NUMERICAL ANALYSIS ON CHARACTERISTIC FEATURES OF SOLAR WIND FLOW AROUND MAGNETIC SAIL

Daisuke AKITA and Kojiro SUZUKI

Dept. of Advanced Energy, Graduate School of Frontier Sciences
University of Tokyo
7-3-1 Hongo, Bunkyo, 113-0033 Tokyo, JAPAN
+81-3-5841-6573
akita@sonic.t.u-tokyo.ac.jp

ABSTRACT

The magnetic sail utilizes the momentum of the solar wind (high-speed plasma flow) to produce the thrust by an applied magnetic field. Since the momentum flux from the sun is transformed into the thrust like a solar sail, no propellant is required and infinite Isp performance is available. Hence, it has a potential to reduce drastically the duration for a deep space mission because it can accelerate the spacecraft up to its theoretical limit, that is, the speed of solar wind. In the present study, its acceleration performance and scales of the phenomena in relation to the solar wind flow around the magnetic sail are roughly estimated. Based on the estimation, the interaction of the solar wind with the applied magnetic field is numerically simulated by the full particle (PIC) method. Fundamental features of the flow field and the induced electromagnetic field around the magnetic sail are clarified. Force acting upon the magnetic sail is also estimated by considering the momentum change of the solar wind.

1. INTRODUCTION

Outside of the heliosphere, which is formed by the interaction between solar wind and the interstellar medium, is unexplored region about which we know very little. Several recent estimates place this region at 100 to 150 AU from the sun. Sending a spacecraft beyond the heliosphere to explore the interstellar region will be one of the greatest scientific enterprises and will bring us the keys to understand the universe. One mission for such deep space exploration is the Interstellar Probe [1] mission proposed by NASA/JPL.

In order to complete such missions in practical duration, say 10 years, various technological innovations are required, for example, advanced propulsion system that works for very long time, the development of highly automated spacecraft system and very long range communication systems. The Voyager1, 2 and Pioneer probes are now traveling into the interstellar region, with speeds between 2.2 and 3.5 AU/year (10.5 and 16.6 km/sec) gained by gravity assist. However, the Voyager 1 has been reaching only about 94 AU from the sun after 27 years' flight. A far higher ΔV capability of over 20 AU/years (95 km/sec) is required to complete a mission within 10 years. Such large ΔV is beyond the ability of the chemical propulsion even when the gravity assist technique is used together. As for electric propulsion, the power requirements would be of the order of Mega Watts, which seems to be prohibitively large at the present state of the art. Therefore, necessary technologies for reducing drastically the mission duration should be pursued not in improvements of the conventional space propulsion systems, but in a breakthrough in the propulsion systems.
One attractive propulsion concept is the magnetic sail by Zubrin and Andrews [2], [3] (1989), who proposed to use the momentum of the solar wind to produce the thrust. Since the momentum flux originated from the sun is used to accelerate the spacecraft like a solar sail, propellant is not required for the magnetic sail. Although the thrust of the magnetic sail is very small due to very low density of the solar wind, the spacecraft is continuously accelerated as long as there is a difference between the solar wind velocity and the spacecraft velocity. Therefore, the magnetic sail has a potential to produce large ΔV and to reduce drastically the mission duration compared with the conventional space propulsion systems.

In the magnetic sail system, the thrust is generated by the interaction of the ambient solar wind with an artificial magnetic field applied by the spacecraft. In the original papers of Zubrin and Andrews, the applied magnetic field is generated by a loop current, which is provided by the power system of the spacecraft. If the loop cable is made from a superconducting material, the power requirements can be negligible, except for the electric power to initiate the loop current and to maintain the cryogenic system to keep the cable in the superconducting state. The conceptual diagram of the magnetic sail is illustrated in Fig. 1, in which the diameter of the loop cable is planned to be tens of kilometers. The solar wind is a high-speed rarefied plasma flow mainly composed of protons and electrons. Its speed and number density are approximately 400km/sec and 2–10/cm³, respectively, at the earth’s orbit. They vary with the distance from the sun. If the solar wind blows with some relative velocity to the spacecraft, trajectories of the charged particles of the solar wind are deflected by Lorentz force according to the magnetic field applied by the loop current of the spacecraft. The momentum change of these particles produces the drag force on the magnetic sail loop in the direction to the incoming solar wind. The drag force continuously accelerates the spacecraft in the direction of the incoming solar wind relative to the spacecraft’s motion, and the maximum speed available would be that of the solar wind itself.

The magnetic sail has the potential advantages in several key areas in comparison with the conventional space propulsion systems. First, no propellant is required to produce the thrust so that infinite Isp performance is available. Second, the thrust is generated just by putting a current on the loop cable without complicated mechanical equipments and structures, since the shape of the loop is kept automatically by the presence of hoop stress acting on it. Third, the magnitude of the thrust can be easily varied by regulating a current of the cable. Forth, the applied magnetic field is also expected to work as a shield to protect the spacecraft from high-energy particles in the solar wind.

In contrast, it also has disadvantages. The magnetic sail requires a huge loop cable whose diameter is tens of kilometers because the density of the solar wind is very low. Figure 2 shows the comparison between the dynamic pressure of the solar wind and the solar radiation pressure that drive the magnetic sail and the solar sail, respectively, as a function of the distance from the sun. Both the dynamic pressure of the solar wind and the radiation pressure decrease inversely proportional to the square of the distance from the sun. However, the former is much smaller than the latter by three orders of magnitude at any distance from the sun. Therefore, if the magnetic sail can generate the magnetic field whose diameter is dozens of times larger than that of solar sail’s foil, the magnetic sail would produce the same magnitude of the thrust as the solar sail. In other words, a magnetic sail whose loop cable is 1km in radius can produce almost the same propulsive force with a solar sail of 100mX100m foil. If the former can be deployed in an easier way than the latter from a viewpoint of the structural dynamics, the magnetic sail seems promising as a space propulsion system for deep space missions in the future.

As for the magnetic sail, however, details of the interaction phenomena between the solar wind and the applied magnetic field has not been well understood, though it is a key process of transforming the
momentum of the solar wind into the propulsive force of the spacecraft. Since the physical mechanism of the plasma flow around the magnetic sail is quite complicated and it involves phenomena of many different characteristic length scales, a fully kinetic approach, which can describe both the small-scale physical processes and the large-scale ones, should be adopted for basic understanding of the phenomena in relation to the magnetic sail.

From the above considerations, the interaction of the solar wind with the applied magnetic field is numerically simulated by the full particle method (PIC method) in the present study. The fundamental features of the solar wind flow and the electromagnetic field around the magnetic sail are investigated. Force acting upon the magnetic sail is also estimated by the considering the momentum change of the solar wind.

![Fig. 1 Conceptual Illustration of Magnetic Sail (Zubrin and Andrews [2], 1989)](image1)

![Fig. 2 Dynamic Pressure of Solar Wind and Radiation Pressure for Distance from the Sun](image2)

2. METHODOLOGY

In order to make an appropriate model for the numerical analysis, the relation between the loop radius and the sail performance, and the length and time scale of the interaction phenomena between the solar wind and the applied magnetic field should be considered beforehand.

ESTIMATION OF MAGNETIC SAIL PERFORMANCE

First, the relation between the loop size and the sail performance will be obtained after the estimation method by Zubrin and Andrews [2], [3]. At the stagnation point of the magnetopause, the dynamic pressure of the incoming solar wind is balanced with the magnetic pressure of the applied dipole magnetic field:

\[
\frac{1}{2} \rho v^2 = \frac{B_{mp}^2}{2\mu_0},
\]

where \( \rho \) and \( v \) are the mass density and mean velocity of the solar wind. \( B_{mp} \) and \( \mu_0 \) are the magnetic flux of the applied magnetic field at the magnetopause and magnetic permeability, respectively. Since the size of the magnetopause is much larger than the radius of the loop, the magnetic field produced by a loop current is assumed to decrease inversely proportional to the cube of the distance from the spacecraft. Then, the magnetic flux at the magnetopause is written by:
where $B_0$ is magnetic flux at the center of the loop current. $r_0$ and $r_{mp}$ are radius of the loop and distance of the magnetopause from the center of the loop, respectively. The magnetic flux at the center of the loop current is given by $B_0 = \mu_0 g A/(2r_0)$, where $A$ and $j$ are the cross-sectional area and current density of the loop cable generating the magnetic field. The loop cable is assumed to be made from the low temperature superconductive material whose critical current density and the mass density are $10^{10}$ A/m$^2$ and 5000 kg/m$^3$, respectively, considering the present state of the art in the superconductive materials. Assuming that the force acting on the magnetic sail is equal to that on a rigid body having the same shape as the magnetopause, the propulsive force $F$ of the magnetic sail is approximately given by:

$$F = \frac{1}{2} \rho v^2 C_D \pi r_{mp}^2,$$

where $C_D$ is the drag coefficient assumed to be unity in the present model.

Consequently, from the eqn. (1)-(3) and specifications of the loop cable, the acceleration $a$ obtained by the magnetic sail is then

$$a \propto \left( \frac{r_0}{A} \right)^{\frac{1}{3}}.$$

Figure 3 shows the variation of the loop radius and loop mass with the acceleration performance ($\Delta v$) of the magnetic sail. In this case, the diameter of the loop cable is set as 2mm. To obtain the $\Delta v$ of about 1 km/sec per month, for example, the radius and the mass of the magnetic sail must be in the order of 10 m and 1 kg, respectively. In the present study, a small magnetic sail of 10 m in radius and 1 kg in mass, which seems to be reasonable size as a test vehicle realizable within the present technology level on the space structure, is considered.

**SCALE OF PHENOMENA**

The solar wind is a high-speed rarefied plasma flow mainly composed of protons and electrons. The mean free path with respect to Coulomb collision (electron to ion) of the solar wind particles is about 1 AU or more at the earth's orbit. Therefore, the solar wind can be considered as the collisionless and perfectly ionized plasma flow. In the present analysis, the solar wind conditions at the earth's orbit [7] are considered as shown in Table 1.

The electron Debye length $\lambda_D$ and the reciprocal of the electron plasma frequency $\omega_p$ of the solar wind particles, which are the characteristic length and time scale for an interaction with an electric field, are in the order of 10 m and $10^4$ sec, respectively. The cyclotron radius $\lambda_c$ and the reciprocal of cyclotron frequency $\omega_c$ of the solar wind particles, which are the characteristic length and time scale for an interaction with a magnetic field, are in the order of 100 km and 0.1 sec for a proton and 100 m and $10^4$ sec for an electron, respectively, at the magnetopause of the applied magnetic field. On the other hand, the characteristic length scale of the interaction between the solar wind and the magnetic sail is assumed to be the standoff distance of the magnetopause from the loop center of the magnetic sail (i.e. $r_{mp} \sim$ hundreds of meters under the condition assumed in preceding section). The characteristic time scale is the time required for the solar wind to pass the characteristic distance at the mean freestream velocity (i.e. $t \sim$ at the order of $10^2$ sec under the same condition). Comparing these parameters:

$$\lambda_D < \lambda_c, r_{mp} \ll \lambda_c, t,$$

$$\omega_p < \omega_c, t < \omega_c^{-1}.$$  

Subscripts $e$ and $i$ denote electron and ion, respectively. $\lambda_c, t$ and $\omega_c^{-1}$ are much larger than $r_{mp}$ and $t$, respectively.
respectively.

Velocity distribution of nonequilibrium collisionless plasma is generally relaxed toward the Maxwellian distribution in time scale of $\omega_{p}^{-1}$, $\omega_{ce}^{-1}$ or $\omega_{ci}^{-1}$ so that a fluid model would be valid only when the characteristic time scale of a phenomenon is sufficiently larger than all of these time scales. Therefore, the particle method is expected to be reasonable compared with the fluid model governed by MHD equations in order to simulate the solar wind flow around the magnetic sail.

Figure 4 shows the variation of the magnetopause standoff distance $r_{mp}$ and the radius of influence on the motion of electron and proton with respect to the intensity of the applied magnetic field $B_{0}$. The former is estimated from eqns. (1) and (2), and the latter is obtained by the distance from the center of the magnetic sail at which their cyclotron radius is comparable to the distance from the center of the magnetic field. The diameter of the loop current is $r_{0}=10m$. In this case, $r_{mp}$ is larger than the radius of influence for the electron's motion, when $B_{0}$ is less than $10^{-3}T$esla. Even if the $B_{0}$ is about 1Tesla, $r_{mp}$ is larger than the radius of influence for the proton's motion. It means that the thrust obtained by the MHD model would be overestimated due to an unsubstantially strong interaction with the applied magnetic field. It should be noted that the affected region by the magnetic field for electron's motion is larger than that for the proton's motion by one order of magnitude. The contribution by the momentum loss of the electron cannot be neglected in the thrust of the magnetic sail, since a much larger number of electrons are affected in their motion by the applied magnetic field, while the mass of the electron is lighter than that of the proton by three orders of magnitude. The difference in the interaction scale of the motion of electron and proton under the applied magnetic field may also lead to the non-uniform potential distribution by the charge separation. The characteristic features of the plasma flow and the thrust by the magnetic sail are expected to be affected by this non-uniform electrostatic potential.

FULL PARTICLE METHOD

Figure 5 shows a flow chart of the particle method used here (PIC method [8]). The numerical method consists mainly of the calculation of the particle's motion and that of the induced electromagnetic field. Since we do not assume the electric neutrality, both an ion and an electron are regarded as particles of different species. The velocity of particles in the freestream is stochastically given by assuming the Maxwellian distribution at the freestream mean velocity and temperature. At the first stage of time stepping, the charge and current densities at the respective computational nodes are calculated from the instantaneous position and velocity of the sample particles by the PIC weighting interpolation. The electric and magnetic field are calculated from the above charge and current densities by solving the Poisson equations for the electrostatic potential and magnetostatic vector potential with the SOR method. The Lorenz force at each particle's position is calculated from the electromagnetic field obtained at the computational nodes by the PIC weighting interpolation. The equations of motion of each sample particle are integrated by the Runge-Kutta method and then the position and velocity are updated for all the sample particles. The updated particle's position and velocity are used to update the calculation of the induced electric and magnetic field. Each particle is assumed to interact individually with the electromagnetic field. The inter-particle collision is not considered, since the mean free path of the solar wind particles is much larger than the magnetopause of the magnetic sail at the earth's orbit. The present study has an interest in time-averaged flow and electromagnetic field properties around the magnetic sail so that the time averaging is started when the number of the sample particles in the computational domain becomes almost constant. The elapsed time is set to be sufficiently long in order to average out the fluctuations of the field properties due to the statistical and plasma oscillation. All the results of the field properties in the following section are thus time-averaged properties. The magnetic field applied by the loop current of the magnetic sail is calculated according to the Biot-Savart law. To save the computation time, when the distance from the
center of the loop is longer than three times as large as the loop radius, the magnetic field is calculated by assuming an equivalent dipole field.

**NUMERICAL CONDITIONS**
The freestream condition of the solar wind at the earth's orbit is used in the analysis and is shown in Table 1. The computational domain has an area of $500\text{m} \times 500\text{m} \times 500\text{m}$ composed of equally-spaced $51 \times 51 \times 51$ nodes (132,651 in total) as shown in Fig. 6, so that the grid spacing of 10.0m is smaller than the electron's Debye length of $\lambda_D = 11.6 \text{m}$. The magnetic sail is located at the center of the computational domain and the axis of the loop is set for the dipole vector of the applied magnetic field to coincide with the opposite direction of the freestream. The loop radius $r_0$ is 10m and a magnetic flux density $B_0$ is $5.0 \times 10^{-4} \text{Tesla}$ at the center of the loop which corresponds to the loop current of about 8kA and the magnetic moment of $|\mu| = 2.5 \times 10^6 \text{Am}^2$. In this case, the thrust estimated by the *eqn.* (3) is in the order of 0.1mN. However, the actual thrust is expected to be smaller than that, since the region of influence for the proton's motion by the magnetic field is smaller than $r_{mp}$ (see Fig. 4) and the momentum loss of the proton will be smaller than that estimated in the *eqn.* (3). As for the boundary conditions for the electromagnetic field, the scalar potential and vector potential of the electromagnetic field are set to be zero assuming that the electromagnetic field near the outer boundary is not disturbed from the freestream condition. The computation time spent is about 10 hours for a case by using the SX-6 supercomputer at Institute of Space and Astronautical Science (ISAS).

<table>
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![Fig. 3 Acceleration Performance and Scale of Magnetic Sail](image1)

![Fig. 4 Interaction Scales of Solar Wind with Magnetic Sail of 20m](image2)
3. RESULTS AND DISCUSSION

Figure 7 shows the electron number density distribution on the plane of symmetry involving the dipole vector with the plot of the three-dimensional streamlines of the electron of the solar wind around the loop current. As observed, the number density distribution is almost axisymmetric. It decreases in the proximity of the loop current, since most of the electrons cannot reach there due to deflection of their trajectories by the applied magnetic field. However, both in front of and in the back of the proximity, increase in the electron number density is observed. In the wake region, it decreases again. The streamlines of the electrons start to be turned in the counterclockwise direction in front of the loop when the distance from the center of the loop decreases to almost the cyclotron radius of the electron in the region (the distance is about 210m in this case). Behind the loop current, however, the electron streamlines are turned in the clockwise direction by the dipole magnetic field. Such tendency is also shown in reference 9. The induced current around the loop is mainly caused by the motion of electrons. It should be noted that the time step size set in this simulation may be larger than the reciprocal of the electron's cyclotron frequency in the proximity of the loop current and that the accuracy on the electrons may be worse than that on the protons.

Figure 8 shows the results for the proton. As can be observed, the number density of proton increases at the region inside the loop current and decreases in the wake region behind it. The wake pattern formed in the number density distribution looks like a tail of a comet. The streamlines of the proton are slightly deflected by the applied magnetic field at the region just ahead of the loop current. The distance at which the proton's motion starts to be affected by the applied magnetic field approximately coincides with the radius of influence estimated in Fig. 4 (~10m). Comparing the results for the electrons and the protons, the length scale of the interaction with the applied dipole magnetic field is significantly different. It is mainly caused by the difference in the particle mass and mean thermal velocity in the freestream between the electron and the proton. The thermal velocity of the electron of about 2000km/sec is much higher than the mean velocity of the freestream, as shown in Table 1.
Hence, in the case of the electron, the region affected by the applied magnetic field is much larger than that of the proton as also shown in Fig. 4.

The number density distribution and streamlines of proton would be affected not only by the applied magnetic field, but also by the induced electromagnetic field. Indeed, the electrostatic potential distribution shown in Fig. 9 indicates the presence of positive charging of approximately 50V around the loop current. It is mainly caused by unbalance of the number density between electrons and protons shown in Figs. 7 and 8. The electrostatic force due to the charge separation would thus also affect the number density distribution and streamlines of the protons and electrons. The effect increases the momentum loss of the proton, although the maximum induced potential is small in comparison with the kinetic energy of proton of about 1 keV.

Figure 10 shows the magnetic flux density distribution of the induced magnetic field in the freestream direction. It is normalized by the value of the applied magnetic field at the center of the loop current. The curved streamlines of the electron around the loop shown in Fig. 7 dominantly generate an induced current and the induced magnetic field is produced. The numerically integrated magnetic field in the direction of the freestream has negative gradient, so that it yields a force acting on the dipole in the direction of the freestream under the condition. In the present numerical conditions, the induced magnetic field is much weaker than the applied one and the effect on the flow field is vanishingly small. However, care must be taken when a force exerted on the loop current is calculated by using the induced magnetic field.

Figure 11 shows the velocity distribution function of the proton in the freestream direction obtained at the region around the loop, comparing with the Maxwellian distribution in the freestream. In this case, the shape of the distribution function of proton is asymmetric with respect to the decelerated mean velocity and significantly different from the Maxwellian distribution. A decrease in population of high-energy particles and increase in that of low-energy particles is observed in comparison with the freestream Maxwellian distribution. Particles at higher velocity are decelerated more significantly in the direction, since the Lorentz force by the applied magnetic field is proportional to the velocity of a charged particle. The result for the electron is also different from the Maxwellian distribution as shown in Fig. 12. The mean thermal velocity of the electron of about 2000km/sec is much higher than
the mean velocity of the freestream. At the region around the loop, the mean velocity of the freestream of the electron is almost zero and a significant decrease in low-energy particles is observed, since they cannot reach there due to the deflection of their trajectories by the applied magnetic field (see Fig. 7). Therefore, these results indicate that the kinetic nonequilibrium effects must be considered for both the electrons and protons at the simulation of the solar wind flow around the magnetic sail of the size assumed here.

![Fig. 9 Electrostatic Potential Distribution on Plane Involving Dipole Vector](image)

![Fig. 10 Induced Magnetic Flux Density Distribution of Solar Direction on Plane Involving Dipole Vector](image)

![Fig. 11 Velocity Distribution Function in Solar Direction of Proton around Loop](image)

![Fig. 12 Velocity Distribution Function in Solar Direction of Electron around Loop](image)

The drag force estimated from the time-averaged momentum loss of the protons in the freestream direction is in the order of $10^{-4}$ mN. The momentum loss of the solar wind is calculated from difference in the momentum of all the sample particles incoming into the computational domain and outgoing from that. As expected, it is lower than the predicted value by eqn. (3) of $10^{-1}$ mN. If the radius in eqn. (3) is the radius of influence for the proton's motion, however, the drag force estimated from the momentum loss of the protons is in the same order of the value by eqn. (3) (not using $r_{mp}$). This means that on the magnetic sail of the size assumed here, the radius of influence for the particle's
motion predominantly determines the interaction length scale of the solar wind with the magnetic sail and thus MHD model is not appropriate for the simulation from the consideration of the drag force on the magnetic sail.

In order to consider the effects of the induced electromagnetic field, a case without the induced electromagnetic field is also computed. In the absence of the induced electromagnetic field, the motions of the protons and electrons are separately calculated with the Lorentz force by the applied magnetic field and they never affect each other. Figures 13, 14 show the number density distribution and the streamlines of electron and proton, respectively, corresponding to the Fig. 7, 8 in which the induced electromagnetic field is considered. Comparing the Fig. 7 to Fig. 13, it is clarified that the number density of the electron around the loop current slightly increases due to the positive potential distribution there (see Fig. 9). However, the effect of the induced electromagnetic field on the flow field is small in this case, especially in the case of the proton as shown in Fig. 8 and Fig. 14. In fact, the drag force calculated by the momentum loss of the proton in the case without the induced electromagnetic field is almost the same as that of the case with it.

![Fig. 13 Number Density Distribution on Plane Involving Dipole Vector and Streamlines of Electron (w/o Induced Electromagnetic Field)](image1)

![Fig. 14 Number Density Distribution on Plane Involving Dipole Vector and Streamlines of Proton (w/o Induced Electromagnetic Field)](image2)

4. CONCLUDING REMARKS

In order to investigate the fundamental features of the solar wind flow around the magnetic sail, the numerical analysis on the high-speed rarefied plasma flow in the presence of the applied magnetic field created by the loop current is performed by the full particle method for the small magnetic sail craft of 10m size. The major conclusions are as follows:

- The length scales of the various phenomena for the interaction of the solar wind with the applied magnetic field of the magnetic sail are roughly estimated. The interaction phenomena are successfully simulated by the full particle method with appropriately defined mesh and time step size to resolve the smallest scale of the phenomena.
- The velocity distribution function obtained at the region around the loop indicates that both the protons and electrons are in thermal nonequilibrium. In this sense, the fluid model is invalid and the kinetic approach should be taken for the analysis of the plasma flow around the magnetic sail.
• A charge separation due to the difference in the length scale of the interaction with the electromagnetic field between the electron and the proton generates the electrostatic potential distribution around the magnetic sail.

• While the induced positive electrostatic potential slightly increases the momentum loss of the proton, the drag force exerted on the magnetic sail mainly depends on that by the applied magnetic field. The radius of influence for the particle’s motion predominantly determines the interaction length scale of the solar wind with the small magnetic sail from the consideration of the drag force on the magnetic sail.

ACKNOWLEDGEMENT
The computation time on SX-6 supercomputer is provided by ISAS/plain center.

REFERENCES