Effects of Surface Motion Difference at Footings on the Earthquake Responses of Large-Span Gable Structures

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Liang SU

ABSTRACT In this paper the spatial distributions of earthquake motions on shallow alluvial valleys with an irregularity in soil layer composition are first checked. Based on the results of analysis, the seismic responses of a large-span gable structure are calculated considering the input difference at the footings due to spatial distributions of the motions. The results of calculation show that the different acceleration inputs will excite an additional vibration associated especially with the second natural symmetric mode as well as the first natural anti-symmetric mode. Furthermore, a method for calculating the seismic responses to the structures is proposed to consider the different inputs based on the results of present paper.

Keywords: Alluvial valley; Input difference; Seismic response; Mode estimation; Response spectrum

1. Introduction

Large-span structures, domes and hangars, behave in their own dynamic characteristics because of their special shapes different from tall buildings. A lot of works had been carried out to find the earthquake response characteristics. In these researches, the earthquake input differences at the structural supporting points, which may be caused by the structural differences of supporting substructures, the different wave propagation path, irregular local soil sites and so on, have also been concerned as one of the important issues. For example, the effects of the earthquake input differences on the dynamic responses of some typical large-span structures are studied by Ishikawa (1997), Fujimoto (1997) and Kato (2000). All these researches reported that earthquake input difference can cause significant effects to the large-span structures. However most of the recent practical design codes still remain unsatisfied with the input difference despite of the results of researches on this subject.

Many earthquake site observations imply that topographical conditions may strongly influence the ground motions at a site. Recent large-magnitude earthquake events, like the 1995 Kobe earthquake, have highlighted the amplification effects produced by the irregular topographies. As a result of such observations, considerable profound works have been done to model and predict these effects (For example, Koketsu, 1991; Ejiri, 1994; Hayashi, 1995; Kawase, 1998; Murono, 1999). On the other hand, the topographical conditions are also the causes of different earthquake inputs to structures because of their irregular soil arrangements. But so far compared with the amplification effect of irregular topographies, the effect of the surface motion difference due to site irregularity on the earthquake responses of large-span structures is still concerned a little.

Accordingly, the present paper focuses on the effect of the surface motion difference due to site irregularity on the seismic responses of large-span structures. The first part of the paper is preliminary to find the basic but approximate information of surface earthquake motion differences due to site irregularity, which may be found in a shallow alluvial valley. Secondly, based on the information from the analysis, the earthquake responses of a steel large-span gable structure which covers 50 meters, taken as one typical example for...
large-span structures, are studied considering the effects of the local site irregularity, leading to a new method to estimate approximately the seismic responses of structures with the acceleration input differences at their supporting points.

2. Local soil irregularity and a structure for analysis

As given for design use in Japanese Design Standard of Railway Structures (1999), a typical locally irregular topography illustrated in Fig. 1 can be always found at an alluvial valley and taken as an example in this paper to include the effects of site irregularities. Local site irregularities in layer composition may be classified into many different types for the real soil topography even in the restricted case of the type shown in Fig. 1. In this paper, the depth, H=20m, is selected as typical for a shallow alluvial deposit, while the slope θ dividing the soft deposit and the hard bedrock is varied as H/L=1/1, 1/2 and 1/4 to investigate the effects of topographical change on surface earthquake motions. The alluvial soft deposit and bedrock layers are both assumed homogenous with the material properties of the density \( \rho_1 \), \( \rho_2 \) and the shear wave velocity \( V_{S1} \), \( V_{S2} \) respectively. Four cases of the impedance \( \rho_1 V_{S1}/\rho_2 V_{S2} \) are considered in analyses as given in Table 1 since the impedance is already found to play a significant role to the amplification of motions. The notation such as L40-H20-VS100 is used to describe the cases, for example L=40m, H=20m and \( V_{S1}=100\) m/s. An approximate analysis of the surface earthquake motions in cases of the above typical irregular deposit is performed with FEM to investigate how the surface motions vary depending on locations, where a steel long-span gable structure shown in Fig. 4 is assumed built with no tie beams between the two footings. At present, very accurate and more detailed FEM analysis has been performed by Ejiri (1994), Hayashi (1998), Murono (1999) and others to investigate the dynamic behaviors of the irregular soil deposits and they can give us much more reliable information, however, this research adopts such an approximate procedure since the first part of the present study aims to find the difference of the input earthquake motions at footings of a structure.

As an example, Fig. 2 shows the FEM analysis model for L40-H20-VS100 type. In analysis, 20-node isoparametric 3-D finite elements under plane strain conditions in the y direction are used to model soils. Furthermore, the dynamic soil responses are treated by

![Fig. 1 Local irregular layer topography in the present paper](image)

### Table 1 Topographical shape data and soil material properties of all the analysis cases

<table>
<thead>
<tr>
<th>Analysis cases</th>
<th>( \rho_1 V_{S1}/\rho_2 V_{S2} )</th>
<th>( V_{S1} ) (m/s)</th>
<th>( V_{S2} ) (m/s)</th>
<th>Shape ratio</th>
<th>L(m)</th>
<th>H=20m</th>
<th>( \rho_1=1800) kg/m(^3)</th>
<th>( \rho_2=2000) kg/m(^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L20-H20-VS100</td>
<td>1/5</td>
<td>100</td>
<td>450</td>
<td>1/1</td>
<td>20</td>
<td>\rho_1</td>
<td>\rho_2</td>
<td></td>
</tr>
<tr>
<td>L20-H20-VS150</td>
<td>3/10</td>
<td>150</td>
<td>450</td>
<td>1/1</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L20-H20-VS200</td>
<td>2/5</td>
<td>200</td>
<td>450</td>
<td>1/1</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L20-H20-VS250</td>
<td>1/2</td>
<td>250</td>
<td>450</td>
<td>1/1</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L40-H20-VS100</td>
<td>1/5</td>
<td>100</td>
<td>450</td>
<td>1/2</td>
<td>40</td>
<td>\rho_1</td>
<td>\rho_2</td>
<td></td>
</tr>
<tr>
<td>L40-H20-VS150</td>
<td>3/10</td>
<td>150</td>
<td>450</td>
<td>1/2</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L40-H20-VS200</td>
<td>2/5</td>
<td>200</td>
<td>450</td>
<td>1/2</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L40-H20-VS250</td>
<td>1/2</td>
<td>250</td>
<td>450</td>
<td>1/2</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L80-H20-VS100</td>
<td>1/5</td>
<td>100</td>
<td>450</td>
<td>1/4</td>
<td>80</td>
<td>\rho_1</td>
<td>\rho_2</td>
<td></td>
</tr>
<tr>
<td>L80-H20-VS150</td>
<td>3/10</td>
<td>150</td>
<td>450</td>
<td>1/4</td>
<td>80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L80-H20-VS200</td>
<td>2/5</td>
<td>200</td>
<td>450</td>
<td>1/4</td>
<td>80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L80-H20-VS250</td>
<td>1/2</td>
<td>250</td>
<td>450</td>
<td>1/4</td>
<td>80</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
an equivalent linearization method to include their elasto-plasticity and the damping due to cyclic plasticity. In the linearization, the effective shear strain of each soil element is calculated by $\gamma_{\text{eff}} = 0.65 \gamma_{\text{max}}$, where $\gamma_{\text{max}}$ is the maximum shear strain. The relationships between the effective strain and both stiffness decrease coefficient and damping ratio are illustrated in Fig. 3 quoted for sand deposits according to the detail for design use of the Notification 1457 (2000) of the Ministry of Construction of Japan. $G_{\text{eq}}$ and $h_{\text{eq}}$ in Fig. 3 are the equivalent shear rigidity and damping factor corresponding to $\gamma_{\text{eff}}$ while the Poisson's ratio is assumed constant as 0.48 under equivalent linearization. In order to approximately include the far field effects of propagation infinitely downward, Lysmer dashpots (1969) are adopted between topsoil layers and bedrock. In the horizontal direction, as shown in Fig. 2, the wave propagation boundary conditions are adopted also based on the Lysmer dashpots to include the soil infinitive effect in the horizontal directions.

Two-point supported large-span structural type, such as reticular cylindrical domes and arches, is a common structural type for hangars and workshops because of its large-span characteristic. On the other hand, the large-span characteristic makes it difficult to provide a tie-beam between the distantly separated supporting points of these kind structures and makes structures themselves more sensitive to the input differences. Accordingly, in this paper, a steel gable structure of two-point supported large-span shown in Fig. 4 is selected as an object for analysis. The reason for this selection is that the structural characteristics are relatively simple and that this simplicity can make the present research concentrate on the effects and fundamental points of the different inputs. The span of such structures may vary, probably, up to less than 100 meters and the length of 50 meters is adopted here for medium span although studies are requested for different length. The vertical dead design load is assumed 6.0kN/m, giving the bending moments and shear force of the column under dead loads as given in Fig. 4, and, according to design practice for large-span structures in Japan, the seismic base shear coefficient $c_{\text{sv}}=0.5$ is adopted to decide the horizontal earthquake design load although this value seems a little large for allowable stress design. Furthermore, it has been well known that the horizontal earthquake input could excite a considerable large vertical response in large-span structures and this response should not be ignored in the seismic design of large-span structures. Thus in this paper, the vertical earthquake design load due to the horizontal earthquake input is assumed to be in sine-function asymmetrical distribution as shown in Fig. 4 and the maximum vertical seismic coefficient $c_{\text{v}}=0.5$ is assumed based on a preliminary study.
section area $A$ and the second moment of inertia $I$ of each element are given in Table 2. Structure-to-foundation is pin connection and the mass of structure is based on the lumped mass concept. The first and second natural periods of the structure are $T_{S1}=0.68s$ and $T_{S2}=0.49s$, respectively. Fig. 4 also illustrates the mode shapes of the first two modes. To be noticed, the first mode shape is an anti-symmetrical deformation that could be excited by a corresponding anti-symmetrical input such as the uniform horizontal earthquake input, while the second mode shape is symmetrical to be excited by the symmetrical dynamic inputs such as the before-mentioned input difference.

In the dynamic analysis, the behavior of structure is assumed elastic and the generalized Rayleigh damping is used to assume the damping of structure and damping coefficients, $h_1$ and $h_2$, for the first and second eigenmodes of structures are assumed 0.02.

As for the integration, an average acceleration method of Newmark $\beta$ scheme is used to solve the dynamic system with $\beta=1/4$ and the time increment $\Delta t=0.01s$ based on a preliminary computation using several different time increments.

### Table 2 Properties of structural elements

<table>
<thead>
<tr>
<th>Elements</th>
<th>$I\times10^{-2}\text{m}^2$</th>
<th>$A\text{cm}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column 1</td>
<td>0.27</td>
<td>50.0</td>
</tr>
<tr>
<td>Column 2</td>
<td>0.50</td>
<td>90.0</td>
</tr>
<tr>
<td>Beam 1</td>
<td>0.46</td>
<td>85.0</td>
</tr>
<tr>
<td>Beam 2</td>
<td>0.27</td>
<td>50.0</td>
</tr>
<tr>
<td>Beam 3</td>
<td>0.19</td>
<td>35.0</td>
</tr>
<tr>
<td>Beam 4</td>
<td>0.19</td>
<td>35.0</td>
</tr>
</tbody>
</table>

### 3. Incident earthquake acceleration for analysis

The incident earthquake acceleration is assumed a SH inplane wave transmitted vertically from downward far-field. The acceleration is artificially simulated according to the Notification 1461 (2000) with an intensity of the ‘damage’ limit, which is defined as the state that stresses remain elastic. The design acceleration spectrum for the damage limit is given in Fig. 5 with an artificially simulated acceleration response spectrum. The acceleration is produced based on a work by Oosaki (1996) with the use of the phase angles of El-Centro NS earthquake record when represented by Fourier series. The time history of simulated wave for 40s is given in Fig. 6, where the peak value is found 71.3cm/s$^2$ as at outcropping rock.

The present acceleration is used as twice the incident wave $\ddot{u}_g(t)$ from the bedrock, accordingly, $2\ddot{u}_g(t)$ means the wave at an outcropping rock.

![Fig. 4 Properties of the structure for analyses](image)

![Fig. 5 Acceleration response spectra $S_A$ for damage limit](image)
4. Free field surface motion characteristics on the irregular soil sites

As it has been reported in the previous researches (e.g. Murono, 1999), soil irregular arrangement induce additional waves at the inclined interface. These additional wave transmissions will either de-amplify or amplify the surface motion at the different soil sites on an alluvial valley compared with the regular soil sites. Thus as shown in Fig. 7, the surface motion \( S(x,t) \) on the alluvial valley can be treated conceptually as the sum of the following two parts: \( F(t) \), which is the surface earthquake motion at the regular soil sites; \( G(x,t) \), which is the additional earthquake wave motion caused by the irregular soil arrangement. Here both \( S(x,t) \) and \( G(x,t) \) are the function of \( x \) and \( t \). Based on these concepts, the basic properties of \( S(x,t) \), \( F(t) \) and \( G(x,t) \) are focused on to present the calculated results of the surface motion in the following analyses. Of course, the soil irregularity will also cause relatively small vertical dynamic motions at the soil surface (Murono, 1999). In the present paper, only the horizontal component is focused on in order to make the results easy to be accessed.

Since the soil irregularities affect a little on the motions of bedrock (Murono, 1999) and in the practice the structure can be rarely constructed at the site above the inclined interface, in this paper the range of \( x>0 \) is selected to be focused on for the analysis. Here, as shown in Fig. 2, the point of \( x=0 \) is assumed to be O point, which is the beginning point of the flat part of alluvial valley on the soil surface. R point refers to the one that locates on the soil surface far away from the effects of irregular sites. Thus the properties of earthquake motions at R point are just as same as free field responses on the regular soil sites.

4-1. Acceleration amplitude ratio \( \alpha(x) \)

The acceleration amplitude is a simple but important value to estimate the intensity of a complex earthquake phenomenon. For the structural design practice, the acceleration amplitude ratio defined by \( \max[G(x,t)]/\max[F(t)] \), calculated approximately by Eq. (1), is adopted in the design code of Japanese Design Standard of Railway Structures (1999) to estimate the effect of soil irregularity on the surface motions despite of the complicated pattern of \( G(x,t) \) due to many factors. In this paper, \( \alpha(x) \) of Eq.(1) is also used to check the present results based on FEM to approximately evaluate the effect of soil irregularity.

\[
\alpha(x) = 0.40 \times \exp\left(-\frac{7.0}{\theta}\right) \times \sqrt{1/\chi} \times \exp\left(-0.44(x/H)\right)
\]

Here, \( \chi \) is the impedance considering the elasto-plasticity of soils; \( \theta \) is the slope angle of the inclined bedrock as shown in Fig. 1.

Although the comparison between the FEM results of this paper and the value calculated by Eq. (1) shows some slight difference, an analysis of \( \alpha(x) \) in Fig. 8 gives the following common observations: (1) With
the increase of $x$, the effects of soil irregularity on soil surface motion decreases since the amplitude of $G(x, t)$ is a decrease function of $x$ in each case. The maximum value of $\alpha(x)$ occurs at the site of $x=0$ and it can be larger even than 1.0 in case of L20-H20-VS100. (2) $\alpha(x)$ decreases when $L$ increases. (3) When the stiffness difference between the bedrock and the alluvial deposit is small, the value of $\alpha(x)$ also becomes small. In other words, for the case of larger impedance, the effect of soil irregularity tends less.

4-2. Surface motion difference

In the analysis of Section 4-1, it has been found that the additional waves caused by soil irregularity spatially change. This spatial change will certainly imply a cause of different earthquake input to structures. Since in this paper the structure to be concerned with is a 50m large-span gable structure supported on two footings without any tie beams, the differences between the two footings of the structure

<table>
<thead>
<tr>
<th>Analysis Cases</th>
<th>Max. of $\frac{A_{fs}(t)}{A_f(t)}$ (cm/s²)</th>
<th>Max. of $(A_{fs}(t) - A_f(t))$ (cm/s²)</th>
<th>Max. of $(D_{fs}(t) - D_f(t))$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L20-H20</td>
<td>41.0/52.2</td>
<td>54.83</td>
<td>1.13</td>
</tr>
<tr>
<td>L40-H20</td>
<td>41.3/49.1</td>
<td>44.48</td>
<td>0.93</td>
</tr>
<tr>
<td>L80-H20</td>
<td>54.6/44.7</td>
<td>28.52</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Table 3 Difference maximum of the calculated results ($Lo=0$)
are checked in all the cases. Furthermore, the case of the location with respect to \( L_0=0 \) is selected to be analyzed since the soil irregularity will have the largest effect on this area as described in Section 4-1. Here \( L_0 \) is the distance between \( O \) point and the left footing of the gable structure as shown in Fig. 1.

Table 3 presents the maximum values of the surface responses. Here, \( A_K(t) \) and \( D_K(t) \) respectively denotes absolute acceleration and absolute displacement of \( K \) point. \( F_L \) and \( F_R \) show the left and right footing points of the structure. The results show that the acceleration difference can be considerable large. For example, the maximums of acceleration difference between the two footing points are even larger than the acceleration maximum of these two footing points in some cases such as L20-H20-VS100. On the other hand, the displacement difference seems still small in engineering sense, because the largest one is 1.1 cm, and decreases along with the increase of \( \rho_1V_{30}\rho_2V_{40}\) ratio.

The response spectrum at a construction site, especially for the value around the period of structures to be built on the site, is always important in design practice of a structure. Fig. 9 and Fig. 10 then present the response spectra of both acceleration average and acceleration difference of the two footings, under the condition that the damping coefficient is 2%. In order to compare the results, the response spectrum of \( R \) point is also presented in these figures. For the response spectra of acceleration average, it can be found that the results of acceleration average are almost the same as that of \( R \) point despite of the effects of soil irregularity. The structure to be dealt with in the later sections is a gable structure of 50m span whose natural period ranges approximately in most cases from 0.3s to 1.0s according to the previous research data (Kato, 2002) despite of some special exceptions. Due to the resonance effect it can be expected that the responses of such kind of structures under the average acceleration excitations of the two footing points will be relatively large on the sites of VS150, VS200, VS250 since the natural period of these three soil layers, which is 0.80s, 0.56s, 0.42s respectively, falls in the structural period range from 0.3s to 1.0s.

Meanwhile, although it is hard to precisely determine the acceleration difference property because of the complexity of the soil behavior, from Fig. 10 we make the following observations: (1) The peak of response spectrum of the acceleration difference occurs at the

![Graphs](image-url)
(2) The response spectra of acceleration difference decrease gradually with the increase of L although their frequency properties are very close to each other.

5. Earthquake responses of a large-span gable structure under different earthquake inputs

In this section how the different inputs on the alluvial valley will affect the earthquake responses of structures is investigated and the earthquake responses of structure under the different inputs are first presented. Here, the left footing point of the structure is assumed at O point, x=0, on the alluvial valley shown in Fig. 1, since the soil irregularity will have the largest effect in this area as described in Section 4-1. The responses of the structure on regular soil sites are also calculated and compared in the present paper.

Fig. 11 shows the responses expressed by Fourier spectrum of the vertical acceleration at Node 5 illustrated in Fig. 4. The period range of the figure is focused on the range between 0.4s to 0.8s since the value of Fourier spectrum at the other period range is very small. From these figures, a basic concept can be built in mind that, compared with the responses at the regular sites, the different inputs will excite a considerable large additional vibration at the second natural period $T_{S2}=0.49s$ as well as the vibration at the first natural period $T_{S1}=0.68s$. Fig. 12 gives the results of the vertical acceleration distribution when the vertical acceleration at Node 5 reaches its maximum. Same as the value of Fourier spectrum, these figures also give us a large discrepancy between the different input cases and the uniform input one: With the additional vibration at $T_{S2}$, compared with the uniform input condition, the acceleration distribution of the roof in the vertical direction does not keep in anti-symmetrical shape again. For example, in the case of L20-H20-VS 100, the vertical acceleration at Node 5 is about 2.0 times that of regular soil site.

Fig. 13 presents the calculated results as the envelope for the maximum bending moments of the structure obtained in the time history analysis. It can be found that due to the additional vibration caused by the different inputs, the maximum bending moment,
Fig. 11 Fourier spectra of vertical acceleration at Node 5

Fig. 12 Vertical acceleration distributions when vertical acceleration at Node 5 reaches its maximum
especially for the bending moment at the column head, will obviously increase in the irregular cases compared with the uniform input cases. These additional vibrations also lead large shear forces within columns as shown in Fig. 14 of L40 Cases for example. In the case of L40-H20-VS100, the right columns are subjected to shear forces and at the same time to bending moments larger almost by 150 percentages than that in case of regular soil site. These results indicate that in the earthquake design practice of the large-span gable structures, the soil irregularity must be considered as an important factor if it exists.

6. Analyses of earthquake responses under different earthquake inputs on the alluvial valley

In order to get a clearer understanding of the results calculated above, the different inputs at two footings are divided into two parts as shown in Fig. 15. The first part noted as AE (Anti-symmetrical Excitation) is the uniform earthquake input with the intensity of $(\frac{\text{InpL}(t)+\text{InpR}(t)}{2})$, where $\text{InpL}(t)$ and $\text{InpR}(t)$ are the input motions at left and right footings respectively. For the second part noted as SE (Symmetrical Excitation), the earthquake inputs at the two footings have the same intensity of $\frac{(\text{InpL}(t)-\text{InpR}(t))}{2}$ but their input directions are opposite to each other. The overall responses of the structure subjected to the different inputs then can be calculated by a simple sum of the responses of the above two excitations since the structure is assumed to be elastic.
6-1. Earthquake responses of structure under anti-symmetrical excitation

The dynamic responses of the large-span gable structures excited by uniform earthquake inputs have been described in detail in the previous papers (Kato, 2002) and it had been reported that the dynamic characteristics of such structures can be judged to mainly vibrate as a single or two degrees of freedom on the basis of the first and/or third mode, both being anti-symmetric modes. Here the natural period of the third mode is $T_{S3} = 0.22s$ in the present case. Furthermore, the maximum accelerations of each mode can be calculated by using Eq. (2).

$$\{\text{Acc}\}_i = \beta_i \times S_{AE}(T_i, h_i) \times \{X_i\}$$  \hspace{1cm} (2)

where $\beta_i$, $S_{AE}(T_i, h_i)$ and $\{X_i\}$ are the participation factor, acceleration response spectrum of AE part and mode shape for the $i^{th}$ mode.

Fig. 16 presents the maximum acceleration distribution comparison of the roof between results based on the time history analyses and the mode estimation, Eq. (2), of the first mode and third mode. Since the similar results can be obtained in the other regular soil site cases and the natural period 0.80s of VS150 soil site is nearest to $T_{S1} = 0.68s$ as it has been mentioned in Section 4-2, as an example, only Case VS150 is selected to be illustrated here. From Fig. 16 it can be found that, compared with vibration of the first mode, the third mode will occupy not so large a part in the calculated cases. Furthermore the maximum acceleration both at horizontal and vertical direction at Node 5 can reach approximately 300cm/s$^2$. Thus the base shear coefficients $c_{oh}=0.5$ and $c_{ov}=0.5$ as adopted in the present paper seem a little safe enough for the allowable stress design of the structure in case of no earthquake input differences.

As obtained in Section 4-2 and shown in Fig. 9, the response spectra of AE part, average response in other words, of the different inputs caused by the soil irregularity of alluvial valley are almost same as those of regular sites. Thus combined with the structural properties described above, in the design of large-span gable structure, we can simply use the results on the regular sites to determine the structural earthquake maximum responses caused only by AE part of the different inputs on the alluvial valley.

6-2. Earthquake response of structure under symmetrical excitation due to input differences

Aiming to calculate the responses of SE input part, the expansion theorem based on the properties of the normal modes is applied in this paper.
whose responses are to be calculated. And the
dynamic equation of the whole structural system is
given by Eq. (3) for the earthquake motions.

\[
\begin{bmatrix}
[M] & 0 \\
0 & [M]
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_r \\
\ddot{x}_s
\end{bmatrix} + 
\begin{bmatrix}
[C] & [C] \\
[C] & [C]
\end{bmatrix}
\begin{bmatrix}
\dot{x}_r \\
\dot{x}_s
\end{bmatrix} + 
\begin{bmatrix}
[K] & [K] \\
[K] & [K]
\end{bmatrix}
\begin{bmatrix}
x_r \\
x_s
\end{bmatrix} = 
\begin{bmatrix}
0 \\
f_r
\end{bmatrix}
\end{equation}

Here, [M], [C] and [K] are the mass, damping and
stiffness matrices.

Eq. (3) can be divided into the following two
equations.

\[
\begin{bmatrix}
[M] & 0 \\
0 & [M]
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_r \\
\ddot{x}_s
\end{bmatrix} + 
\begin{bmatrix}
[C] & [C] \\
[C] & [C]
\end{bmatrix}
\begin{bmatrix}
\dot{x}_r \\
\dot{x}_s
\end{bmatrix} + 
\begin{bmatrix}
[K] & [K] \\
[K] & [K]
\end{bmatrix}
\begin{bmatrix}
x_r \\
x_s
\end{bmatrix} = 
\begin{bmatrix}
0 \\
f_r
\end{bmatrix}
\end{equation}

Thus, if absolute displacement \(\{x_r\}\) and absolute
velocity \(\{\dot{x}_r\}\) at the support points are known, the
responses of whole structure can be easily calculated
by using Eq. (4). In this paper, \(\{x_r\}\) and \(\{\dot{x}_r\}\) at each
point are those calculated results in the first part
considering soil irregularity. The reaction forces at
support points can be obtained by Eq. (5).

By performing the modal transformation assuming

\[
\{x_r(t)\} = \sum_{k=1}^{N} \{X_k\} q_k(t)
\]

Eq. (7) then can be obtained from Eq. (4). Here, \(\{X_k\}\)
is the \(k^{th}\) mode shape of the structure.

\[
\sum_{k=1}^{N} \left[\begin{bmatrix}
[M] & [C] \\
[C] & [K]
\end{bmatrix} \{X_k\} q_k(t) + \{K_{rs}\} \{X_k\} q_k(t)\right] = 
\sum_{k=1}^{N} \left[-\{C_{rs}\} \{\dot{x}_r(t)\} - \{K_{rs}\} \{x_r(t)\}\right]
\]

Using the orthogonal property between mode
shapes, the response of \(q_k(t)\) can be calculated as a
single degree of freedom system from Eq. (7).

\[
\ddot{q}_k(t) + 2\zeta_k \omega_k \dot{q}_k(t) + \omega_k^2 q_k(t) = 
\frac{\{X_k\}^T \{C_{rs}\} \{\dot{x}_r(t)\} V_{se}(t) + \{X_k\}^T \{K_{rs}\} \{x_r(t)\} D_{se}(t)}{\{X_k\}^T [M_{rs}] \{X_k\}}
\]

Here, in case of the present paper, motions of SE part
are calculated by Eq. (9) assuming a vector of

\[
\{\xi\}^T = [1 \ -1]
\]

\[
\{\ddot{x}_{SE}(t)\} = \{\xi\} \cdot A_{SE}(t)
\]

\[
\{\dot{x}_{SE}(t)\} = \{\xi\} \cdot D_{SE}(t)
\]

\[
\{\ddot{x}_{SE}(t)\} = \{\xi\} \cdot V_{SE}(t)
\]

where \(A_{SE}(t), D_{se}(t)\) and \(V_{se}(t)\) are the time histories of
acceleration, velocity and displacement of the input
intensity of \((\text{InpL}(t)-\text{InpR}(t))/2\).

Considering that the earthquake component with a
frequency being close to the structure frequency \(\omega_k\)
will predominate the vibration of the \(k^{th}\) mode of
structure and that \(V_{se}(t)\) and \(D_{se}(t)\) can be simply
assumed here by integrating the absolute acceleration
\(A_{se}(t)\) although \(V_{se}(t)\) and \(D_{se}(t)\) should be those
obtained in time history analysis considering soil
irregularity, the value of the right side of Eq. (8) can
be evaluated. Table 4 presents the comparison
between \(\gamma^C_k\) and \(\gamma^K_k\) to check the effects of \([C_{rs}\]
part and \([K_{rs}\] part in the right side of Eq. (8) on the
dynamic response of structures under SE input part.

\[
\gamma^C_k = \frac{\{X_k\}^T \{C_{rs}\} \{\xi\} / \omega_k}{\{X_k\}^T [M_{rs}] \{X_k\}}
\]

\[
\gamma^K_k = \frac{\{X_k\}^T \{K_{rs}\} \{\xi\} / \omega_k^2}{\{X_k\}^T [M_{rs}] \{X_k\}}
\]

<table>
<thead>
<tr>
<th>Tse (Sec.)</th>
<th>(\gamma^C_k)</th>
<th>(\gamma^K_k)</th>
<th>(\gamma^C_k / \gamma^K_k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First mode</td>
<td>0.68</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Second mode</td>
<td>0.49</td>
<td>13.92</td>
<td>230.25</td>
</tr>
<tr>
<td>Third mode</td>
<td>0.22</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Fourth mode</td>
<td>0.20</td>
<td>39.26</td>
<td>191.31</td>
</tr>
</tbody>
</table>

From the results of Table 4, it can be obtained that
the value of \(\gamma^C_k\) is relatively small compared with
that of \(\gamma^K_k\) especially at the second mode since the
ratio between them is only 6%. The reason for this
result is that the damping coefficient for the steel
structures is very small as 2% in the consideration.

Similarly to the results obtained under the uniform
input, Fig. 18 shows the maximum acceleration
distribution comparison of the roof between the results
based on direct time history integration and the mode
Fig. 18 Comparisons between time history analyses and mode estimations under the symmetrical excitation
Fig. 19 Acceleration comparisons between time history analyses and Eq. (13) under the different inputs.
estimation by using Eq. (12) under excitation of SE part in all the cases.

\[
\{Acc\}_2 = \gamma^2 \times S_{AE(SE)}(T_2, h_2) \times \{X_2\} \tag{12}
\]

From the results given in Fig. 18, we draw a conclusion that the maximum responses of the large-span gable structures under excitation of SE part can be estimated effectively based only on the second mode by using Eq. (12). Also it can be found that the excitation of SE part can excite a large vibration in the structure. For example, in the case of VS150, the maximum vertical acceleration is about 300cm/s² just equal to the maximum acceleration value of AE part.

6-3. Proposal to calculate the maximum seismic responses considering the different inputs

The time history responses of the structure subjected to the different earthquake inputs can be calculated by a simple sum of the responses of the AE and SE excitations since the structure is assumed to be elastic. Meanwhile, as it has been obtained above, the maximum responses to AE part can be estimated effectively based on the first and third modes by using Eq. (2) and those to SE part based on second mode using Eq. (12). Considering that the maximum responses of AE and SE part can rarely happen simultaneously in the time history responses, in the present paper, Eq. (13) is proposed to approximately calculate the maximum seismic responses considering the different inputs on the alluvial valley.

\[
\{ACC\}_\text{max} = \sqrt{\{ACC\}_1^2 + \{ACC\}_2^2 + \{ACC\}_3^2} \tag{13}
\]

Fig. 19 shows the good agreement between the maximum acceleration of the results based on direct time history integration and that of the mode estimation by using Eq. (13). Fig. 20 and Fig. 21 then show those comparisons of the maximum moment distributions and maximum shear forces within columns between them. The figures indicate that in some cases the results based on direct time history integration may be larger than the estimation of Eq. (13). On the other hand, when the results based on direct time history integration are larger than the estimation of Eq. (13), their differences can be quite
small. Accordingly, the estimation of Eq. (13) can be assumed safety in the practice application.

**Conclusion**

The surface motion difference at typical alluvial valleys and its effect on the earthquake responses of large-span gable structure are checked above, followed by comparisons and discussions. Based on the results obtained, the following conclusions are drawn:

The irregular soil arrangement induces an additional wave transmission in the alluvial valley. This additional wave, which is affected by slope angle $\theta$ and impedance ratio, also causes different inputs to structures.

The different inputs at the supporting points of the large-span gable structure excite a considerable large additional vibration at the second natural period as well as the vibration at the first natural period compared with the uniform input condition. The maximum acceleration responses under the different input on the alluvial valley can be estimated effectively by using a simple combination formula based on the first three modes of the structure.

**References**


