo

Albedo in each pixel of the MSS is estimated from:

\[ \alpha' = f(\lambda_i) \]  \quad (24)

where \( \alpha' \) is the estimated albedo, and \( \lambda_i \) the reflectance in the visible wavelength band. The function f (x) is obtained by regression analysis using the albedo and CCT count from airborne MSS data. Albedo is observed using two solarimeters in and around the ground truth center.

(c) Net radiation

Net radiation (S) in each pixel of MSS data can be calculated from Eq. (25) by substituting the heat balance terms.

\[ S = (1 - \alpha') R + (-F) \]  \quad (25)

where the sign of the second term (F) of Eq. (25) is minus because the volume of longwave energy flux
from the earth's surface is larger than that of downward flux in the daytime, and the heat balance theory stipulates that the flux from the earth's surface has a minus sign.

d\(S\) and d\(T_s\) are estimated from Eqs.(26) and (27), respectively:
\[
dS = S_{\text{max}} - S_{\text{min}} \quad \text{(26)}
\]
\[
dTs = T_{S_{\text{max}}} - T_{S_{\text{min}}} \quad \text{(27)}
\]

where d\(S\) and d\(T_s\) are the daily range of net radiation and surface temperature, respectively. Each subscripted max and min indicates the maximum and minimum of \(S\) and \(T_s\). The minimum value was obtained in the early morning (6:15-6:22) and the maximum value was obtained at midday (12:58 -13:00).

(2) Normalized albedo and relative humidity

Albedo is defined as the ratio of reflected energy shorter than 3 \(\mu\)m to global solar radiation. Albedo was observed both in the field at the National Institute for Environmental Studies and on the Kujukuri coastal plain. Albedo is an important heat balance term, and it is also used to estimate the relative humidity of the soil surface. Figure 1 shows the relationship between albedo and total water storage near the earth's surface (depth of soil layer, about 1 cm) based on data obtained at the above locations. The incidence angle at noon during August in 1986 varied from 22 to 19 degrees. The angle at noon was 56 degrees on November 15, 1985.

It is well known that the albedo varies with time and season and with different incidence angles of solar radiation. Therefore, although the influence of season is unavoidable, the albedo was sampled at noon to avoid variations of time. It is also well known that albedo increases exponentially with a decrease of soil moisture (Maruyama, 1967). Excluding the part of the curve of high soil moisture, the curve in Fig. 1 agrees with the data of Maruyama.

In airborne remote sensing, relative humidity (\(\mu\)) at the soil surface cannot be calculated by the heat balance method because latent heat flux (IE) in Eq. (8) is an unknown quantity. Therefore, surface relative humidity (\(\mu\)) was substituted by estimated relative humidity (\(\mu'\)).

According to observations in the field, saturation of the soil took only 24 hours of rainfall, and total water storage was about 21.0 mm. Thus, soil moisture at field capacity for loamy soil is assumed to be over 21.0 mm.

The minimum albedo of the wettest soil, which is 21.0 mm at field capacity, was about 6%. Conversely, maximum albedo was approximately 24% in dry conditions, which is equivalent to a total water storage of 1.2 mm. The albedo was well correlated with soil moisture, as shown in Fig. 1. The estimated albedo (\(\alpha'\)) obtained using airborne MSS data is assumed to vary between the maximum and minimum values. The albedo (\(\alpha'\)) minus the value of minimum albedo (0.06) was divided by the value of the total variation or range of albedo (0.18). Thus, the normalized albedo of the soil surface varies from 0.0 to 1.0, and this can be substituted for soil surface relative humidity in airborne MSS measurements. Therefore, relative humidity of the soil surface can be estimated using Eq. (28):

![Figure 1](image-url)
where $\mu'$ is the estimated relative humidity of the soil surface, $\alpha'$, the albedo estimated from MSS data, $\alpha_{\text{min}}$, the minimum albedo of the wettest soil, and $\alpha_{\text{max}}$, the maximum albedo of the driest soil.

Thus, the estimated relative humidity ($\mu'$) at the soil surface varies from 0.0 under the driest conditions to 1.0 under the wettest. This parameter ($\mu'$) of the soil surface can be used in place of the relative humidity ($\mu$) based on the heat balance method in airborne MSS measurements.

(3) Diffusion velocity

Soil moisture is physically and closely correlated with thermal inertia ($I_{\text{cr}}$) as described the previous chapter. The diffusion coefficient ($K$) is an inherent parameter for the modeling, and controls heat flux to the air. It can be estimated from wind velocity at different heights.

The diffusion velocity, which has a similar effect on heat flux, is well known in heat balance theory. It is almost independent of atmospheric conditions such as stability or instability. Therefore, it is an adequate parameter for determining sensible and latent heat flux (Uchijima, 1974). Thus, diffusion velocity ($D$) between two heights, 10 and 110 cm above the ground, was used in place of the diffusion coefficient ($K$). The parameter ($D$) is calculated by the following well-known heat balance method:

$$D_{10-110\text{cm}} = \frac{1}{E} \left( \frac{0.622}{p} \frac{1}{\rho} \left( e_{10} - e_{110} \right) \right) \cdots (29)$$

$$D_{10-110\text{cm}} = \frac{H}{(C_p \rho \left( T_{10} - T_{110} \right))} \cdots (29')$$

As mentioned above, heat balance terms such as $dS$, $dT_s$, $D$ (in place of $K$), $L$, $U$, $F$, $\mu'$ and $q$ were obtained from the airborne MSS and ground truth measurements. Terms such as $dS$, $dT_s$, $\mu'$ and $q$ in each pixel can be easily substituted into Eq. (17). Then, thermal inertia in each pixel of the MSS data was determined using Eq. (17) and heat balance terms obtained from the airborne MSS and ground truth data.

Figure 2 Regression of thermal inertia and total water storage of sandy soil obtained from an experiment conducted in the Kujukuri coastal plain, central Japan.

(4) Thermal inertia model for estimation of soil moisture

The thermal inertia can be calculated using the heat balance terms at the ground truth center and from the airborne MSS data, as mentioned in the previous section. Figure 2 and Eq. (30) obtained from airborne MSS and ground truth measurements in the Kujukuri coastal plain show linear relationships. The thermal inertia model for estimating soil moisture is:

$$\text{Soil moisture (mm)} = -9.679 + 306.586 \frac{1}{I_{\text{cr}} \cdots (30)}$$

($R^2: 0.90$, standard error of the residuals: 2.9, $n=20$)

where soil moisture is expressed in terms of total water storage (mm), and $I_{\text{cr}}$, thermal inertia.

4. Evaluation of the Thermal Inertia Model

For thermal modeling, Watson (1975) developed a model of thermal inertia based on a one-dimensional heat transfer equation with periodic heating which is used in place of the soil heat flux term in the heat balance equation. His model was derived without knowledge of the amounts of both the latent and sensible heat fluxes, which depend on atmospheric conditions (Watson, 1975). Therefore, Watson's model overestimates the incident flux into the soil.
layer, and leaves no room for the relative humidity at the earth's surface.

Kahle(32,34,35) solved the heat transfer equation with heat balance terms such as latent heat, sensible heat and net radiation. Though the heat transfer equation was used to calculate the heat conduction into soil, it was solved numerically using the finite difference technique. As a correction parameter for numerical solving of latent heat flux, Kahle introduced the parameter of surface relative humidity (Kahle, 1977)(32). Later, in analysis of a desert area, lack of latent heat flux was assumed intuitively (Kahle, 1985)(35). This assumption led to incorrect estimation of thermal inertia.

Price's thermal inertia(40,41) was also based on the one-dimensional heat transfer equation which was used in place of the soil heat flux term. For simplification of the heat balance equation, lack of soil heat flux was assumed by averaging over 24 hours (Price, 1980)(41). This elimination of the soil heat flux term precedes that of the thermal inertia, since thermal inertia is a heat conductance parameter inherent in soil heat flux.

Thermal inertia was also estimated using a calibration chart derived from simulation results, and such simulation results for thermal properties of soil such as porosity and soil type were discussed previously for remote sensing (Pratt and Elyett, 1979)(38). For soil heat flux estimation, the heat balance equation was not solved analytically, but the iterative technique and a method for numerical solution of sensible and latent heat fluxes was introduced. In the numerical solution, some previous models included many input parameters, and some of them lacked essential parameters of the heat balance equation. In the analytical solution, the heat transfer equation was used in place of the soil heat flux term in heat balance equations, and the other heat balance terms were evaluated numerically(25,32,38). In some studies(31,38), albedo was handled simultaneously with thermal inertia. Thermal inertia is derived from calculation of heat balance parameters such as albedo, solar radiation and so on. In other words, the albedo, which determines the ratio of absorption of shortwave radiation, is one of the parameters governing the net radiation in equation (17). Therefore, it cannot be used simultaneously with thermal inertia. The use of albedo as a means of correcting thermal inertia shows that the previous thermal inertia models could not be solved by analytic examination.

In this study, all of the heat balance terms, such as latent heat $\mathbf{L}$, sensible heat $\mathbf{H}$ and soil heat flux $\mathbf{B}$ on the left hand side of Eq. (1) were analyzed by the heat transfer method, and thus a mathematical solution for evaluation of thermal inertia was developed by incorporating the heat transfer equation into the heat balance model(51)-55).

The concept of thermal inertia is well developed and widely accepted in heat transfer theory (Oga, 1931)(52). Laiktman (1961)(53) analyzed the relationships between daily amplitude of surface temperature and heat flux at the earth's surface based on both heat balance and heat transfer theories. However, he did not derive a solution using integration of heat flux in a half cycle. In the present study, an analytical solution of thermal inertia was obtained on the basis of both theories and integration in a half cycle, because of the brevity of the mathematical solution and the efficiency of the remote sensing experiment. Thermal inertia is a fundamental parameter and is basic for physical evaluation of the earth's surface.

This analytic solution requires neither the iteration method nor simplification of the heat balance model such as that involving elimination of the latent heat flux/soil heat flux term(25,32,38), nor statistical analysis (Wetzel et al., 1984)(41) etc.). From this viewpoint, the author's thermal inertia model derived by integration in a half cycle has several significant advantages. It can also eliminate the noise or outlier derived from measurement errors of practical remote sensing of soil moisture because of the large dynamic range of net radiation and surface temperature.

As for surface relative humidity, Nappo (1975)
clarified that his moisture availability term, improved from previous parameters of latent heat flux, had larger sensitivity to moisture variations\(^3\). Same parameters related to humidity were introduced into the estimation of latent heat flux (Kahle, 1977; Carlson et al., 1981\(^3\),\(^4\)). Carlson et al., (1981\(^4\)) also estimated sensible and latent heat fluxes, temperature and thermal inertia by adapting moisture availability in place of surface relative humidity. Thus, for precise estimation of latent heat flux, surface relative humidity should be included in the model. In the present study, it was also introduced for thermal inertia modeling.

As for the relationships between thermal inertia and soil moisture, the author confirmed the fundamental equation derived by previous investigators. Each of the soil components such as thermal conductivity, specific heat and specific gravity related to thermal inertia were clarified as a function of soil moisture. Thus, soil moisture can be derived from a reversed function of thermal inertia.

Some ambiguities are still present in this thermal inertia model, because soil heat properties such as soil type, porosity and mineral components vary with time and position. For example, the degree of porosity in soil changes abruptly after plowing even in the same field. Usually, accurate information on soil physical parameters can be obtained not by remote sensing but by field observation at each point and area.

As mentioned above, some ambiguities such as overestimation of latent heat, assumption of lack of soil heat flux or lack of latent heat flux were eliminated in the present model. As this model is thus fundamental for thermal inertia modeling, it can be applied to soil moisture remote sensing by adapting other subjective parameters such as atmospheric effects, incident light angle, surface inclination and soil physical parameters such as porosity, ratio of mineral components in soil and specific heat.

Restriction on the depth of soil moisture sensing based on thermal inertia is obvious in the solution of heat transfer equation of the earth's crust. The daily amplitude of soil temperature at a depth of 37 cm is negligibly small (5 % of surface temperature variation), and that at a depth of 57 cm is only 1 % (Oga, 1931\(^5\)). An analogy for the soil layer can be recognized from the results of measurements of soil temperature. Therefore, thermal inertia can be used to estimate soil moisture in a shallow soil layer and root zone of about 15 cm. After solution of the thermal inertia model, the author expanded the model to airborne remote sensing for soil moisture evaluation. For this expansion, a procedure for estimation and normalization of albedo, and its application for estimation of surface relative humidity were proposed. An equation for the evaluation of soil moisture was developed using thermal inertia derived from airborne MSS and ground truth data for several points in the Kujukuri coastal plain. Some variables in the equation still need to be confirmed experimentally using further airborne and satellite remote sensing data and ground truth data. In addition, repeated measurements of thermal inertia are expected to yield many fundamental data under varying conditions of soil moisture for different kinds of soil in the laboratory.

### 5. Conclusion

The author has examined analytically a thermal model of the earth's surface, and constructed a fundamental thermal inertia model for soil moisture sensing. Since the model was derived by introducing the relative humidity of the soil surface, overestimation of latent heat flux is avoided in this model.

The advantage of this thermal inertia model is that it is based on physical parameters such as heat transfer and heat balance, and mathematical solutions. After derivation of the model it was applied practically to airborne MSS experiments centered on the Kujukuri coastal plain, central Japan. This model is a fundamental one, which may be improved for practical airborne and spaceborne remote sensing.
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