A technique to analyse scattered waves from forest fire scars and its application to estimate its scars thickness in central Borneo using a SAR data

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1. Introduction and study area

A simple analysis technique to solve scattered waves from forest fire scars using the transmission line method was developed by Tetuko (2002), Tetuko et al. (2003a) and Tetuko et al. (2003b), where the roughness of targeted surface was ignored. In these reports, the scattered wave was modelled by using a simple two-dimensional three layers of flat medium. This model is almost different to the real condition of study area because the soil surface actually has various roughness types. Therefore, in this research, the scattered wave from the three layers of medium with two rough surfaces is developed to obtain the relationship between the medium thickness and the backscattering coefficient. One of the application of this technique is to estimate the thickness of forest fire scars with the study area as explained below.

The study area is the district of south Barito and Kapuas, central Borneo (Kalimantan), Indonesia (114°23'–114°45'E, 2°23'–2°47'S) where a huge forest fire occurred in this area from June to September 1997 (see figure 2). The ground data that included the vegetation types, the soil types and their conditions, was collected from 1995 to 1997. The main vegetation in this area is Tengkawang (Dipterocarp-
paceae spp.), and purun grass (see figure 1 (B)) that find in peat swamp area. Figure 1 (A) and (C) show the main vegetation found at the study areas before and after the forest fires. Especially figure 1 (C) shows the conditions of forest fire scars in the study area.

The altitude of this area is ranging from 9m to 14m above the sea level or relatively flat surface from north to south area (Java sea). The Barito river (east) and the Kapuas river (west) surround this area, and the vegetation type of this area is a tropical forest. This area is mainly covered by peatland (soil with about 60% of coal content) and peat swamp (peatland area with high water content) (Nuraini 1999, Hadi et al. 2000a, Hadi et al. 2000b, Sarbini 2000, Radjagukguk 1997). The climate on Borneo is wet through the whole year with an average annual rainfall of around 3500mm to 4500mm, while the relative humidity varies between 70% and 90%.

Figure 2 shows the SPOT HRV images of fire events in the study area prior fires (6 June 1997), during the fires (29 July 1997 and 7 August 1997) and post fires (8 September 1997). This area called ‘One million hectares peatland project’ district A (PLG-A) at central Borneo, Indonesia (CSAR 1997, Sarbini 2000, Radjagukguk 1997) was opened by the Indonesian government as agricultural sites. The artificial forest fires spots can be seen in the figures 2 (B) and (C), with the fire haze of these fire spots widespread to the Asian countries. Figure 3 shows the distribution of forest fire scars thickness in the study area collected in the ground survey from 1995 to 1997.
The acquired ground data shows that the forest fire scars surface in the study area had standard deviation $\delta$ of roughness about 0.3m (Nuraini 1999). It means $\delta$ is larger than wavelength of JERS-1 SAR that was operating at 1.275GHz. Thus the assumption in this paper could be simplified using the stationary-phase approximation (Ulaby et al. 1986). Here the method is applied to obtain analytical solution of scattered wave from two layers of rough surfaces as shown in figure 4, where the burnt tree trunk is not considered in this model.

In section 2, firstly, the numerical analysis of scattered waves from the rough forest fire scars is discussed. Then the confirmation of the analysis result by comparing the result of the proposed method with that of previous simple analysis is discussed in section 3. Section 4 shows the application of the proposed analysis to estimate the thickness of forest fire scars in the study area using JERS-1 SAR data. Finally, conclusions are made in section 5.
2. Analysis

Figure 4 and figure 5 show the geometry of the scattered wave analysis and the flow-chart of the fields calculation in each medium shown in figure 4, respectively. The geometry of the scattered wave analysis show that the analysis model is composed by three media; medium 1 (air), medium 2 (forest fire scars) and medium 3 (peatland). The flow-chart shows the flow of fields calculation in each medium, where the number in the brackets indicates the
section in this paper, \( n \) is the loop number necessary to find the optimum value in analysis, \( p \) and \( q \) are the polarization symbols (horizontal \( h \) or vertical \( v \)).

Here the Kirchhoff or Physical Optics formulation is applicable to surfaces with gentle undulations where the average horizontal dimension is large compared with the incident wave (Ulaby et al. 1986). As a result, it is assumed that the total field at any point on the surface can be computed as if the incident wave is impinging upon an infinite plane tangent to the point. The vector formulation of the Kirchhoff method is based upon the vector second Green’s theorem, which states that the scattered field at any point within a source-free region bounded by a closed surface can be expressed in terms of tangential fields on the surface (Fung et al. 1992). A mathematical statement of this fact formulated by Stratton and Chu (Stratton 1941) and modified for the far zone by Silver (Silver et al. 1947) is expressed by

\[
E^s = -\nabla \times (\hat{n} \times E - \eta \hat{n} \times (\hat{n} \times H))e^{jkx \cdot x}dS
\]  

where a time factor of the form \( e^{j\omega t} \) is understood and

\[
K = -jk\varepsilon_0 \varepsilon / (4\pi R) \quad \hat{n}_s, \hat{n}, \eta, k_s, R, E, \text{ and } H
\]

are unit vector in the scattered direction, unit vector normal to the interface inside the medium in which scattering is considered, intrinsic impedance of the medium in which \( E^s \) is evaluated, wavenumber of the medium in which \( E^s \) is evaluated, range from the centre of the illuminated area to the point of observation, total electric and magnetic fields on the interface, respectively. In a local frame of reference, the tangential fields \( \hat{n} \times E \) and \( \hat{n} \times H \) are calculated to compute the scattered field \( E^s \).

The total electric and magnetic tangential fields are obtained as (Tetuko 2002)

\[
\hat{n}_i \times E = (1 + R_i)(\hat{a} \cdot \hat{i})(\hat{n} \times \hat{i})
\]

\[
- (1 - R_i)(\hat{n} \cdot \hat{n}_i)(\hat{a} \cdot \hat{d})\hat{i} E_o
\]

\[
\eta_i(\hat{n}_i \times H) = - (1 - R_i)(\hat{n} \cdot \hat{n}_i)(\hat{a} \cdot \hat{i})\hat{i}
\]

\[
+ (1 + R_i)(\hat{a} \cdot \hat{d})(\hat{n} \times \hat{i})E_o
\]

where \( \hat{a}, \hat{n}_i, \text{ and } k_i \) are unit polarization vector, unit vector in the incident direction, and wavenumber in medium 1 (air), respectively. \( \hat{d} \) and \( \hat{i} \) are the unit vector for vertically and horizontally polarization, respectively. \( \hat{n}_s, \eta, \hat{n}_r \) are unit normal vector to the interface in medium 1, the intrinsic impedance of medium 1, the unit vector in the reflected field.
Figure 5 Flow-chart that shows the flow of field calculation in each medium.

direction, respectively. \( R_1 \) and \( R_0 \) are the Fresnel reflection coefficient for horizontal and vertical polarization, respectively. Due to the continuity of the tangential fields at a surface boundary, the scattered field in either medium 1 or medium 2 can be computed in terms of (2) and (3). In the same way, the tangential fields in terms of medium 2 parameters are obtained as

\[
\sum_{m} E_{m}^{s} = \text{Finish}
\]

\[
\text{Initial parameters } m = 0
\]

Fields on interface 1

Fields in medium 1 (2.1.1) \* \( E_{11}^{s} \)

Fields in medium 2 (2.1.2) \* \( E_{21}^{s} \)

Fields on interface 2 (2.2) \* \( E_{22}^{s} \)

Fields in medium 1 (2.3.2) \* \( E_{12}^{s} \)

Fields in medium 2 (2.3.1) \* \( E_{21}^{s} \)

Yes \( m \leq n \) No

\[
\sigma^{e} = \text{Summed over illuminated area with phase factor, exp}\left(-jk_{1}n_{1}\eta_{1}\varphi_{s1}\right)
\]

\[
\eta_{1}(\hat{n}_{1}\times E) + T_{1}(\hat{n}_{1}\times \hat{E}) = \left(T_{1}\hat{n}_{1}\times \hat{E}\right)_{\eta_{1}/\eta_{1}} E_{0}
\]

\[
\text{where } \hat{n}_{1} \text{ is the normal in medium 2. } \hat{n}_{1} \text{ is the unit vector in the transmitted field direction. } \eta_{1} \text{ is the intrinsic impedance of medium 2, and } T_{1}, T_{0} \text{ are the Fresnel transmission coefficient for horizontal and vertical polarizations, respectively, with } T_{1} = 1 + R_{1} \text{ and } T_{0} = 1 + R_{0}. \text{ Finally, we obtained (2) and (3) and (5) to calculate the scattered fields in medium 1 and 2, respectively.}
\]

2.1 Scattered fields on forest fire scars surface (1) or interface 1

2.1.1 Scattering field in medium 1

The scattered fields in medium 1 (air) is obtained by substituting (2) and (3) into (1) as

\[
1E_{1}^{s} = 1K_{1}\hat{n}_{1}\times [\hat{n}_{1}\times E - \eta_{1}\hat{n}_{1}\times (\hat{n}_{1}\times H)] \exp\left[jk_{1}(\hat{n}_{1}\times \hat{r})\cdot \hat{r}'\right] dS'
\]

where \( 1K_{1} = -jk_{1}\exp(-jk_{1}R)/(4\pi R) \) and \( R \) is the range from the centre of the illuminated area to the point of observation. Where \( \hat{n}_{1} = \vec{x}\sin\theta_{1}\cos\phi_{1} + \vec{y}\sin\theta_{1}\sin\phi_{1} - \vec{z}\cos\theta_{1} \) and \( \hat{n}_{s1} = \vec{x}\sin\theta_{s1}\cos\phi_{s1} + \vec{y}\sin\theta_{s1}\sin\phi_{s1} + \vec{z}\cos\theta_{s1} \). \( \theta_{1} \) and \( \theta_{s1} \) are the incident and scattered angles on (1) in figure 4, respectively. (6) shows that surface fields can be summed over the illuminated area with the phase factor, exp\(-jk_{1}\hat{n}_{1}\cdot \hat{r}'\), of the incident wave. The next approximation is considered to find the analytical solution of (6). It means that scattering can occur only along directions for which there are specular points on the surface. The approximation relations are obtained from the phase \( Q \) of (6), i.e.,

\[
Q = k_{1}(\hat{n}_{1} \cdot \hat{r} - \hat{n}_{1} \cdot \hat{r}') = q_{1} + q_{2} + q_{3} + q_{4}
\]

where \( q_{1} = k_{1}((\sin\theta_{s1}\cos\phi_{s1} - \sin\theta_{1}\cos\phi_{1}), q_{2} = k_{1}((\sin\theta_{s1}\sin\phi_{s1} - \sin\theta_{1}\sin\phi_{1}), q_{3} = k_{1}(\cos\theta_{s1} - \cos\theta_{1}). \) The phase \( Q \) is said to be stationary at a point if its rate of change is zero at the point. Hence, the partial derivations of the surface slopes can be replaced by the components of the phase as \( \partial Q/\partial x = -q_{1}/q_{4} \) and \( \partial Q/\partial y = -q_{3}/q_{4}. \) By substituting the components into (6), the expression for \( 1E_{1}^{s} \) can be rewritten as

\[
1E_{1}^{s} = 1K_{1}\hat{n}_{1}\times [\hat{n}_{1}\times E - \eta_{1}\hat{n}_{1}\times (\hat{n}_{1}\times H)] I_{1}
\]

where \( I_{1} = \int \exp(jk_{0}(\hat{n}_{1} - \hat{n}_{s1})\cdot r') dS' \). Let \( \vec{e} \) and \( \hat{h} \) be the unit polarization vectors for the incident vertical and horizontal waves respectively. Let \( \vec{v}_{s1} \) and \( \hat{h}_{s1} \) be the corresponding polarization vectors for the scattered waves that are chosen to coincide with \( \hat{h} \) and \( \vec{e} \) in a standard spherical coordinate system. Thus \( \hat{h}_{s1} \) and \( \vec{v}_{s1} \) are the horizontal and vertically polarised unit vector are \( \hat{h}_{s1} = -\hat{x}\sin\phi_{1} + \hat{y}\cos\phi_{1} \) and \( \vec{v}_{s1} = \theta = \hat{h}_{s1} \times \hat{n}_{s1} = \hat{x}\cos\theta_{s1}\cos\phi_{s1} + \hat{y}\cos\theta_{s1}\sin\phi_{s1} - \hat{z}\sin\theta_{s1} \).
where \( \mathbf{E}\cdot \mathbf{n} \). When the incident wave is vertically polarized, the scattered fields are obtained from (9) and (10) by an interchange of \( \mathbf{E}\) with \( \mathbf{H}\) and \( \mathbf{H}\) with \( \mathbf{E}\):

\[
\mathbf{E}_{\text{scrv}}^z = \mathbf{E}_{\text{scrv}}^z \quad \text{and} \quad \mathbf{H}_{\text{scrv}}^z = \mathbf{H}_{\text{scrv}}^z. \tag{11}
\]

2.1.2 Scattering field in medium 2

In the same way, the scattered field in medium 2 (forest fire scars) is obtained by substituting (4) and (5) into (1):

\[
\mathbf{E}_{\text{scrv}}^z = \mathbf{E}_{\text{scrv}}^z \times (\mathbf{n}\times \mathbf{E} - \eta_2 \mathbf{n}\times (\mathbf{n}\times \mathbf{H})) \exp \left( -j k_2 z - j \mathbf{q}_{\text{scrv}} \cdot \mathbf{n}\right) dS, \tag{13}
\]

where \( k_2 = -j k_2 \exp(-j k_2 R_1)/(4\pi R_1) \) and \( \xi \) is the thickness of forest fire scars. \( \mathbf{q}_{\text{scrv}} = \mathbf{\nabla}\times \mathbf{D}_{\text{scrv}} \) in the stationary-phase approximation in (13) with \( Q = (k_2 \mathbf{n}\times \mathbf{E} - k_2 \mathbf{n}\times \mathbf{H}) \). Thus we obtained \( \mathbf{q}_{\text{scrv}} = k_2 \sin \theta_2 \cos \phi_2 \mathbf{E}_{\text{scrv}}^z - k_2 \sin \theta_2 \sin \phi_2 \mathbf{H}_{\text{scrv}}^z - \cos \theta_2 \mathbf{B}_{\text{scrv}}^z \). Then the surface slopes can be replaced by \( \partial \mathbf{q}_{\text{scrv}}/\partial x' = -\partial \mathbf{q}_{\text{scrv}}/\partial y' = 0 \). By this replacement, \( \mathbf{n}\times \mathbf{H} \) and \( \mathbf{n}\times \mathbf{E} \) become independent to the integration variables and (13) can be rewritten as

\[
\mathbf{E}_{\text{scrv}}^z = \mathbf{E}_{\text{scrv}}^z \times (\mathbf{n}\times \mathbf{E} - \eta_2 \mathbf{n}\times (\mathbf{n}\times \mathbf{H})) \exp \left( -j k_2 z - j \mathbf{q}_{\text{scrv}} \cdot \mathbf{n}\right) dS. \tag{15}
\]

2.2 Scattering field on peatland surface or interface 2

To solve the problem of scattering from a perfectly conducting random surface, first, the determination of the surface current is done. To simplify the calculation, the surface current estimation is shown as (Gotoh et al. 1993)

\[
\mathbf{J}(r) = \mathbf{n}\times \mathbf{H}^m, \tag{20}
\]

where \( m = 2, 4, 6, \ldots \). \( \mathbf{n}\) is the unit normal vector to the surface of medium 3 or perfectly conducting medium (see figure 2), \( \mathbf{H}^m \) is the incident magnetic field. The integration is over the illuminated area of the surface. To obtain an expression for the surface current, consider incident horizontally and vertically polarized fields of the form

\[
\mathbf{E}^m = \mathbf{y}^m \mathbf{E}^m_{\text{exp}}(\mathbf{r}) = \mathbf{y} \mathbf{E}^m_{\text{exp}}(\mathbf{r}) \exp(-j k_2 x \cos \theta_m - k_2 z \sin \theta_m), \tag{21}
\]

where \( \mathbf{y} \) is the horizontal or vertical magnetic field at the surface of medium 3, \( \mathbf{y} \) is the unit vector to the surface of medium 3 or perfectly conducting medium (see figure 2), \( \mathbf{H}^m = \mathbf{E}^m_{\text{exp}} - j \mathbf{k} \times \mathbf{E}^m_{\text{exp}} \) is the incident magnetic field. To obtain an expression for the surface current, consider incident horizontally and vertically polarized fields of the form

\[
\mathbf{E}^m = \mathbf{y}^m \mathbf{E}^m_{\text{exp}}(\mathbf{r}) = \mathbf{y} \mathbf{E}^m_{\text{exp}}(\mathbf{r}) \exp(-j k_2 x \cos \theta_m - j k_2 z \sin \theta_m). \tag{22}
\]

Thus, the partial derivatives of the surface of medium 3 are obtained from the scattered fields in medium 2, refer to (16) and (17). \( \theta_m \) is the angle of incidence, \( k_2 \) is the wavenumber in medium 2 or forest fire scars, and \( \mathbf{z} \) is the magnetic field at the surface of medium 3 or perfectly conducting medium. Next, let \( \mathbf{n}\times \mathbf{H} \) and \( \mathbf{n}\times \mathbf{E} \) become independent to the integration variables and (13) can be rewritten as

\[
\mathbf{E}_{\text{scrv}}^z = \mathbf{E}_{\text{scrv}}^z \times (\mathbf{n}\times \mathbf{E} - \eta_2 \mathbf{n}\times (\mathbf{n}\times \mathbf{H})) \exp \left( -j k_2 z - j \mathbf{q}_{\text{scrv}} \cdot \mathbf{n}\right) dS. \tag{15}
\]

\[
\mathbf{J}(r) = \mathbf{n}\times \mathbf{H}^m. \tag{20}
\]

It is effectively the Kirchhoff approximation to the surface-current density (Gotoh et al. 1993). The expression for the surface current density for horizontal polarisation is

\[
\mathbf{J}_h = \mathbf{J}_h = \mathbf{E}^m_{\text{exp}} \times \mathbf{z} \left[ -Z_x \sin \theta_m Z_x \sin \theta_m + \cos \theta_m Z_y \cos \theta_m \right] \exp(-j k_2 x \cos \theta_m - j k_2 z \cos \theta_m) \tag{24}
\]

and for vertical polarisation is

\[
\mathbf{J}_v = \mathbf{J}_v = \mathbf{E}^m_{\text{exp}} \times \mathbf{z} \left[ -Z_x \sin \theta_m Z_x \sin \theta_m + \cos \theta_m Z_y \cos \theta_m \right] \exp(-j k_2 x \cos \theta_m - j k_2 z \cos \theta_m) \tag{24}
\]
In accordance with the Stratton-Chu integral (Stratton 1941), the far-zone scattered fields from medium 3 is shown as

\[ nE_{3m} = -C_m \eta_3 \hat{n}_{3m} \times \int_{A_0} J_{3m} \exp(\imath k_3 \hat{n}_{3m} \cdot r) \, dx \, dy \tag{26} \]

where \( C_m = (j \kappa_2 / 4 \pi R_m) \exp(-j k_2 R_m) \) and \( R_m = \xi \sec \theta_m \). \( \xi \) is the thickness of forest fire scars that is measured from forest fire scars surface \((z=0)\) to depth of a perfectly conducting medium \((z=R_m)\). \( \hat{n}_{3m} \) is the unit vector pointing in the direction of observation, \( A_0 \) is the illuminated area, \( \eta_3 \) is the intrinsic impedance of the forest fire scars, \( J_3 \) is either \( J_h \) or \( J_v \), and \( k_2 \) is the wave number in medium 2. That can be rewritten as

\[ nE_{3m} = -C_m \eta_3 \int_{A_0} \exp(-j k_{3m} x - j k_{3m} z) \, dx \, dy \tag{27} \]

For horizontal polarisation,

\[ nE_{3v} = \tilde{y} \cdot nE_{3h} \tag{28} \]

Integrating by parts in (28) and ignoring the edge effects, we obtain

\[ nE_{3h} = C_m \eta_3 \sec \theta_m \int_{A_0} \exp(-2j k_{3m} x - 2j k_{3m} z) \, dx \, dy \tag{29} \]

Similarly, we can show that for the vertically polarised case the scattered field is

\[ nE_{3v} = C_m \eta_3 \sec \theta_m \int_{A_0} \exp(-2j k_{3m} x - 2j k_{3m} z) \, dx \, dy \tag{30} \]

Upon comparing (29) and (30), these equations are similar, it is indicating that for a perfectly conducting surface, there is no polarisation difference between the vertically and horizontally polarised fields. The cross-polarised scattered fields \( nE_{3v} \) and \( nE_{3h} \) are computed by taking \( \tilde{y} \cdot nE_{3i} \) and \( \sec \theta_m \hat{x} \cdot nE_{3i} \) respectively, where respectively \( J_3 \) and \( J_4 \) are used in the field expression to obtain the fields. Hence \( nE_{3h} = nE_{3v} = 0 \) is obtained. Now the polarised scattered fields were obtained and will be used to calculate the scattered fields at the boundary of medium 1 and 2 or interface 1 with distance \( R_m \) from the surface of perfectly conducting medium as discussed in the next section.

### 2.3 Scattering field on forest fire scars surface (2)

Or interface 1

#### 2.3.1 Scattering field in medium 2

The far-zone scattered fields in medium 2 can be shown as

\[ nE_{2m} = nE_{2i} \hat{n}_{2m} \times \int_{A_0} \hat{n}_{2m} \times \hat{E}_{4m} \, dx \, dy \tag{31} \]

where \( nE_{2m} = \hat{n}_{2m} \times \hat{E}_{4m}(\hat{n}_{2m} \times \hat{H}) \),

\[ \exp(\imath k_2 (\hat{n}_{2m} - \hat{n}_{4m}) \cdot r') \, ds' \tag{32} \]

where \( K_2 = -\imath \kappa_2 \exp(-\imath k_2 R_m) / (4 \pi R_m) \), \( m = 3, 5, \ldots \),

\( \hat{n}_{2m} = \hat{x} \sin \theta_m \cos \phi_{4m} + \hat{y} \sin \theta_m \sin \phi_{4m} - \hat{z} \cos \theta_m \), and

\( \hat{n}_{4m} = \hat{x} \sin \theta_m \cos \phi_{4m} + \hat{y} \sin \theta_m \sin \phi_{4m} + \hat{z} \cos \theta_m \). Similarly with the previous derivation in section 2.1.1 and 2.1.2, we must simplify (31) using the stationary-phase approximation, where phase \( Q \) of (31):

\[ Q = k_2 (\hat{n}_{2m} - \hat{n}_{4m}) \cdot r' = \hat{q}_{2m} x - \hat{q}_{2m} y + \hat{q}_{2m} z \tag{33} \]

where \( \hat{q}_{2m} = k_2 (\hat{x} \sin \theta_m \cos \phi_{4m} - \hat{z} \cos \theta_m) \), \( \hat{q}_{4m} = k_2 (\hat{x} \sin \theta_m \sin \phi_{4m} - \hat{z} \cos \theta_m) \), and \( \hat{q}_{2m} = -k_2 (\hat{z} \cos \theta_m + \hat{z} \cos \theta_m) \). By considering the stationary condition of the phase \( Q \), the partial derivatives of the surface slopes can be replaced by the components of the phase as \( \partial x' / \partial x = -q_{2m} / q_{2m} \) and \( \partial y' / \partial y = -q_{2m} / q_{2m} \). Then the expression for \( nE_{2m} \) can be rewritten under such approximation as

\[ nE_{2m} = K_2 \hat{n}_{2m} \times (\hat{n}_4 \times \hat{E} - \hat{n}_2 \hat{n}_{2m} \times (\hat{n}_4 \times \hat{H})) \tag{34} \]

where \( I_2 = \int \exp(\imath k_2 (\hat{n}_{2m} - \hat{n}_{4m}) \cdot r') \, ds' \). Let define \( \hat{v}_m \), \( \hat{h}_m \), and \( \hat{v}_m \), \( \hat{h}_m \) as the unit polarisation vectors for the incident and scattered vertical and horizontal wave respectively as \( \hat{v}_m = \hat{x} \sin \phi_{4m} + \hat{y} \cos \phi_{4m} \), \( \hat{v}_m = \theta_m = -(\hat{x} \cos \theta_m \cos \phi_{4m} + \hat{y} \cos \theta_m \sin \phi_{4m} + \hat{z} \sin \theta_m) \), \( \hat{h}_m = \hat{v}_m = \hat{x} \sin \phi_{4m} + \hat{y} \cos \phi_{4m} \), and \( \hat{v}_m = \hat{x} \cos \theta_m \cos \phi_{4m} + \hat{y} \cos \theta_m \sin \phi_{4m} - \hat{z} \sin \theta_m \). The scattered polarised and depolarised fields are obtained

\[ nE_{2v} = nE_{2h} = nE_{2v} = nE_{2h} = 0 \tag{35} \]

Where \( nE_{2v} = nE_{2h} = nE_{2v} = nE_{2h} = 0 \). When the incident wave is vertical
cally polarised, the scattered fields can be obtained from (34) and (35) by an interchange of $\hat{v}_m$ with $\hat{h}_m$ and $\hat{v}_m$ with $\hat{h}_m$:

\[ nE_{\varphi \varphi} = nM_2(R_1(\hat{v}_m \times \hat{n}_m))(\hat{v}_m \times \hat{n}_m) \]

\[ + R_1(\hat{h}_m \times \hat{n}_m)(\hat{h}_m \times \hat{n}_m) = nK_2 n_1^{\text{m}} E_{\varphi \varphi} D_{\text{vuv}} \]  

(36)

\[ nE_{\varphi h} = nM_2(R_1(\hat{v}_m \times \hat{n}_m))(\hat{v}_m \times \hat{n}_m) \]

\[ - R_1(\hat{v}_m \times \hat{n}_m)(\hat{h}_m \times \hat{n}_m) = nK_2 n_1^{\text{m}} E_{\varphi h} D_{\text{vuv}} = 0 \]  

(37)

2.3.2 Scattering field in medium 1

In the same way with the section above, the far zone scattered fields in medium 1 (air) is obtained

\[ nE_s = nK_1 n_m \times \langle (\hat{n}_i \times E - n_t \hat{n}_s m \times (\hat{n}_i \times H)) \rangle \exp(j(k_i n_m - k_m n_s) \cdot r') \]  

\[ \exp(j(k_i n_m - k_m n_s)) \]  

(38)

where $m = 3, 5, \cdots$ and $n_s m = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi$. By applying the stationary-phase approximation in (38) with

\[ Q = (k_i n_m - k_m n_s) \cdot r' = q_{x zm} x' + q_{y zm} y' + q_{z zm} z' \]  

where $q_{x zm} = k \sin \theta \cos \phi, q_{y zm} = k \sin \theta \sin \phi, q_{z zm} = k \cos \theta$. Then the surface slopes can be replaced by $\partial \alpha/\partial x = -q_{x zm} q_{zm}$ and $\partial \alpha/\partial y = -q_{y zm} q_{zm}$. Then the expression for $nE_s$ can be rewritten under the approximation as

\[ nE_s = nK_1 n_m \times \langle (\hat{n}_i \times E - n_t \hat{n}_s m \times (\hat{n}_i \times H)) \rangle n_1 \]  

(40)

where $n_1 = \exp(j(k_i n_m - k_m n_s) \cdot r')$.

Let we define $\hat{v}_m$, $\hat{h}_m$ and $\hat{v}_m$, $\hat{h}_m$ as the unit polarisation vectors for the incident and scattered vertical and horizontal wave respectively.

\[ nE_{s h} = -nM_1(T_1(\hat{h}_m \times \hat{n}_m))(\hat{h}_m \times \hat{n}_m) \]

\[ + T_1(\hat{h}_m \times \hat{n}_m)(\hat{n}_m \times \hat{n}_m) \]

\[ = nK_1 n_1^{\text{m}} E_{s h} U_{\text{vuv}} \]  

(41)

\[ nE_{s h} = nM_1(T_1(\hat{v}_m \times \hat{n}_m))(\hat{v}_m \times \hat{n}_m) \]

\[ - T_1(\hat{v}_m \times \hat{n}_m)(\hat{n}_m \times \hat{n}_m) \]

\[ = nK_1 n_1^{\text{m}} E_{s h} U_{\text{vuv}} \]  

(42)

\[ nE_{s h} = -nM_1(T_1(\hat{h}_m \times \hat{n}_m))(\hat{h}_m \times \hat{n}_m) \]

\[ + T_1(\hat{h}_m \times \hat{n}_m)(\hat{n}_m \times \hat{n}_m) \]

\[ = nK_1 n_1^{\text{m}} E_{s h} U_{\text{vuv}} \]  

(43)

\[ nE_{s h} = nM_1(T_1(\hat{v}_m \times \hat{n}_m))(\hat{v}_m \times \hat{n}_m) \]

\[ - T_1(\hat{v}_m \times \hat{n}_m)(\hat{n}_m \times \hat{n}_m) \]

\[ = nK_1 n_1^{\text{m}} E_{s h} U_{\text{vuv}} \]  

(44)

where $\hat{v}_m = \hat{x} \cos \theta \sin \phi + \hat{y} \cos \theta \sin \phi + \hat{z} \cos \theta$, $\hat{h}_m = \hat{x} \sin \phi \cos \theta + \hat{y} \sin \phi \cos \theta + \hat{z} \cos \theta$, $n_1 = nK_1 n_1^{\text{m}} E_{\varphi h} (k_i - k_m n_s m) / (q_m D_{mv} D_m)$, $q_m = q_{zm}$.

+ $q_{zm}$, $D_m = q_m D_m / |q_m|$, and $D_m = |\hat{n}_m \times \hat{n}_s m|$. Now that all the field expressions are available, the average scattered power and scattering coefficient are ready to be computed.

2.4 Scattering coefficient

By derivation of scattered wave in medium 1 and 2 (see figure 4 and figure 5), the total scattered field in medium 1 (air) is defined by their summation and is shown as

\[ E_{\varphi h} = E_{\varphi h 1} + E_{\varphi h 2} + \cdots = \sum_{n=0}^{\infty} E_{\varphi h n} \]  

(45)

Hence, mathematically the scattering coefficient in medium 1 (air) can be written as

\[ \sigma_{\varphi h} = \frac{4 \pi R^2}{\lambda} \frac{\text{Re}(\langle |E_{\varphi h}^2| \rangle)}{\text{Re}(\langle |E_\varphi^2| \rangle)} \approx \frac{4 \pi R^2}{\lambda} \text{Re}(\langle \langle |E_{\varphi h}^2| \rangle \rangle) / \text{Re}(\langle |E_\varphi^2| \rangle) \]  

(46)

where $A_o$ is the illuminated area, $R$ is the distance from the point of observation to the center of $A_o$, $\text{Re}(\cdots)$ is the real part operator, and $\langle \cdots \rangle$ is the symbol for the ensemble average. To compute $\sigma_{\varphi h}$, for different polarization states, it is necessary to calculate the ensemble average of $|E_{\varphi h}^2|$. For example, the first term in (46), since the two integrals are similar, it is sufficient to show the computation of $|E_{\varphi h}^2|$ for a Gaussian-distributed random surface with surface height distribution of medium 2 or forest fire scars:

\[ \rho(z) = (2 \pi \sigma^2)^{-1/2} \exp(-z^2/2 \sigma^2) \]  

(47)

where $\sigma^2$ is the variance of surface heights of medium 2 or forest fire scars. We have

\[ \langle |E_{\varphi h}^2| \rangle = \int \int \langle \text{exp}(j \theta_1 (\hat{n}_1 - \hat{n}_2) \cdot (\hat{r}' - \hat{r})) \rangle \times \text{Re}(\langle |E_{\varphi h}^2| \rangle) / \text{Re}(\langle |E_{\varphi h}^2| \rangle) \]  

(48)

to express (48) in the rectangular coordinates shown in figure 4, note that

\[ dS' = dx' dy'/(|\hat{n}_1 \cdot \hat{z}|) = q_{1x} dx' dy' / |q_{2x}| \]  

(49)

Hence

\[ \langle |E_{\varphi h}^2| \rangle = \frac{q_{2x}^2}{q_{2x}} \int \int \text{exp}(j \theta_1 (x' - x))^2 + j \theta_1 (y' - y) \times \text{Re}(\langle |E_{\varphi h}^2| \rangle) \]  

(50)
The factor $<\cdots>$ in (50) is recognized as the joint characteristic function of $z(x', y')$ and $z(x'', y'')$. By assuming $z(x, y)$ to be a stationary Gaussian random process with zero mean, variance $\sigma^2$, and correlation coefficient $\rho$, the characteristic function is given by (Wu et al. 1988).

\[ <\cdots> = \exp(-q_s^2\sigma^2(1-\rho_i)) \]  

The correlation coefficient of a random process is a function of spatial variables. For stationary process, it depends only on the difference variables, $u = x'-x''$, $v = y'-y''$. Assume that the size of the illuminated area is $2L \times 2L$. In terms of the difference variables, (50) becomes

\[ <\cdots> = \exp(-q_s^2\sigma^2(1-\rho_i)) \]  

This equation can be simplified by interchanging the order of integration, which leads to the following identity:

\[ \int_{-2L}^{2L} \int_{-2L}^{2L} \exp(jq_{s1}u + jq_{s1}v) \]  

The use of the above identity permits (52) to be written as

\[ <\cdots> = \frac{2\pi}{\sigma^2} \frac{q_s^2}{q_{s1}^2} \]  

Further simplification requires additional assumptions. Two commonly used assumptions are that (a) the surface roughness is isotropic (b) $(q_{s1}\rho_i)^2$ is large so that the contribution to the integrals in (54) is significant only for small values of $u$ and $v$. Then $\rho_i$ can be approximated by the first two terms of its Taylor series expansion at the origin. In this case, it is advantageous to change $u$, $v$ to polar coordinates $r$ and $\phi$. Upon ignoring $|u|$, $|v|$ in comparison with $2L$ and integrating $\phi$, (54) reduces to

\[ <\cdots> = \frac{2\pi q_s^2}{q_{s1}^2} (2L)^2 J_0(r(q_{s1} + q_s^2)) \]  

where $J_0(\cdots)$ is the zeroth-order Bessel function, $\rho_i(0)$ is the second derivative of $\rho_i$ evaluated at the origin, and $(2L)^2$ is the illuminated area $A_0$. Note that in (55), $\sigma^2\rho_i(0)$ corresponds to the mean squared slope of the surface. Since the integrand of (55) is negligible for large values of $r$, no significant error results if we extend the upper limit to infinity. With this change in limit, the integrated result of (55) is

\[ <\cdots> = \frac{2\pi A_c q_s^2}{q_{s1}^2\sigma^2(\rho_i(0))} \exp\left[-\frac{q_{s1}^2 + q_s^2}{2q_{s1}\sigma^2(\rho_i(0))}\right] \]  

Upon substituting (56) into the product in the scattered-field expression for medium 1, we obtain

\[ <\cdots> = \frac{k_1E_0 U_{pq1}}{(4\pi R)^2} <\cdots> \]  

Substituting (57) into (46), we obtain the reflected bistatic-scattering coefficient in medium 1 as

\[ <\cdots> = \frac{(k_1E_0 U_{pq1})^2}{(4\pi R)^2} \]  

Similarly, by considering the scattered field from medium 3 (peat as a perfectly conductor) or interface 2, the scattering coefficient in $m=3, 5, 7, \cdots$ is obtained as

\[ <\cdots> = \frac{(k_1E_0 U_{pq1})^2}{(4\pi R)^2} \]  

where $C\sigma^2 = \int dx dy exp(-2k_1\tilde{\mathbf{n}}_{m-1} \cdot \mathbf{r})$. Upon substituting (58) into (46), we obtain the reflected bistatic-scattering coefficient in medium 1 as

\[ <\cdots> = \frac{(k_1E_0 U_{pq1})^2}{(4\pi R)^2} \]  

where $\langle m_{pq} \rangle <\cdots> = \int dx dy exp(-4j k_1\tilde{\mathbf{n}}_{m-1} \cdot \mathbf{r})$. By referring to (58), $<m_{pq} \rangle_{pq}$ can be derived in the same manner. Then to calculate the average scattered power of the factor $<m_{pq} \rangle_{pq}$, the major interest here is in the incoherently scattered power, which is the usually measured parameter. This means that we should subtract the mean or the coherent field from the expression given by (29) and (30) before we form the power expression. However, this is equivalent to compute the factor $<m_{pq} \rangle_{pq}$ in (59) and then subtract an appropriate mean product or mean squared quantity from each other. In what follows, we shall proceed by considering only the final factor in (59). For the purpose of illustration, we shall...
assume a Gaussian height distribution for the surface of medium 3 (peat) under consideration. The mean squared of (29) and (30) are

\[
\langle E_{xx} E_{yy} \rangle = |C_m|^2 \langle E_{xx}^* \rangle \text{sec}^2 \theta_{m-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp(2j\kappa_x (x-x')) \exp(2j\kappa_y (y-y')) dx dy \]

where \( \xi = x-x', \ y = y-y', \ \rho_s(\xi, \eta) \) is the surface-height autocorrelation function and \( \sigma_2^2 \) is the variance of the surface of medium 3. To obtain the incoherent power, we subtract the mean-squared value \( \langle E_{xx} E_{yy} \rangle \) from it, yielding

\[
\langle E_{xx} E_{yy} \rangle - \langle E_{xx} \rangle^2 = |C_m|^2 \langle E_{xx}^* \rangle \text{sec}^2 \theta_{m-1} A_o \exp(-4k_0^2 \sigma_1^2) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp(2j\kappa_x \xi) \exp(4k_0^2 \sigma_2^2 \rho_2 - 1) \]

where \( A_o \) is the illuminated area. Hence the backscattering coefficient for the factor \( f_{3pq}^h \) of (59) can be obtained multiplying the power expression (62) by the factor \( 4\pi R/A_o \text{Re}(1 + \varepsilon^2_{spq}) \). Denoting the factor of backscattering coefficient by \( f_{3pp}^h \) and \( f_{3pp}^v \) for horizontal and vertical cases respectively, we have

\[
f_{3pq}^h \approx \frac{2k_0^2}{R_{m-1}^{-\text{cos}^2 \theta_{m-1}}} \left[ \exp(-4k_0^2 \sigma_1^2 \text{cos}^2 \theta_{m-1}) \sum_{n=1}^{\infty} \left( 4k_0^2 \sigma_1^2 \text{cos}^2 \theta_{m-1} \right)^n / n! \right] W^{(n)}(2k_0 \text{sin} \theta_{m-1}, 0)
\]

or

\[
f_{3pq}^v \approx 8k_0 \text{exp}(-2k_0^2 \sigma_1^2 \text{cos}^2 \theta_{m-1}) \sum_{n=1}^{\infty} \left( 4k_0^2 \sigma_1^2 \text{cos}^2 \theta_{m-1} \right)^n / n! \ W^{(n)}(2k_0 \text{sin} \theta_{m-1}, 0) \exp(-2k_0^2 \sigma_1^2 \text{cos}^2 \theta_{m-1})
\]

where \( W^{(n)}(U, V) \) is the roughness spectrum of the surface related to the \( n \) th power of the surface correlation function by Fourier transform as follows:

\[
W^{(n)}(U, V) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\xi d\eta \exp(-jU\xi - jV\eta) \rho^{(n)}(\xi, \eta)
\]

\( n = 1, 2, \cdots \)

As an example for a Gaussian correlation function \( \rho(\xi, \eta) = \exp(-(\xi^2 + \eta^2)/l) \), where \( l \) is the pulse width, (63) takes the form

\[
f_{3pq}^h \approx \frac{2k_0 l}{(R_{m-1}^{-\text{cos} \theta_{m-1}})^2} \sum_{n=1}^{\infty} \left( 4k_0 \sigma_1 \text{cos} \theta_{m-1} \right)^n / n! \exp(-2k_0^2 \sigma_1^2 \text{cos}^2 \theta_{m-1})
\]

To simplify the computation, in the derivation of \( \sigma_{pq}^h \), the effects of shadowing and multiple scattering have been ignored. Hence the backscattering coefficient for \( m = 3, 5, 7, \cdots \) is

\[
\sigma_{pq}^h \approx \frac{(k_0 q_{pq} A_{pq}^m)^2}{2q_{pq} \sigma_1^2 \rho_1(0)} \exp(-\frac{q_{pq}^2}{2q_{pq} \sigma_1^2 \rho_1(0)} - \frac{q_{pq}^2}{2q_{pq} \sigma_1^2 \rho_1(0)} - \frac{q_{pq}^2}{2q_{pq} \sigma_1^2 \rho_1(0)} - \frac{q_{pq}^2}{2q_{pq} \sigma_1^2 \rho_1(0)})
\]

where \( A_{pq} \) is the illuminated area. Hence the backscattering coefficient and the thickness of forest fire scars is obtained. The computation results and its confirmation are discussed in the next section.

3 . Results and discussion

To obtain the correlation between the like-polarised backscattering coefficient \( \sigma^o \) and the thickness of forest fire scars \( \xi^o \), parameters of the forest fire scars are the specific permeability \( \mu_r = 1 \), the dielectric constant of forest fire scars in 1.275GHz (JERS-1 SAR) is \( \varepsilon_r = 2.5 - j0.1 \) (Tetuko 2002, Tetuko et al. 2003a, Tetuko et al. 2003b), the wavelength
Figure 6  Relationship between the backscattering coefficient and the thickness of forest fire scars.

\[ \lambda = 23.5 \text{cm}, \text{ the incident angle } \theta_i = 38.7^\circ, \text{ and } \xi \text{ varied from 0 to 1m. The standard deviation } \sigma_1 \text{ of medium 2 (forest fire scars) and medium 3 (peat) } \sigma_2 \text{ surfaces are assumed to be } 0.3 \text{m (Nuraini 1999). By substituting these variables in (68), the backscattering coefficient } \sigma_0 \text{ with respect to each thickness of forest fire scars } \xi \text{ is obtained. This result is illustrated in figure 6 (hh-polarisation and vv-polarisation). This figure shows that increment in the thickness } \xi \text{ of forest fire scars is directly proportional to reduction in backscattering coefficient } \sigma_0. \text{ It implies that the forest fire scars absorbed wave energy. In the same figure, the previous result is also shown (previous method) (Tetuko et al. 2003a, Tetuko et al. 2003b). This result shows that both polarisations have different intensity for the same thickness, because in this study, the roughness of forest fire scars is considered. On the contrary, this matter was ignored in the previous analysis. The results also prove that the roughness of targeted surface plays an important role in the analysis of surface scattering. Additionally, the result shows that hh- and vv-polarisations in L-Band wave scattering have different intensity about } -10 \text{dB between each other. In the next section, the results are applied to estimate the thickness } \xi \text{ of forest fire scars in the study area.} \]

4. Application

A JERS-1 SAR data (path 95, row 305) that was acquired on 29 July 1997 (dry season and during fire events), is used to estimate the thickness of forest fire scars in the study area. The data processing is done using the same process that was discussed in (Tetuko 2002, Tetuko et al. 2003a and Tetuko et al. 2003b). The backscattering coefficient of each forest fire scars class is shown in figure 7 and its value is depicted in table 1. By plotting the backscattering coefficient value into graph of ‘proposed method’ (hh-polarization) in figure 6, \[ \xi \text{ of each class is acquired (see table 1), where the } \xi \text{s in the study area are between 0.16 and 0.21m. The result was confirmed by ground data (figure 3 and figure 7), where the obtained forest fire scars A and B area in figure 3 have thickness of forest fire scars between 0 to 0.5m. Therefore, the estimated results are obtained in this range value or the results show that the fires reached 0.21m in depth (thickness).}

Previous method was developed by assuming that the scattered field as a corner reflected wave that emitted to the SAR sensor after reflected by the
burnt tree trunk (see figure 1 (C), Tetuko et al. 2003a, Tetuko et al. 2003b). Therefore the previous method shows a higher backscattering coefficient comparing to the proposed method that the model is assumed by three layer media with two rough surfaces without the burnt tree trunk on the interface between air and forest fire scars.

The other reason of the error results occurring in the previous method and the proposed method is the strong dependency of microwave dielectric constant of soil to the soil moisture content (Ulaby et al. 1986). The soil moisture content could be used to determine the magnitudes of the scattering coefficient for air–forest fire scars interface, as well as the penetration depth in the forest fire scars layer. For the purpose to know the tendency of backscattering coefficient, we employed only the sample of forest fire scars whose dielectric constant is relatively lower than the natural one in the study sites that suppose to have higher soil moisture contents. The natural forest fire scars that found in the post forest fire actually have high soil moisture content, because the relative humidity in the study area is high (70% to 90%). Hence, in this research, the effect of the dielectric properties of the near surface layer on surface scattering and volume scattering are ignored. Therefore the backscattering coefficient obtained are lower than the previous method and the ground survey data. However the estimation result obtained is half of the estimated value using previous method, it is still in range of ground data (0 to 0.5m).

5 . Conclusions

Numerical analysis was conducted to analyse the relationship between the backscattering coefficients $\sigma^\circ$ and forest fire scars thickness $\xi$. This analysis result was confirmed by previous result that was derived using a transmission line method. Proposed result show about half value of previous estimated result. It is due to the roughness of forest fire scars surface without the consideration of reflected wave from burnt tree trunk in the analysis model. Then this result was applied in estimating the forest fire scars thickness in central Borneo, Indonesia. The analysis result was confirmed by the ground data that was collected by ground survey from 1995 to 1997, and it shows that the fires penetrated forest fire scars to 0.21m in depth (thickness).

Further, application of this result can be used to estimate the forest fire scars thickness whose information could be employed in the process of extinguishing the forest fire effectively and accurately in the study area in which forest fires happen annually. This technique can also be applied to monitor the post-crisis management of fire events by estimating the damages suffer by the peat soil and the gas (carbon, nitrous oxide etc) release into the atmosphere. The previous techniques (Hadi et al. 2000a, Hadi et al. 2000b, Radjagukguk 1997, and Sudiana et al. 2003) developed to estimate the damages did not include the consideration of physical conditions of forest fire scars and burnt peat layer as the source haze or gases. Therefore, it is supposed that the employment of the proposed technique in this research would allow the obtention of other information relating the surface conditions (phenomenons) of the study area.

In this research, many parameters are still ignored. Hence, in the next research, burnt tree trunk model, shadowing and multiple scattering will be
considered to obtain more accurate results.

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