Development of An Intuitive Drawing Method
—A Case of Three-point Perspective

Summary
The present author has dealt with the development of drawing methods for three-dimensional design, interpreting conventional projective perspective drawing. As a part of this research, a new method for two-point perspective as well as by-products were reported.

Due to its complexity of three-point perspective drawing, its development was tentatively left out by this report. It was, however, found that all cube images in three-point perspective drawing can be reduced to nine configurations of three principle edges according to their spatial locations, and thus the configurations are consistently similar to those in two-point perspective drawing. Since the configurations were proved to function as a starting point of drawing, a drawing method can then be proposed. In comparison with conventional drawing methods, which rigidly defines a drawing situation at the very initial stage of drawing, this method is simple to understand, perceive and use. Three-point perspective drawing now becomes a user-friendly method.

1. Introduction
Since the discovery of illusionistic spatial representation, perspective drawing, in the fourteenth century in Italy, seems to have been perfected as a system in the field of descriptive geometry. This means of representation has interesting aspects which are unique and cannot be perceived in other spatial representation systems; this system is based on the geometry of simulating human vision. This discovery and the system's uniqueness ultimately comes down to the simulation, as follows:

1. The system is logical and our vision of space is quite different from observation and experience. McKim claims that analogically this is a matter of cooperation between the eye, brain, and hand.

2. It is well recognised that the representation of space is impossible to learn the rules and conventions of this system cannot be fully understood through observation and experience, so it is essential to learn the rules and conventions.

3. Dubery and Willats show that a great variety of artists' drawings from very many periods and cultures can be classified in terms of various systems of projective geometry, and Willats also shows that a similar approach can be applied to children's drawings.

4. Numerous researchers agree that the development follows a sequential order in the process of creating perspective drawings through oblique and axonometric techniques. Perspective drawing is the final stage of development and is not automatically achieved.

A review of this literature review has convinced the present author that one of the main issues involved when studying drawings is in the development of drawing systems, in which McKim's three elements can simultaneously function as a whole. In other words, it is essential to use a
system which satisfies both the rules and conventions of geometry and human vision.

A major area of drawing study is art education, where several proposals have been made on the basis of psychological or empirical studies. Catterson-Smith, for instance, applied the effect of his 'shut-eye' technique to drawing from memory. Salome investigated the important role of perception in drawing and proposed a visual perception training method of contour line drawing for drawing from life. His studies were done on the basis of the perceptual studies of Hebb and Attneave. Hebb pointed out that people respond to parts of a perceived figure by noting the direction of the lines, the angles, and the distance between points, while Attneave pointed out that they also respond to the contours of an object or pattern, especially at points of direction change or where lines are formed by texture or colour changes.

Furthermore, Maire taught his analytical drawing method (in Simmons) at a foundation course at the School of Design, Basel, Switzerland. Doblin simplified the conventional perspective drawing system and published a well-known book exclusively for product designers. Nagata and Nagata and Minato proposed an overall picture of a super-ordinate system of perspective drawing and drawing device (in Freeman's term) to bridge the gap between geometry and mental activity in drawing.

2. Problem of perspective drawing system
Numerous three-dimensional designers feel that there is not only inconvenience but also contradiction in the conventional perspective drawing system. Moorhouse, for instance, reported from his study that descriptive geometry is not regarded with enthusiasm by most students and that there are many with demonstrated ability in mathematics and science who find it extremely difficult. Furthermore, Moorhouse suspects that the difficulties encountered may well be perceptual ones (in Poole and Stanley).

The problem involved with the conventional perspective drawing system concerns the system itself. As stated above, the system simulates human vision of the three-dimensional world but not the human way of pictorial representation. The method of drawing consists of three stages: preparatory, projection, and completion.

In the preparatory stage, the draughtsman is required to define a space to draw.
1. Definition and preparation of the drawing space by the views of plan, elevation, and others,
2. Setting-up of the drawing situation defining view point, picture plane, viewing distance and orientation of the space,
3. Checking the cone of vision to avoid extreme distortion, and adjusting the set-up if necessary.

Following the preparatory stage, the projection of space is then begun. Identification of piercing points on the picture plane then proceeds by the joining of viewing points and all necessary major points of space in both plan and elevation views. Finally, in the completion stage, the projected image is constructed. All piercing points on the picture plane are joined to each other by straight lines considering the spatial location of point in terms of the front or the back. Since perspective drawing in the three stages can automatically proceed once the space has been defined in the first step, the definition is extremely important. In other words, the definition at the first step is equivalent to its projected image. More importantly, the perceptual cycle of exploration described by Neisser does not function at all in this procedure.

When considering the procedure from the viewpoint of visualisation, its definition has critical defects: First, the definition is an essential requirement in the process without any visible images in three-dimensional space. Second, when considering the procedure from a designing viewpoint, the definition means the completion of designing and does not have a marginal latitude for the acceptance of visual thinking. Third, in the nature of projective geometry, the perspective image can be constructed at the third step by the children's puzzle 'connect the dots', where the consciousness of space seems to be low because, for instance, the convergence of lines is not defined by the draughtsman himself but by the projection. Fourth, the entire procedure of drawing in this method is extremely laborious, so that it sometimes does not have the free atmosphere needed when designing.

To solve these issues, Nagata translated the conventional drawing system from the viewpoint of geometry/perception stated above and proposed a drawing
system for two-point perspective drawing. Due to the complexity of three-point perspective drawing, however, it remained as an unsolved problem.

3. Three-point perspective drawing
3.1 Configurations of principal edges of cube image

In three-point perspective drawing, no true length appears in the projected image. Odaka\(^{21}\) tackled the complexity of three-point perspective drawing giving fundamental equations, where variables are given by projected lengths of the edges of a cube, the angles formed by these edges, and the distances of the three vanishing points.

It has been found that projected images of the cube in three-point perspective drawing can be classified into nine configurations as shown in Figure 1. These are defined in terms of the lengths of edges of a cube and the angles formed by these edges.

We here assume that N is the nearest corner of the cube in Figure 1, while A and B are the left and right edges of cube, respectively, and C is the vertical edge which is located in front of the viewer. The centre line of vision is set up to the corner and the picture plane is at right angles in the visual line of sight. In this situation the cube is always located in front of the viewer, there is less distortion and misleading images can be avoided. In this situation, the vertical edge of the nearest corner always appears as a vertical line on the plane (i.e., the vertical line on paper), so that a distortion of the projected image is less likely. The cube is placed on the ground plane and swung laterally at an angle (\(\alpha\)). A vertical angle (\(\beta\)) is then swung from the picture plane. Let us call the three edges of the cube from the N point, "the Principal Edges."

This situation examined how the lengths of the principal edges of A, B and C change with \(\alpha\) and \(\beta\). It was also calculated that the angles defined by A and B at N, form a horizontal line on the picture plane. The diagram shows a summary and schematic pattern of the changes of the lengths and angles.

3.2 Comparison of angles and relative lengths

In the diagram, there is one special case involved with these three principal edges A, B and C. This is when they are of equal length and form a 120° angle symmetrically.

Actually, \(\alpha\) and \(\beta\) are 45° and 35°16' (\(\cos^{-1}\sqrt{2/3}\)) respectively. Placing the formation in the centre, the lengths of A, B and C vary according to angles \(\alpha\) and \(\beta\). Therefore, the larger \(\alpha\) is, the longer A becomes and the shorter B becomes.

The length of C is independent of the angle \(\alpha\), but is dependent on the angle of \(\beta\); the bigger \(\beta\) is, the shorter C becomes.

From these results we can conclude that,
(a) The closer to the picture plane the edge is, the longer the image of the principal edges become, and vice versa,
(b) The closer to the picture plane the edge is, the smaller the angles of images become, and vice versa,
(c) The length of C is independent of A and B, and
(d) At \(\alpha=45^\circ\), the lengths of A and B are identical, and angles of \(\alpha=\beta\).

The larger \(\beta\) is, the slimmer the Y-shaped configuration becomes, while the smaller \(\beta\) is, the wider the Y-shaped configuration becomes. Figure 2 demonstrates how the proportional lengths of edges of A, B and C vary according to the angles of \(\alpha\) and \(\beta\). In this figure, the three equal length edges of A, B, and C emerge at \(\alpha=45^\circ\) and \(\beta=35^\circ16'\). Similarly, Figure 3 shows the angles formed by A, B and C, where, at 120°, angles formed by A, B, and C are identical.

4. Proposed Method

In the previous section on perspective drawing, the conventional drawing procedure was segmented into three stages; preparatory, projection, and completion. Nagata\(^{22}\) however, interpreted the process as disassembling a
completed image of a cube into 12 edges and reassembling them simulating a normal depiction of a cube in a step-by-step manner. He found that in this process that a set of four principal edges can be defined without any geometric constraints; however, the other eight edges can be depicted only by following geometric constraints. The definition of the first four edges indicates an approximate spatial location; above or below eye level and right or left orientation. This free definition of spatial location of an object by means of the four principal edges of a cube differs from plan and elevation views in conventional projection.

Using this knowledge, the following method of three-point perspective construction can be proposed.

The construction consists of six steps which mainly include definitions of the principal edges and a diagonal line of top surface, and construction of the top and side surfaces. These steps are:

- Step 1: Imagine a completed cube in three-point perspective and define an edge configuration representing the three principal edges (as shown in Figure 1) stemming from the nearest corner (N).
- Step 2: Define a diagonal line across the top surface by

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Figure 2 Proportional lengths of three principal edges of cube in three-point perspective drawing in various lateral and vertical rotational angles

Figure 3 Angles formed by three principal edges of cube in three-point perspective drawing in various lateral and vertical rotational angles
using five sub-steps. This method is based on a geometric fact that a right-angled triangle inscribes a semi-circle.

1. Draw a horizontal line to intersect the principal edges.
2. Swing an arc through the intersections.
3. Join one of the intersected points with an intersected point formed by the arc and vertical edge.
4. Draw line 4, forming an angle of 45° with line 3 and extend line 4 to intersect line 1 (see Figure 4).
5. Join point N to the point of intersection on line 1. This line 5 becomes a diagonal line receding to a diagonal vanishing point.

- Step 3: Draw two lines to meet on the diagonal line from the both ends of the horizontal line drawn in Step 2. The quadrangle thus formed is a miniature of the top surface of the cube. To enlarge the top surface, draw parallel lines from the right and left corners of the top surface. With this operation, the lengths of the initially defined principal edges of the top surface become firmly determined.

- Step 4: Definition of vertical edges.
 Join the right and left corner of the miniature top surface to the lower end of the vertical edge (one of the miniature vanishing points). In this operation, the entire configuration of three-point perspective can be seen in a small scale (lines 1 and 2, Step 4, Figure 4).

Draw parallel lines receding to the vanishing point from right and left corners of the top surface (lines 3 and 4, Step 4, Figure 4).

- Steps 5 and 6: Finally, draw the bottom lines of the cube so that they recede to the right and left vanishing points. Sets of similar triangles may be simple to use. Thus, completion of the cube construction.

5. Discussion

As demonstrated above, three-point perspective drawing was completed and used a procedure consistent with that of two-point perspective. As stated earlier, the difference between conventional projective drawing and the proposed drawing methods is in the first stage of procedures (i.e., the conventional manner uses a rigid definition of the drawing object while the proposed method uses a loose definition). The former is appropriate for visualisation of a given situation, while, on the other hand, the latter is good for construction of an imaginary space such as needed in designing. The latter can be exemplified by photography, where a cameraman does not always pay attention to distance, angle, and so, but on the image in the viewfinder. Therefore, these elements of distance and angle will always work out if the image itself is visualised appropriately. Consequently, draughtsmen and cameramen need to have a sensitive eye to decide how a drawing or photograph should be represented.

Furthermore, the method demonstrated earlier uses the assumption that the frontal edge (C of Figure 1) of the three principal edges appears perpendicular on the picture plane. To avoid this assumption, there are two possible solutions; (1) expansion of the cube to any direction or (2) tilting the cube by means of rotating the paper in any direction. In the former case, the line joining the vanishing points on the right and left is still horizontal, and in the latter case, the line can be tilted accordingly.

6. Conclusion

The problem raised earlier in the present study was solved and proved by computation of the length of the principal edges and the angles formed by these edges. The present author has taught this method in his drawing class for three years. It also has proved to be that simpler and faster to use than the conventional one, and there have been no complaints about it.

However, the problem that clearly emerged was the draughtsmen's perceptive ability. As demonstrated earlier, this method accepts the draughtsman's drawing
intentions in the process of construction as much as possible at Steps 1, 3 and 4. In Step 4, in particular, the length of the vertical edge should be carefully determined because an extremely tall definition makes the cube image look like a column, which would be a distortion of the perspective drawing. The only factor that will prevent this from happening is the draughtsman’s perceptual ability, which is beyond the present study.

Notes and References

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\begin{aligned}
\frac{a^2}{b^2} - \frac{ca \cos \beta - ab \cos \gamma + bc \cos \delta}{b^2 - ba \cos \gamma + bc \sin \alpha + ca \cos \beta} &= \left[ \frac{d}{b-e} \right]^2 \\
\frac{b^2}{e^2} - \frac{ab \cos \gamma - bc \cos \alpha + ca \cos \beta}{e^2 - ba \cos \alpha - ca \cos \beta + ab \cos \gamma} &= \left[ \frac{f}{e-c} \right]^2 \\
\alpha + \beta + \gamma &= 4R \angle
\end{aligned}
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