NUMERICAL INVESTIGATION OF MAGNETO PLASMA SAIL USING IDEAL MAGNETOHYDRODYNAMIC EQUATIONS

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ABSTRACT

The interaction between the solar wind and the magnetic field created around the spacecraft is investigated by utilizing two-dimensional ideal magnetohydrodynamic (MHD) equations to estimate the thrust of Magneto Plasma Sail (MPS). In this propulsion system, the dynamic pressure of the solar wind is converted into the aerodynamic drag force to the magnetosphere created around the spacecraft through the MHD interaction. When the dynamic pressure of the solar wind is 0.753 [nPa] and the magnetic dipole moment is 5000 [Tm^3], the estimated thrust of MPS is 1.45 [N], which is close to the theoretically estimated values.

Nomenclature

- \( \rho \) Density
- \( \mathbf{v} \) Velocity
- \( v_f \) Fast Alfvén wave speed
- \( \mathbf{B} \) Magnetic field vector, \( \mathbf{B} = (B_x, B_y, B_z)^T \)
- \( c \) Speed of sound
- \( D \) Drag force to the magnetosphere
- \( e \) Total energy per unit weight
- \( M_a \) Mach number
- \( m \) Magnetic dipole moment
- \( \mathbf{n} \) Unit vector normal to the cell interface
- \( p \) Static pressure
- \( R_{MS} \) Radius of magnetosphere
- \( \gamma \) Specific heat ratio, 5/3

Subscript

- \( L/R \) Left / Right at cell interface
- \( i+1/2,i-1/2 \) Index for numerical flux
- \( MS \) Magnetosphere
- \( n \) Normal to the cell interface
- \( SW \) Solar wind

1. INTRODUCTION

Various kinds of deep space mission to outer planets such as Jupiter, Saturn, and Pluto are now being planned. For such missions an innovative propulsion system is required to reduce the mission term and cost. One of the innovative propulsion systems is Magneto Plasma Sail (MPS) originally proposed by Winglee (Ref. 1). In this propulsion system, the solar wind, high-speed plasma flow, is deflected by the magnetic field expanded around the spacecraft and the dynamic pressure of solar wind is converted into the aerodynamic drag force to the magnetic field, that is, the thrust of the
spacecraft.

Based on a simple Newtonian flow theory, the thrust of 8.2 [N] can be extracted from the momentum of solar wind when we can prepare the magnetic field with a radius of 25 [km] and the dynamic pressure of solar wind of 2.1 [nPa] (Ref. 2). On this estimation, the thrust is dependent on the size of the magnetic field. In general, the shape of the magnetic field is changed by the plasma flow when the magnetic Reynolds number is large. Thus, it is very difficult to estimate the size and shape of the magnetic field. In other words, in order to estimate accurately the thrust of this system, we need to understand how the magnetic field around the spacecraft is formed.

For that purpose, we performed CFD analyses to estimate the thrust of MPS. Especially, we focus on the size of the magnetic field created around the spacecraft. In the present study, we selected ideal magnetohydrodynamic (MHD) equations as a governing equation because the problem which we are considering now is expected to be similar to the flow structure in the interaction between the magnetic field of the Earth and the solar wind.

2. GOVERNING EQUATION

In the present analysis, we used ideal MHD equations. The flow is assumed to be two-dimensional one for simplicity. The equation is composed of the following equations;

- Conservation of mass (1 equation)
- Conservation of momentum (3 equations)
- Faraday's law (3 equations), and
- Conservation of energy (1 equation).

This set of equations can be written in the following vector form.

\[
\frac{\partial U}{\partial t} + \nabla \cdot F = S
\]  \hspace{1cm} (1)

where \(U, F,\) and \(S\) are vector of conservative (non-dimensional) variables, flux vectors, and source term, respectively as shown below.

\[
U = \begin{pmatrix} \rho \\ \rho v \\ B \\ \rho e \end{pmatrix}, \quad F = \begin{pmatrix} \rho v \\ \rho v^2 + I(p + B \cdot B/2) - BB \\ Bv - vB \\ (\rho e + p + B \cdot B/2)v - (v \cdot B)B \end{pmatrix}, \quad S = \begin{pmatrix} 0 \\ B \\ v \\ v \cdot B \end{pmatrix}
\]  \hspace{1cm} (2)

where \(I\) is the unity matrix and \(e\) is the total energy per unit weight defined as follows

\[
\rho e = \frac{p}{\gamma - 1} + \frac{1}{2} \rho v \cdot v + \frac{1}{2} B \cdot B.
\]

The source term, \(S,\) reveals when the constraint of the divergence free condition for the magnetic field is not imposed. This indicates that the divergence of magnetic field is not identically zero. By using this formulation, we could keep the numerical stability and remove the process of cleaning the magnetic field in the CFD analysis (Ref. 3).

3. NUMERICAL METHODS

In this study, we selected TVD Lax-Friedrich scheme as shown in the following form.
\[ F'(U_L, U_R) = \frac{F(U_L) + F(U_R) - (v_n + c_{n}^{f}) U_R - U_L}{2} \] (3)

where \( v_n \) is the velocity normal to the cell interface and \( c_{n}^{f} \) is the fast Alfvén wave speed. Thus \( |v_n| + c_{n}^{f} \) is the largest wave speed in the direction normal to the cell interface. This numerical flux is one of the most dissipative numerical fluxes. In order to keep the spatial accuracy we need to utilize a reconstruction method. For that purpose, MUSCL formulation is very popular in typical CFD analyses.

In the present study, we could not obtain the converged solution due to the numerical instability around the shock wave when we used typical limiter functions such as Van Albada limiter. In order to conserve both the numerical stability and the spatial accuracy we employed the cubic limiter. The limiter function is expressed as follows.

\[
\phi_{i+1/2,L} = \phi_i + \Psi(r_i)(\phi_i - \phi_{i-1})/2, \quad \phi_{i+1/2,R} = \phi_{i+1} - \Psi\left(\frac{1}{r_{i+1}}\right)(\phi_{i+2} - \phi_{i+1})/2
\]

where \( r \) is the change of variable between the neighboring two cells. This limiter function is relatively dissipative among the typical limiter functions for the MUSCL formulation but can conserve the shock properties such as a pressure jump.

Equation (1) is discretized in the generalized coordinate by using the finite volume method. The right hand side of eq. (1) is treated as a source term. And eq. (1) is integrated in time by Euler explicit method. A CFL number is calculated by the largest wave speed shown in eq. (3). A CFL number is set to be between 0.1 and 0.3.

4. RESULTS AND DISCUSSIONS

Assessment of the code

At first, we performed test calculations to evaluate the accuracy of the code. The test calculation is a supersonic plasma flow around a cylinder with the magnetic field created by a line dipole. The computational grid is created around a cylinder in the cylindrical coordinate. The computational grid has 121 points along the surface and 251 points in the direction normal to the body surface. The inflow condition is that the density is 1.0 and the static pressure is 0.5. The inflow Mach number is 2.5. The specific heat ratio of the gas is 5/3. The magnetic field vector is expressed as follows.

\[ B = m\nabla\left(\frac{y}{x^2 + y^2}\right) \] (5)

where \( m \) is the dipole moment. In this calculation, \( m \) is set to be 350.

As the boundary conditions, the inflow condition is fixed at the initial values. At the outflow boundary, the values are determined by 0th order extrapolation. On the surface of the cylinder, the static pressure and the density is determined by the extrapolation. The magnetic field vector is fixed at the initial value. The radius of the cylinder is set to be 10. The radius of the outer boundary of the computational grid is set to be 400.
Fig. 1, Distribution of static pressure (top left), Mach number (top right), density (bottom left), and magnetic field strength (bottom right) in the case of m=350. The bow shock wave and the magnetosphere are created in front of the cylinder.

Figure 1 shows the distribution of static pressure, Mach number, density, and the magnetic field strength around the cylinder. From the distribution of static pressure and Mach number, we can observe that the bow shock and the magnetosphere are created in front of the cylinder. The magnetic field is compressed and the strong magnetic field area is created around the stagnation region of the cylinder. This area is called a magnetosphere.

As you can see, inside the magnetosphere, the Mach number is very small. This means that the solar wind does not come into the magnetosphere and the magnetosphere behaves like a solid wall to the solar wind. Thus, as shown in Fig. 2, the shock stand-off distance is larger than the one in the case that the magnetic field is not applied. Additionally, the density inside the magnetosphere is very low due to the same reason.
Fig. 2, Distribution of the static pressure around the cylinder with (top) and without magnetic field (bottom)

Figure 2 shows the comparison of the static pressure distribution for the cases of with and without the magnetic field. When the magnetic field is applied, the magnetosphere is created around the cylinder. Therefore, the shock stand-off distance in the case that the magnetic field is applied is much larger than that in the case that the magnetic field is not applied. The change of the shock stand-off distance leads to the change of the thrust of MPS.

Figure 3 shows the distribution of static pressure and density along the stagnation line of the windward side of the cylinder. From this figure, we can observe that both the bow shock and the magnetosphere are captured very clearly. And the pressure jump at the bow shock is well calculated even with the limiter function used in our code.

In order to assess the accuracy of our code, we estimated the size of the magnetosphere. The size of the magnetosphere is dependent on the dynamic pressure of the inflow gas and the dipole moment. The relationship is as follows,

$$\rho_{sw}v_{sw}^2 = \frac{2m^2}{R_{MS}^4}$$

(6)

The subscript SW means the value of the solar wind and the subscript MS means the value at the magnetosphere. The radius of the magnetosphere can be estimated by using eq. (6).

$$R_{MS} = \left( \frac{2 \cdot m^2}{M_\rho^2 (\gamma P / \rho)} \right)^{\frac{1}{4}} = \left( \frac{2 \cdot 350^2}{2.5^2 (5 / 3 \cdot 0.5)} \right)^{\frac{1}{4}} = 15$$

From Fig. 3, the size of the magnetosphere obtained by our code agrees well with the theoretical value. Thus, we concluded that our code can predict the size of the magnetosphere and can reproduce the flow features in the interaction between the supersonic plasma flow and the magnetic field.
**Thrust estimation of MPS**

The calculation condition for estimation of the thrust of MPS is shown in Table 1. In this section, we investigated the effect of the dipole moment on the thrust of MPS. The dipole moment is varied between 0.5 and 5000 [Tm^3]. When the dipole moment is strong, the magnetic field is extended widely. Thus, the more solar wind is captured and converted into the thrust. In this calculation, the spacecraft is replaced by a cylinder. When the dipole moment is very large, the magnetic field near the spacecraft is very strong and the calculation cannot be finished due to the numerical instability related to the strong magnetic field near the dipole. Thus, the inner radius of the cylinder is changed so that the magnetic field on the surface of the cylinder is kept constant according to the change of the dipole moment. The outer radius of the cylinder (the outer boundary of the computational domain) is also varied from 100 [km] to 400 [km]. These values are considered to be large enough to cover the shock wave created around the cylinder.

Figure 4 shows the distribution of static pressure around the spacecraft (cylinder) for the cases of m=0.5 and 5. As shown in the previous section, the bow shock and the magnetosphere are created around the spacecraft. As expected, the smaller the dipole moment is, the smaller the magnetosphere is. The balance between the dipole moment and the dynamic pressure of the solar wind determines the size of the magnetosphere.

**Table 1, Calculation condition**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number Density [m^-3]</td>
<td>5.0e6</td>
</tr>
<tr>
<td>Velocity [km/s]</td>
<td>300</td>
</tr>
<tr>
<td>Temperature [eV]</td>
<td>20</td>
</tr>
<tr>
<td>Static Pressure [nPa]</td>
<td>0.032</td>
</tr>
<tr>
<td>Dynamic Pressure [nPa]</td>
<td>0.753</td>
</tr>
</tbody>
</table>
Fig. 4, Distribution of static pressure around the spacecraft with the dipole moment of 0.5 (top) and 5 (bottom) \([\text{Tm}^3]\). The size of the magnetosphere is proportional to the dipole moment.

Fig. 5, Relationship between the magnetic dipole moment and the thrust of MPS

Figure 5 shows the relationship between the dipole moment and the thrust. The dipole moment has a direct impact on the size of the magnetosphere. Thus, it is natural that the thrust is dependent on the dipole moment. The thrust estimated by Newtonian flow theory is calculated by the following formula.
Table 2. Calculation results for estimation of thrust of MPS

<table>
<thead>
<tr>
<th>Dipole Moment [Tm3]</th>
<th>Thrust [N]</th>
<th>R_{MS} [km]</th>
<th>Size of spacecraft (inner radius) [km]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1.86e-4</td>
<td>0.20</td>
<td>0.090</td>
</tr>
<tr>
<td>5</td>
<td>1.87e-3</td>
<td>0.66</td>
<td>0.285</td>
</tr>
<tr>
<td>50</td>
<td>1.82e-2</td>
<td>2.14</td>
<td>0.900</td>
</tr>
<tr>
<td>5000</td>
<td>1.45</td>
<td>22.14</td>
<td>9.000</td>
</tr>
</tbody>
</table>

\[ D = \rho_{SW} \nu_{SW}^2 (2R_{MS}) \cdot (2R_{MS}) \]  
\( (7) \)

And the thrust by the present CFD analyses is calculated by the integration of the momentum along the outer boundary of the computational domain and the radius of the magnetosphere as shown in the following formula.

\[ D = \oint \rho \nu (v \cdot n) dl \cdot (2R_{MS}) \]  
\( (8) \)

The results are summarized in Table 2. The radius of the magnetosphere is determined from the magnetic field distribution on the stagnation line as shown in Fig. 2. The radius of the magnetosphere is almost proportional to the square root of the magnetic dipole moment. It is reasonable because the magnetic field strength decreases inversely proportional to the square root of the distance from the magnetic dipole when the line dipole is used. As for the thrust of MPS, the thrust is almost proportional to the dipole moment. In order to confirm the thrust dependence on the magnetic dipole moment, using eq. (6) and (7), the theoretical relationship between the magnetic dipole moment and the thrust of MPS is derived in the following form,

\[ D = 4 \sqrt{2} \left( \rho_{SW} V_{SW}^2 \right)^{1.5} \times m \]  
\( (9) \)

, where \( m \) is the magnetic dipole moment. The thrust calculated by eq. (9) is also plotted in Fig. 5. The agreement between CFD result, Newtonian theory, and theoretical relationship eq. (9) is relatively well. These results indicate that the thrust of MPS is dependent on the size of the magnetosphere. It should be noted that the present CFD simulation is performed in two dimensions. Generally speaking, the drag coefficient (in other words, thrust) of cylinder is larger than that of the sphere. Thus, the thrust of MPS estimated by our present CFD simulations may be overestimated.

5. CONCLUSIONS

In this study, we investigated the thrust of MPS by using ideal MHD equations. We found that the thrust of MPS is almost proportional to the dipole moment. This is because the thrust of MPS is dependent on the size of the magnetosphere created around the spacecraft and the balance between the dynamic pressure of solar wind and the dipole moment determines the size of the magnetosphere. This indicates that the size of the magnetosphere is dependent on the magnetic dipole moment. To confirm our CFD results, we derived the theoretical relationship between the magnetic dipole moment and the thrust of MPS as shown in eq. (9). This relationship gives us a rough but good estimation of the thrust of MPS. The thrust of MPS estimated by our CFD analysis is 1.45 [N] with the magnetic dipole of 5000 [Tm^3]. The agreement among CFD analyses, Newtonian theory, and eq. (9) is relatively good.

In the present study, we did not take account of the inflation of the magnetic field, which is a key problem of this propulsion system from the viewpoint of feasibility of this system. In the future, for practical evaluation of this propulsion system, we should perform CFD analyses including the inflation of the magnetic field by injection of plasma.
REFERENCES

