上下辺に一樣圧縮荷重を受ける段ボール箱型容器（正方形筒）の 弾性曲げ応力解析（等方性胴の場合）

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Elastic Stress Analysis on Corrugated Fiberboard Container of a Box Shape under Uniform Compressive Loading in Upper and Lower Edges (On Isotropic Side Plates)

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An elastic formulation of bending deformation for side plates of the case (width 350 mm, height 300 mm and thickness 5.44 mm, longitudinal coefficients 291 N/mm² and Poison's ratio 0.10) of the corrugated fiberboard box shape made by applied uniform compression loading of width direction strain zero in upper and lower edges was expressed by the application to reasonable stress function combination. And from this formulation, bending stresses were calculated and these characteristic behaviors were discussed.

Absolute maximum value of bending normal stress ($\sigma_{bxmax} = -0.0652$ N/mm²) in width direction is at plate centers and maximum values of bending normal stress ($\sigma_{bymax} = 0.262$ N/mm²) in height direction, principal stress ($= 0.262$ N/mm²) and deflection ($= 3.36$ mm) are at centers of side edges. And absolute maximum value of bending shear stress ($\tau_{bxy\max} = \pm 0.163$ N/mm²) and principal shear stress ($= \pm 0.163$ N/mm²) are at plate corners. Ratios of bending stresses and plane stresses (σ_{xmax} , σ_{ymax} , $\tau_{xy\max}$) are $\sigma_{bx\max}/\sigma_{xmax} = 6.6$, $\sigma_{by\max}/\sigma_{ymax} = 2.6$, $\tau_{bxy\max}/\tau_{xy\max} = 33.5$ and those large ratio values are made of bending deformation easiness for the shape of wide and thin side planes.

Keywords : Computational Mechanics, Structure Analysis, Elastic Bending, Case of Corrugated Fiberboard, Elastic Stress Analysis, Structure Strength, Numerical Analysis

分類 : T_s 段ボール, Q₆ 板紙試験法

1. 緒 言

段ボール箱は、軽量で緩衝性があり、生産性に優れ

ており、現在包装用容器として主要な役割をなしている¹⁾²⁾。よって、圧縮荷重下における段ボール箱型容器の弾性曲げ応力強度およびたわみ状況を明らかにする

ことは、段ボール箱の強度設計上、また包装工学上重要なことである。

段ボール箱型容器の強度を基礎的な力学的立場より議論したものととして、一樣圧縮を受ける段ボール箱型容器の応力解析³⁾⁴⁾、変形解析⁵⁾⁶⁾に関する研究、段ボール箱型容器の変形強度に関する研究⁷⁾があり、一樣圧縮荷重を受ける異方性段ボール箱型容器の応力解析⁸⁾⁹⁾がある。上下辺に一樣圧縮荷重を受ける異方性段ボール箱型容器の曲げ応力解析についての基礎的研究がある¹⁰⁾。これは妥当な曲げ作用力であるせん断力の際のものであるが、その変形表示の導出処理が複雑であるために、表示因子および表示形式の妥当性について更に検討することが必要があると思われる。そして、妥当な作用力を考慮した変形表示を議論することは重要なことである。また、一樣圧縮荷重を受ける段ボール箱型容器の曲げ変形破壊は、生じる総最大応力（平面応力+曲げ応力の最大値）によって生じるものと考えられ、曲げ時の主応力状態を明らかにするのに必要なことであり、座屈強度の議論に必要なことであると考えられる。最近、段ボール箱の変形と座屈との関係の議論を試みたものがある¹¹⁾。

そこで、本報告では、前報⁹⁾¹⁰⁾の応力表示を基にして、上下辺に一樣圧縮荷重が働く際の段ボール箱型容器（等方性正方形筒）のより妥当な曲げ応力、主応力の表示を導出し、それら応力の特性を明らかにすることを試みた。

2. 解析方法

段ボール箱の変形は、段ボール¹²⁾¹³⁾の原紙および構成から異方性を示すものと考えられるが、議論を容易にするために等方一樣性胴段ボール箱型容器の弾性曲げ変形表示導出の議論をおこなう。

2.1 支え板の変形条件

本変形の側板は薄くて広いものとし、その変形は平面応力状態であると考え、側板の面方向の応力成分は近似的に零であると考え。そして、議論する容器側板の座標は、一隅の厚さ中央を原点Oとし、幅、高さ、厚さの方向をx, y, z軸にする（Fig.1）。

一樣圧縮荷重 p_{y0} (<0) を受ける段ボール箱型容器の側板（幅L、高さh、厚さT）の変形は上下辺（Fig.1）に連なる耳板（flap plate）による変形制限に基づく上下辺幅方向変位が近似的に零なるものとする。すなわち前報⁹⁾にならい、側板上下辺のx方向のひずみ ε_x は近似的に

$$\varepsilon_x(x, y=0 \text{ or } h)=0 \quad \cdots(1)$$

であるとする。

Fig.1 Corrugated fiberboard box and x and y axes

そして、一樣圧縮によって生じる側板のx, y方向の垂直応力を σ_x , σ_y 、せん断応力を τ_{xy} で、y方向の垂直ひずみを ε_y 、せん断ひずみを γ_{xy} で表すと、側板の弾性方程式¹⁴⁾は近似的に

$$\varepsilon_x = (\sigma_x - \nu \sigma_y) / E \quad \cdots(2a)$$

$$\varepsilon_y = (\sigma_y - \nu \sigma_x) / E \quad \cdots(2b)$$

$$\gamma_{xy} = \tau_{xy} / G \quad \cdots(2c)$$

で表されるものと考えられる。Eは縦弾性係数、Gは横弾性係数、 ν はポアソン比である。

上下辺に一樣圧縮荷重 p_{y0} が働く際、荷重に応じた σ_x , σ_y および τ_{xy} が生じるものと考えられ、両側辺の σ_x は近似的に幾何学的条件に基づき零になり、

$$\sigma_x(x=0 \text{ or } L, y)=0 \quad \cdots(3)$$

また4隅の τ_{xy} も

$$\tau_{xy}(x=0 \text{ or } L, y=0 \text{ or } h)=0 \quad \cdots(4)$$

になるものと考えられる。上下辺に生じる σ_y は荷重条件により

$$\sigma_y(x, y=0 \text{ or } h)=p_{y0} \quad \cdots(5)$$

で表せ、 σ_x は式(1)より

$$\sigma_x(x, y=0 \text{ or } h)=-\nu p_{y0} \quad \cdots(6)$$

で表されるものと考えられる。

側辺域の曲げ変形は対称性および側板面の分布荷重零であると考えられる。また、 $\tau_{xy}T$ によって生じる隣接側板の曲げ作用力は作用力成分を配慮すると M_y , Q_y , M_{xy} が考えられるが、曲げ変形の対称性および曲げ作用力の保存性を考えると M_y と M_{xy} によって表される Q_y が妥当な曲げ作用力であると考えられる。すると、 Q_y と $\tau_{xy}T$ との関係は

$$Q_y = \frac{\partial M_y(x=0 \text{ or } L, y)}{\partial y} + \frac{\partial M_{xy}(x=0 \text{ or } L, y)}{\partial x} = T \tau_{xy}(x=0 \text{ or } L, y) \quad \dots\dots(7)$$

で表されるものと考えられる¹⁵⁾。

2.2 支え板の応力表示

板変形の応力を表す応力関数 $F(x, y)$ の適用条件式は

$$\frac{\partial^4 F}{\partial x^4} + 2 \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0 \quad \dots\dots(8)$$

で表され、応力 σ_x , σ_y , τ_{xy} と応力関数 F との関係は

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 F}{\partial x^2}, \quad \tau_{xy} = - \frac{\partial^2 F}{\partial x \partial y} \quad \dots\dots(9)$$

で表される¹⁴⁾。

上述の変形制限、荷重条件および作用力に対する容器全体の力の釣り合い状態、また応力関数 $F(x, y)$ の適用条件式を考慮して、前報⁹⁾と同様に応力関数 F を

$$F = \sum_{i=1, 2, 3, \dots} \left[\sin(\alpha_i x) f_{li} + \sin(\beta_i y) f_{2i} \right] + \frac{P_{y0}}{2} x^2 \quad \dots\dots(10 a)$$

$$\alpha_i = \frac{\pi}{L}(2i+1), \quad \beta_i = \frac{\pi}{h}(2i+1) \quad \dots\dots(10 b)$$

$$f_{li}(y) = c_{li} \cosh \left[\alpha_i \left(y - \frac{h}{2} \right) \right] + c_{2i} \alpha_i \left(y - \frac{h}{2} \right) \sinh \left[\alpha_i \left(y - \frac{h}{2} \right) \right] \quad \dots\dots(11 a)$$

$$f_{2i}(y) = c_{3i} \cosh \left[\beta_i \left(x - \frac{L}{2} \right) \right] + c_{4i} \beta_i \left(x - \frac{L}{2} \right) \sinh \left[\beta_i \left(x - \frac{L}{2} \right) \right] \quad \dots\dots(11 b)$$

で表す。ただし、 c_{li} , c_{2i} , c_{3i} , c_{4i} は係数である。

周知の応力関数による σ_x , σ_y , τ_{xy} の式(9)を用いると、 σ_x , σ_y , τ_{xy} は式(10 a)～(11 b)より

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} = \sum_{j=1, 2, 3, \dots} \left[\alpha_j^2 \sin(\alpha_j x) \left[c_{1j} \cosh \left[\alpha_j \left(y - \frac{h}{2} \right) \right] + c_{2j} \left(2 \cosh \left[\alpha_j \left(y - \frac{h}{2} \right) \right] + \alpha_j \left(y - \frac{h}{2} \right) \sinh \left[\alpha_j \left(y - \frac{h}{2} \right) \right] \right) \right] - \beta_j^2 \sin(\beta_j y) \right]$$

$$\left[c_{3j} \cosh \left[\beta_j \left(x - \frac{L}{2} \right) \right] + c_{4j} \beta_j \left(x - \frac{L}{2} \right) \sinh \left[\beta_j \left(x - \frac{L}{2} \right) \right] \right] \quad \dots\dots(12 a)$$

$$\sigma_y = \frac{\partial^2 F}{\partial x^2} = \sum_{j=1, 2, 3, \dots} \left[-\alpha_j^2 \sin(\alpha_j x) \left[c_{1j} \cosh \left[\alpha_j \left(y - \frac{h}{2} \right) \right] + c_{2j} \alpha_j \left(y - \frac{h}{2} \right) \sinh \left[\alpha_j \left(y - \frac{h}{2} \right) \right] + \beta_j^2 \sin(\beta_j y) \left[c_{3j} \cosh \left[\beta_j \left(x - \frac{L}{2} \right) \right] + c_{4j} \left(2 \cosh \left[\beta_j \left(x - \frac{L}{2} \right) \right] + \beta_j \left(x - \frac{L}{2} \right) \sinh \left[\beta_j \left(x - \frac{L}{2} \right) \right] \right) \right] + p_{y0} \right] \quad \dots\dots(12 b)$$

$$\tau_{xy} = \frac{\partial^2 F}{\partial x \partial y} = - \sum_{j=1, 2, 3, \dots} \left[\alpha_j^2 \cos(\alpha_j x) \left[c_{1j} \sinh \left[\alpha_j \left(y - \frac{h}{2} \right) \right] + c_{2j} \left(\sinh \left[\alpha_j \left(y - \frac{h}{2} \right) \right] + \alpha_j \left(y - \frac{h}{2} \right) \cosh \left[\alpha_j \left(y - \frac{h}{2} \right) \right] \right) \right] + \beta_j^2 \cos(\beta_j y) \left[c_{3j} \sinh \left[\beta_j \left(x - \frac{L}{2} \right) \right] + c_{4j} \left(\sinh \left[\beta_j \left(x - \frac{L}{2} \right) \right] + \beta_j \left(x - \frac{L}{2} \right) \cosh \left[\beta_j \left(x - \frac{L}{2} \right) \right] \right) \right] \right] \quad \dots\dots(12 c)$$

で表される³⁾⁷⁾。そして、 c_{li} , c_{2i} は

$$c_{li} = \frac{2 \nu p_{y0} K_{2i}}{\pi \alpha_i^2 (2i+1) K_{li}^2} \quad (i=0, 1, 2, 3, \dots i_0) \quad \dots\dots(13)$$

$$c_{2i} = \frac{K_{li}}{K_{2i}} c_{li} \quad (i=0, 1, 2, 3, \dots i_0) \quad \dots\dots(14)$$

$$K_{li} = \cosh \left(\frac{\alpha_i h}{2} \right), \quad K_{2i} = \left(\frac{\alpha_i h}{2} \right) \sinh \left(\frac{\alpha_i h}{2} \right) \quad (i=0, 1, 2, 3, \dots i_0) \quad \dots\dots(15)$$

で表され⁹⁾, c_{3i} , c_{4i} は

$$c_{3i} = \frac{\alpha_i^2 \left[(c_{1i} + c_{2i}) K_{3i} + c_{2i} K_{4i} \right] K_{6i}}{\beta_i^2 \left[K_{6i} K_{7i} - K_{5i} (K_{7i} + K_{8i}) \right]} \quad (i=0, 1, 2, 3, \dots i_0) \quad \dots\dots(16)$$

$$K_{3i} = \cosh \left(\frac{\beta_i L}{2} \right), \quad K_{4i} = \left(\frac{\beta_i L}{2} \right) \sinh \left(\frac{\beta_i L}{2} \right) \quad \dots\dots(17 a)$$

$$K_{5i} = \sinh \left(\frac{\alpha_i h}{2} \right), \quad K_{6i} = \left(\frac{\alpha_i h}{2} \right) \sinh \left(\frac{\alpha_i h}{2} \right) \quad \dots\dots(17 b)$$

$$K_{7i} = \sinh \left(\frac{\beta_i L}{2} \right), \quad K_{8i} = \left(\frac{\beta_i L}{2} \right) \sinh \left(\frac{\beta_i L}{2} \right) \quad \dots\dots(17 c)$$

$$c_{4i} = -(K_{5i}/K_{6i})c_{3i} \quad (i=0, 1, 2, 3, \dots, i_0) \quad \dots\dots(18)$$

で表される⁹⁾。

2.3 たわみ、曲げ応力表示

たわみ w および曲げ応力 σ_{bx} , σ_{by} , τ_{bxy} の表示を議論する。

本報の応力表示は容器の形態上変形の条件によって複雑に生じること¹⁰⁾により、曲げ表示はより複雑なものと考えられる。

上述の式(7)によると、曲げ作用力は隣側板側辺に働く $T_{\tau_{xy}}$ によって生じる。

Fig. 2 に示すように、隣側板との曲げモーメントの連続性に基づき隣接板の $T_{\tau_{xy}}$ による曲げモーメント M_x , M_y は周辺条件として

$$M_x(x=0 \text{ or } L) = 0 \quad \dots\dots(19a)$$

$$M_x(y=0 \text{ or } h) = 0 \quad \dots\dots(19b)$$

$$M_y(y=0 \text{ or } h) = 0 \quad \dots\dots(20)$$

であることが必要であり、たわみ w は変形条件

$$w(y=0 \text{ or } h) = 0 \quad \dots\dots(21)$$

を満たすことが必要であると考えられる。

側辺 ($x=0$ また L) に働く Q_y は、作用力の釣合いとして式 (7) を満たし、

$$Q_y = -D \left(\frac{\partial^3 w}{\partial y^3} + \frac{\partial^3 w}{\partial x \partial y^2} \right) \quad \dots\dots(22)$$

$$D = ET^3/[12(1-\nu)]$$

$$Q_y(x=0 \text{ or } L) = T_{\tau_{xy}}(x=0 \text{ or } L) = \sum_{i=1,2,3,\dots} T_{\tau_{xyi}} \quad \dots\dots(23a)$$

$$\tau_{xyi} = \tau_{xyli} + \tau_{xy2i} \quad \dots\dots(23b)$$

で表される¹⁴⁾。

一方、式(7)、(12c) により τ_{xyli} , τ_{xy2i} は

Fig. 2 Shear stress τ_{xy} in side edge and bending action Q_y in neighborhood side edge.

$$\tau_{xyli} = [\alpha_i^2(c_{1i} + c_{2i})K_{3i} + c_{2i}K_{4i}] \quad \dots\dots(24a)$$

$$\tau_{xy2i} = [\beta_i^2(c_{3i} + c_{4i})K_{7i} + c_{4i}K_{8i}] \quad \dots\dots(24b)$$

で表される。式(23)の作用力の原動力となる τ_{xyli} 項と作用力との連携を考慮すると、前報¹⁰⁾とは異なる $\cos(\gamma_n y)$ のフリエー級数による展開をおこなうことが妥当な計算処方であると考えられる。そこで、積分関係¹⁶⁾

$$\begin{aligned} \int \exp(\alpha_i y) \sin(\beta_i y) dy &= \frac{\exp(\alpha_i y)}{\alpha_i^2 + \beta_i^2} \\ &\quad [\alpha_i \cos(\beta_i y) + \beta_i \sin(\beta_i y)] \\ \int y \exp(\alpha_i y) \sin(\beta_i y) dy &= \frac{y \exp(\alpha_i y)}{\alpha_i^2 + \beta_i^2} \\ &\quad [\alpha_i \cos(\beta_i y) + \beta_i \sin(\beta_i y)] \\ &\quad - \frac{\exp(\alpha_i y)}{(\alpha_i^2 + \beta_i^2)^2} [(\alpha_i^2 - \beta_i^2) \cos(\beta_i y) + 2\alpha_i \beta_i \sin(\beta_i y)] \end{aligned}$$

を用いると級数は

$$T_{\tau_{xyli}} = \sum_{n=1,2,3,\dots} G_{Ain} \cos(\gamma_n y) \quad \dots\dots(25a)$$

$$\gamma_n = \pi n/h \quad \dots\dots(25b)$$

$$\begin{aligned} G_{Ain} &= \frac{4}{\pi} \left[\frac{\alpha_i^3}{(\alpha_i^2 + \gamma_n^2)} \left[(c_{1i} + c_{2i}) L_{li} \right. \right. \\ &\quad \left. \left. + c_{2i} \left[\frac{\alpha_i^3}{(\alpha_i^2 + \gamma_n^2)} L_{2i} - \frac{\alpha_i^3 (\alpha_i^2 - \gamma_n^2)}{(\alpha_i^2 + \gamma_n^2)^2} L_{1n} \right] \right] \right] \quad \dots\dots(26a) \end{aligned}$$

$$L_{1n} = \cosh\left(\frac{\gamma_n L}{2}\right), \quad L_{2n} = \left(\frac{\gamma_n L}{2}\right) \sinh\left(\frac{\gamma_n L}{2}\right) \quad \dots\dots(26b)$$

$$(n=1, 3, 5, \dots, n_0)$$

で表される。式(25a)の積分値は計算処理の際各 n 次の $\sin(\gamma_n y)$ 項値は零となり、また、 n = 偶数次の $\cos(\gamma_n y)$ 項値も零となる。

変形条件式(7)、式(23a)を満たす(12c)を用いると、式(24a)の $T_{\tau_{xyli}}$ の G_{Ain} と τ_{xyli} との関係は

$$T_{\tau_{xyli}}(x=0 \text{ or } L, y=0 \text{ or } h) = \sum_{n=1,2,3,\dots} G_{Ain} \quad \dots\dots(27)$$

$$(n=1, 3, 5, \dots, n_0)$$

で表される。

側板のたわみの適用条件¹⁴⁾

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = 0 \quad \dots\dots(28)$$

および曲げ変形の対称性を満たす w の表示を前報¹⁰⁾とは異なるより妥当な表示、すなわち変形条件に応じた表示

$$w = \sum_{i=1,2,3,\dots} [\sin(\gamma_i y) (C_{1i}^* f_{li}^* + C_{2i}^* f_{2i}^*)] \quad \dots\dots(29)$$

$$f_{li}^* = \sinh\left[\gamma_i \left(x - \frac{L}{2}\right)\right] \quad \dots\dots(30a)$$

$$f_{2i}^* = \left[\gamma_i \left(x - \frac{L}{2} \right) \right] \sinh \left[\gamma_i \left(x - \frac{L}{2} \right) \right] \quad \cdots \cdots (30 \text{ b})$$

$$\gamma_i = \pi (2i+1)/h \quad \cdots \cdots (30 \text{ c})$$

で表す。

式(29)を式(22)に代入すると、 Q_y は

$$\begin{aligned} Q_y &= \sum_{i=1,2,3,\dots} \gamma_i^3 D \left[[(C_{1i}^* + 2C_{2i}^*) f_{3i} + C_{2i} f_{4i}] \cos(\gamma_i y) \right] \\ &= \sum_{i=1,2,3,\dots} \left[\left[\sum_{n=1,2,3,\dots} G_{Ain} \cos(\gamma_i y) \right] + G_{Bi} \cos(\gamma_i y) \right] \end{aligned} \quad \cdots \cdots (31 \text{ a})$$

$$\sum_{n=1,2,3,\dots} G_{Ain} = T \alpha_i^2 [(C_{1i} + c_{2i}) K_{3i} + c_{2i} K_{4i}] \quad \cdots \cdots (31 \text{ b})$$

$$G_{Bi} = \beta_i^2 (c_{3i} + c_{4i}) K_{7i} + c_{4i} K_{8i} \quad \cdots \cdots (31 \text{ c})$$

$$\gamma_n = \beta_n = \pi (2n+1)/L \quad \cdots \cdots (31 \text{ d})$$

で表され、 G_{Ain} の次数 i, n を入替えた値 G_{Ani} を用い、その和 G_{Ai} を

$$G_{Ai} = \sum_{n=1,2,3,\dots} G_{Ani} \quad \cdots \cdots (31 \text{ e})$$

で表す。側辺 $x=0$ また L における作用力 Q_y の保存性を考慮すると、式(12c) の $y=0$ また h の項 τ_{xyi} は

$$\begin{aligned} T \tau_{xyi} &= -\gamma_i^3 D [(C_{1i}^* + 2C_{2i}^*) K_{7i} + C_{2i}^* K_{8i}] \\ &= (G_{Ai} + G_{Bi}) \end{aligned} \quad \cdots \cdots (32)$$

で表される。

M_x の関係¹⁵⁾は式(29)より

$$\begin{aligned} M_x &= -D \left[\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right] \\ &= \sum_{i=1,2,3,\dots} \gamma_i^2 D \left[[(C_{1i}^* + 2C_{2i}^*) f_{3i} + C_{2i}^* f_{4i}] \right. \\ &\quad \left. + \nu (C_{1i}^* f_{3i} + C_{2i}^* f_{4i}) \right] \sin(\gamma_i y) \end{aligned} \quad \cdots \cdots (33)$$

で表される。この関係に式(20)の条件を適用すると、関係

$$[(1-\nu)K_{7i}C_{1i}^* + [2K_{7i} + (1-\nu)K_{8i}]C_{2i}^*] = 0 \quad \cdots \cdots (34)$$

が得られる。式(34)により、関係

$$C_{1i}^* = -\frac{2K_{7i} + (1-\nu)K_{8i}}{(1-\nu)K_{7i}} C_{2i}^* \quad \cdots \cdots (35)$$

が得られる。

式(33)、(35)により、 C_{2i}^* は

$$\begin{aligned} C_{2i}^* &= -\frac{(G_{Ai} + G_{Bi})}{D \gamma_i^3 [(1+A_{0i})K_{7i} + A_{0i}K_{8i}]} \\ A_{0i} &= -\frac{(1-\nu)K_{7i}}{2K_{7i} + (1-\nu)K_{8i}} \end{aligned} \quad \cdots \cdots (36)$$

で表されることがわかる。

M_y, M_{xy} の表示¹⁵⁾および式(29)により、これらの関係は

$$\begin{aligned} M_y &= -D \left[\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right] \\ &= \sum_{i=1,2,3,\dots} \sum_{j=1,2,3,\dots} \gamma_i^2 D [(1-\nu)f_{1i}^* C_{1i}^* \\ &\quad + [(1-\nu)f_{2i}^* + 2f_{1i}^*]C_{2i}^*] \sin(\gamma_i y) \end{aligned} \quad \cdots \cdots (37)$$

$$\begin{aligned} M_{xy} &= -\frac{GT^3}{6} \frac{\partial^2 w}{\partial x \partial y} \\ &= \sum_{i=1,2,3,\dots} \left(\frac{GT^3 \gamma_i^2}{6} \right) \\ &\quad \cos(\gamma_i y) [(C_{1i}^* + C_{2i}^*)f_{1i}^* + C_{2i}^* f_{2i}^*] \end{aligned} \quad \cdots \cdots (38)$$

で表される。

外表面における x, y 方向の曲げ垂直応力 σ_{bx}, σ_{by} , 曲げせん断応力 τ_{bxy} は

$$\sigma_{bx} = -\frac{TE}{2(1-\nu^2)} \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \quad \cdots \cdots (39)$$

$$\sigma_{by} = -\frac{TE}{2(1-\nu^2)} \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \quad \cdots \cdots (40)$$

$$\tau_{bxy} = -TG \frac{\partial^2 w}{\partial x \partial y} \quad \cdots \cdots (41)$$

で表され¹⁵⁾、側板外表面 ($z=T/2$) の各曲げ応力成分は式(35)、(37)~(41)より

$$\sigma_{bx} = \frac{6M_x}{T^2}, \quad \sigma_{by} = \frac{6M_y}{T^2}, \quad \tau_{bxy} = \frac{6M_{xy}}{T^2} \quad \cdots \cdots (42)$$

で表されることがわかる。

2.4 曲げ主応力、曲げ主せん断応力

各部材の破壊は最大応力位置に生じるものと考えられる。部材に生じる最大垂直応力は主応力であり、最大せん断応力は主せん断応力である。したがって、一般に、荷重を受ける部材の曲げ応力による破壊は曲げ主応力 σ_{b0} の最大位置より、塑性変形開始すなわち弾性応力破壊は曲げ主せん断応力 τ_{b0} の最大位置より生じるものと考えられる。よって、 σ_{b0}, τ_{b0} 値を明らかにすることは必ず強度設計上重要であると考えられる。

σ_{b0}, τ_{b0} は

$$\sigma_{b0} = \frac{1}{2} (\sigma_{bx} + \sigma_{by}) \pm \frac{1}{2} \sqrt{(\sigma_{bx} - \sigma_{by})^2 + 4\tau_{bxy}^2} \quad \cdots \cdots (43)$$

$$\tau_{b0} = \pm \frac{1}{2} \sqrt{(\sigma_{bx} - \sigma_{by})^2 + 4\tau_{bxy}^2} \quad \cdots \cdots (44)$$

で表される¹⁷⁾。

3. 結果と考察

一様分布荷重 p_{y0} を受ける本容器の形状は使われている箱に近いものに、また議論を容易にできるように、 $L=350 \text{ mm}, h=300 \text{ mm}, T=5.44 \text{ mm}$ にした。そし

て、 $E=2.91 \times 10^2 \text{ N/mm}^2$ 、 $\nu=0.10$ にした⁷⁾。

式(10 a)～(11 b)の次数 i は無限大までの整数であるが、計算誤差が5%以下となるよう $i_0, n_0=24$ （49次）として計算をおこなった。

3.1 側板の σ_{bx} 、 σ_{by} 、 τ_{bxy} 、 w 分布

本表示式(42)より求めた σ_{bx} 、 σ_{by} 、 τ_{bxy} および w は幅、高さ中心線に対し σ_{bx} 、 σ_{by} および w の分布は対称的に、 τ_{bxy} は反対称的に生じることがわかった。そして、これらの分布概況を所定の x 、 y における $x=0 \sim L/2$ 、 $y=0 \sim h/2$ の分布によって表し、Fig.3～6に示す。

Fig.3より、 x の増加に伴い、 $x=0 \sim L$ 間の σ_{bx} は $y=0$ また h で $\sigma_{bx}=0$ となり、 $y=h/8 \sim 7h/8$ では減少、増加を示し、その間に極小が生じることが、 y の増加に伴い、 $y=0 \sim h$ 間の σ_{bx} は $x=0$ また L で $\sigma_{bx}=0$ となり、 $x=L/8 \sim 7L/8$ では減少、増加を示し、その間に極小が生じることがわかる。

位置 $x=L/2$ 、 $y=h/2$ に $|\sigma_{bx}|$ の最大値 $\sigma_{bx\max}$ （ $=-0.0652 \text{ N/mm}^2$ ）が生じることがわかった。そして、 σ_{bx} は周辺で $\sigma_{bx}=0$ 、 $x=L/2$ 、 $y=h/2$ で負値の $\sigma_{bx\max}$ が生じる凹状分布を示すことがわかる。また、本表示の σ_{bx} 値は辺に関する M_x の変形条件式(19 a, b)を満たすことがわかる。

Fig.4より、 x の増加に伴い、 $x=0 \sim L$ 間の σ_{by} は $y=0$ また h で $\sigma_{by}=0$ となり、 $y=h/8 \sim 7h/8$ 付近では減少、増加を示し、その間に極小が生じることが、 y の増加に伴い、 $y=0 \sim h$ 間の σ_{bx} は増加、減少し、その間に極大が生じることがわかる。

$x=0$ また L 、 $y=h/2$ で σ_{by} の最大値 $\sigma_{by\max}$ （ $=0.262 \text{ N/mm}^2$ ）が生じることがわかった。そして、 σ_{by} は上下辺で $\sigma_{by}=0$ を、 $x=0$ 、 L 、 $y=h/2$ で正值の $\sigma_{by\max}$ を示す鞍状（ y 方向に沿う）の分布であることがわかる。また、本表示の σ_{by} 値は上下辺に関する M_y の変形条件式(20)を満たすことがわかる。

Fig.5より、 x の増加に伴い $x=0 \sim L$ 間の τ_{bxy} は $y=h/2$ で $\tau_{bxy}=0$ となり、それ以外の $y < h/2$ では増加を、 $y > h/2$ では減少を示すことが、 y の増加に伴い、 $y=0 \sim h$ 間の τ_{bxy} は $x=L/2$ では $\tau_{bxy}=0$ となり、 $x < L/2$ では増加を、 $x > L/2$ では減少を示すことがわかる。

側板4隅（ $x=0$ また L 、 $y=0$ また h ）で $|\tau_{bxy}|$ の最大値 $\tau_{bxy\max}$ （ $=\pm 0.163 \text{ N/mm}^2$ ）が生じることがわかる。そして、 τ_{bxy} は幅、高さの中心線で零となり、 $x=0$ 、 $y=0$ 、 $x=L$ 、 $y=h$ で正值の $\tau_{bxy\max}$ 、 $x=0$ 、 $y=h$ 、 $x=L$ 、 $y=0$ で負値の $\tau_{bxy\max}$ となるそり曲げ分布を示すことがわかる。

Fig.3 Relations between σ_{bx} and positions x , y .

($L=350$, $h=300$, $T=5.44$ (mm), $p_{y0}=0.1$, $E=2.91 (\times 10^2 \text{ N/mm}^2)$, $\nu=0.1$, x/L :open sign, y/h :full sign)

Fig. 4 Relations between σ_{by} and positions x, y .

($L = 350$, $h = 300$, $T = 5.44$ (mm), $p_{y0} = 0.1$, $E = 2.91 (\times 10^5 \text{ N/mm}^2)$, $\nu = 0.1$, x/L : open sign, y/h : full sign)

Fig. 5 Relations between τ_{bxy} and positions x, y .

($L = 350$, $h = 300$, $T = 5.44$ (mm), $p_{y0} = 0.1$, $E = 2.91 (\times 10^5 \text{ N/mm}^2)$, $\nu = 0.1$, x/L : open sign, y/h : full sign)

Fig. 6 Relations between w and positions x, y .

($L=350$, $h=300$, $T=5.44$ (mm), $p_{y0}=0.1$, $E=2.91 (\times 10^2 \text{ N/mm}^2)$, $\nu=0.1$, x/L :open sign, y/h :full sign)

Fig. 6 より, x の増加に伴い, $x=0 \sim L$ 間の w は $y=0$ また h で $w=0$ を示すことが, $y=h/8 \sim 7h/8$ では, 減少, 増加を示し, その間に極小が生じることが, y の増加に伴い, $y=0 \sim h$ 間の τ_{bx} は増加, 減少を示し, その間に極大が生じることがわかる。

$x=0$ また $L, y=h/2$ で $|w|$ の最大値 $w_{\max}$ ($=3.36$ mm) が生じることがわかった。そして, w は σ_{bx} の際と同様な鞍状分布を示すことがわかる。また, 本表示の w 値は上下辺に関する式(21)の $w=0$ の変形条件を満たすことがわかる。

また, $\sigma_{x\max}$, $\sigma_{y\max}$, $\tau_{xy\max}$ と $\sigma_{bx\max}$, $\sigma_{by\max}$, $\tau_{bxy\max}$ との比は $\sigma_{bx\max}/\sigma_{x\max}=6.5$, $\sigma_{by\max}/\sigma_{y\max}=2.6$, $\tau_{bxy\max}/\tau_{xy\max}=33.5$ となり, 曲げ応力が非常に大きいことがわかる。このように曲げ応力が大きくなることは, 側板が薄く, 広い形状であることによるものであり, 曲げ作用力 $T \tau_{xy}$ による側板の曲げ変形が容易に生じることによるものと考えられる。すなわち, 長棒の引張り, 圧縮変形より長棒の曲げたわみが容易に大きく生じることと同様である。本報の応力, w の分布状態は前報¹⁰⁾と同様であるが, 値は応力は $1/2$, w は $1/3$ 程度であることがわかった。

3.2 側板の σ_{b0} , τ_{b0} 分布

式(43), (44)より求めた σ_{b0} は主応力の定義¹⁶⁾に基づ

き側板の幅, 高さの中心線に対し対称に, τ_{b0} は反対称に生じることがわかった。

そこで, 得られた σ_{b0} , τ_{b0} と x, y との関係を Fig. 7, 8 に各示す。

Fig. 7 より, x の増加に伴い, $x=0 \sim L$ 間の σ_{b0} は $y=0$ また hL 付近で減少, 増加を示し, その間に極小が生じることがわかり, y の増加に伴い, 増加, 減少を示し, その間に極大が生じることがわかる。

側板中央に $|\sigma_{b0}|$ の最大値 $\sigma_{b0\max}$ ($=0.261 \text{ N/mm}^2$) が生じることがわかった。

Fig. 8 より, x の増加に伴い, $x=0 \sim L$ 間の τ_{b0} は $y=0$ で減少を, $y=L$ で増加を示し, $y < h/2$ では減少を, $y > h/2$ では増加を示すことがわかり, y の増加に伴い, $x=0 \sim h$ 間の τ_{b0} は $x=0$ 付近で減少を, $x=L$ 付近では増加を示し, $x < L/2$ では増加を, $x > L/2$ では減少を示すことがわかる。

4 隅 ($x=0$ また $L, y=0$ また h で $|\tau_{b0}|$ の最大値 $\tau_{b0\max}$ ($=\pm 0.163 \text{ N/mm}^2$) が生じることがわかった。

なお, 幅, 高さの中心線上で σ_{b0} 値は連続性であるが, その接線は必ずしも連続ではないことが, τ_{b0} 値も連続ではないことがわかる。

Fig.7 Relations between σ_{b0max} and positions x, y .

($L = 350$, $h = 300$, $T = 5.44$ (mm), $p_{y0} = 0.1$, $E = 2.91 (\times 10^2 \text{N/mm}^2)$, $\nu = 0.1$, x/L :open sign, y/h :full sign)

Fig.8 Relations between τ_{b0max} and positions x, y .

($L = 350$, $h = 300$, $T = 5.44$ (mm), $p_{y0} = 0.1$, $E = 2.91 (\times 10^2 \text{N/mm}^2)$, $\nu = 0.1$, x/L :open sign, y/h :full sign)

4. 結 言

近似的に、上下辺の x 方向の変位および側辺の x 方向の垂直ひずみ ε_x を零であるとして、側板（原点は1隅、 x , y 軸は幅 L , 高さ h 方向）上下辺に一樣分布荷重 p_{y0} が働く際の等方性正方形胴段ボール箱型容器側板（基準：幅 350 mm, 高さ 300 mm, 厚さ 5.44 mm）の曲げ変形表示を求めた。そして、側板の諸応力成分、たわみ w および曲げ主応力 σ_{b0} , 曲げ主せん断応力 τ_{b0} を求め、これらの分布状況および諸応力および w の最大値の特性を議論した。ただし、縦弾性係数 E , ポアソン比 ν は $2.91 \times 10^4 \text{ N/mm}^2$, 0.10 にした。

1) x の増加に伴い, $x=0 \sim L$ 間の幅方向の垂直 σ_{bx} は $y=0$ また h で $\sigma_{bx}=0$ となり, $y=h/8 \sim 7h/8$ では, 減少, 増加を示し, その間に極小が生じ, y の増加に伴い, $y=0 \sim h$ 間の σ_{bx} は $x=0$ また L で $\sigma_{bx}=0$ となり, $x=L/8 \sim 7L/8$ では減少, 増加を示し, その間に極小が生じる。 x の増加に伴い, $x=0 \sim L$ 間の高さ方向の垂直 σ_{by} は $y=0$ また h で $\sigma_{by}=0$ となり, $y=h/8 \sim 7h/8$ 付近では減少, 増加を示し, その間に極小が生じる。 y の増加に伴い, $y=0 \sim h$ 間の σ_{bx} は増加, 減少を示し, その間に極大が生じる。 x の増加に伴い, $x=0 \sim L$ 間のせん断応力 τ_{bxy} は $y=h/2$ で $\tau_{bxy}=0$ となり, $y < h/2$ では増加を, $y > h/2$ では減少を示し, y の増加に伴い, $y=0 \sim h$ 間の τ_{bxy} は $x=L/2$ で $\tau_{bxy}=0$ となり, $x < L/2$ では増加を, $x > L/2$ では減少を示す。

2) 上下辺中央に $|\sigma_{bx}|$ の最大値 ($= -0.0652 \text{ N/mm}^2$), σ_{by} の最大値 ($= 0.262 \text{ N/mm}^2$) が, 側板4隅に $|\tau_{bxy}|$ の最大値 ($= \pm 0.163 \text{ N/mm}^2$) が生じる。

3) x の増加に伴い, $x=0 \sim L$ 間の w は $y=0$ また h で $w=0$ を示し, $y=h/8 \sim 7h/8$ では減少, 増加を示し, その間に極小が生じ, y の増加に伴い, $y=0 \sim h$ 間の w は増加, 減少を示し, その間に極大が生じる。側辺中央に w の最大値 $w_{\max}$ ($= 3.36 \text{ mm}$) が生じる。

4) x の増加に伴い, $x=0 \sim L$ 間の σ_{b0} は減少, 増加を示し, その間に極小が生じ, y の増加に伴い, $y=0 \sim h$ 間の σ_{b0} は増加, 減少を示し, その間に極大が生じる。 x の増加に伴い, $x=0 \sim L$ 間の τ_{b0} は $x < L/2$ では減少を, $y > L/2$ では増加を示し, y の増加に伴い, $x=0 \sim h$ 間の τ_{b0} は $x=0$ で減少を, $x=L$ では増加を示し, $x < L/2$ では増加を, $x > L/2$ では減少を示す。

5) 側板中央に $|\sigma_{b0}|$ の最大値 ($= 0.262 \text{ N/mm}^2$)

が, 4 隅に $|\tau_{b0}|$ の最大値 ($= \pm 0.163 \text{ N/mm}^2$) が生じる。幅, 高さの中心線上で, σ_{b0} の値は連続であるが, その接線は必ずしも連続ではなく, τ_{b0} の値も連続ではない。

6) σ_{bx} , σ_{by} および w の分布は対称的に, τ_{bxy} は反対称的に生じ, $\sigma_{bx\max}$, $\sigma_{by\max}$, $\tau_{bxx\max}$ の値は一樣圧縮下の幅, 高さ方向の垂直応力 $\sigma_{x\max}$, $\sigma_{y\max}$, およびせん断応力 $\tau_{xx\max}$ より各 6.5, 2.6, 33.5 倍大きい。このように曲げ応力が大きくなるのは側板が薄く, 広いことによるものである。

以上のように, 上下辺に一樣分布圧縮荷重を受ける段ボール箱胴部（正方形筒）の弾性曲げ変形表示によってその諸曲げ応力, たわみの概況を議論した。その結果は, 要求する諸変形条件を満たし, 妥当なものであると考えられる。このことより, 本表示は段ボール箱の圧縮強度の基本的議論に役立つものと考えられる。

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1. Introduction

Corrugated fiberboard is light in weight, and is a good buffer against impact. It is efficient in productivity, and is commonly used as a packaging material¹⁾²⁾. So, clarifying the elastic bending stress and strain of a corrugated container of a box shape is very necessary for designing an optimal corrugated container and is important in the engineering of packaging.

There were papers discussing strength properties of corrugated boxes in a theoretical and mechanical view point. They were stress analysis under uniform compression³⁾⁴⁾, research on deformation analysis⁵⁾⁶⁾, fundamental research on deformation of a corrugated box⁷⁾ and stress analysis under uniform compression on an anisotropic corrugated box⁸⁾⁹⁾.

There was also a paper on the theoretical analysis of a box of anisotropic character which was under uniform load at the center of its top and bottom faces¹⁰⁾.

In that paper, shear force was regarded as a major bending force, and deducting deformation equations was complicated. So, it is necessary to take into consideration more reasonable working forces to study bending deformation in more details. When a corrugated container of a box shape under uniform com-

pression load is collapsed, it is caused by the total maximum stress (stress in a plane plus maximum bending stress). So, calculating distributions of principal stresses under a bending mode is necessary to discuss its collapsible strength. Recently, there was a paper discussing a relation between deformation of corrugated box and its collapse¹¹⁾.

In this paper, a corrugated container of a box shape which is isotropic in character is taken as a model. Stress equations in preceding papers⁹⁾¹⁰⁾ are taken as a base. Then, they are modified to get more reasonable equations for bending stress and principal stress, which, as results, clarify the characteristics of those stresses.

2. Analytical method

The deformation of a corrugated box is thought to be anisotropic, as it consists of corrugated sheet which is anisotropic in character¹²⁾¹³⁾. However, a model of a box in this paper is assumed to be isotropic to make discussion simpler and more understandable

2.1 Deformation criteria of the side plate

Every side plate of a model box is assumed to be thin and broad enough, and its deformation is caused by plane stress. So, a stress component perpendicular

to the plane is supposed to be zero. One corner of the box is located at the coordinates origin and its width is in x direction (axis) and its height is in y direction (axis) and its depth is in z direction (axis) as in Fig. 1.

The box is under uniform compression load p_{y0} (< 0). The side plate (width : L , height : h , and thickness : T) is assumed to deform as follows. As a flap plate which continues from the side plate restricts deformation of the side plate at its upper and lower edges and their deformations in x direction would be regarded as almost zero. As in the preceding paper⁹⁾, the strain at the upper and lower edges of the side plate in x direction is closely to be as in the equation (1).

$$\varepsilon_x(x, y=0 \text{ or } h) = 0 \quad \cdots \cdots (1)$$

Uniform compression load produces stress and strain in the side plate. They are represented as follows. Normal stress in x axis is σ_x and that in y axis is σ_y . Shear stress is τ_{xy} . Normal strain in y axis is ε_y and shear strain is γ_{xy} . Then, elastic equations on the side plate will closely be as in following equations.

$$\varepsilon_x = (\sigma_x - \nu \sigma_y) / E \quad \cdots \cdots (2a)$$

$$\varepsilon_y = (\sigma_y - \nu \sigma_x) / E \quad \cdots \cdots (2b)$$

$$\gamma_{xy} = \tau_{xy} / G \quad \cdots \cdots (2c)$$

E is Young's modulus and G is a shear modulus ν is Poisson's ratio.

When uniform compression load p_{y0} is applied on the upper and lower edges, σ_x , σ_y and τ_{xy} are produced in the side plate. σ_x on the both side edges of the plate will be zero due to the geometrical condition, and is expressed as in the equation (3). τ_{xy} at the each corner of the side plate will also be as in the equation (4).

$$\sigma_x(x=0 \text{ or } L, y) = 0 \quad \cdots \cdots (3)$$

$$\tau_{xy}(x=0 \text{ or } L, y=0 \text{ or } h) = 0 \quad \cdots \cdots (4)$$

σ_y on the upper and lower edges will be as in the equation (5) due to the loading condition. σ_x from the equation (1) will be as in the equation (6).

$$\sigma_y(x, y=0 \text{ or } h) = p_{y0} \quad \cdots \cdots (5)$$

$$\sigma_x(x, y=0 \text{ or } h) = -\nu p_{y0} \quad \cdots \cdots (6)$$

Bending deformation in the side plate is thought to be symmetrical. Bending force in the adjoining side plate which is caused by $\tau_{xy} T$ will consist of M_y , Q_y and M_{xy} . Among them, Q_y will be reasonable bending shear force, taking into consideration the symmetry in bending deformation and the balance of bending force.

As Q_y is expressed with M_y and M_{xy} , the equation (7) is deduced¹⁵⁾.

$$Q_y = \frac{\partial M_y(x=0 \text{ or } L, y)}{\partial y} + \frac{\partial M_{xy}(x=0 \text{ or } L, y)}{\partial x} \\ = T \tau_{xy}(x=0 \text{ or } L, y) \quad \cdots \cdots (7)$$

2.2 Stresses in the side plate

In deformation of a plate, stress function $F(x, y)$ is restricted by the equation (8).

$$\frac{\partial^4 F}{\partial x^4} + 2 \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0 \quad \cdots \cdots (8)$$

σ_x , σ_y and τ_{xy} are expressed with F as in the equation (9)¹⁴⁾

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 F}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} \quad \cdots \cdots (9)$$

As in the preceding paper⁹⁾, F is expressed as in (10 a), (10 b), (11 a) and (11 b), taking into consideration the restriction in deformation, the balance of forces on the model box and the restriction on F .

$$F = \sum_{i=1,2,3,\dots} \left[\sin(\alpha_i x) f_{1i} + \sin(\beta_i y) f_{2i} \right] + \frac{p_{y0}}{2} x^2 \quad \cdots \cdots (10a)$$

$$\alpha_i = \frac{\pi}{L}(2i+1), \quad \beta_i = \frac{\pi}{h}(2i+1) \quad \cdots \cdots (10b)$$

$$f_{1i}(y) = c_{1i} \cosh \left[\alpha_i \left(y - \frac{h}{2} \right) \right] + c_{2i} \alpha_i \left(y - \frac{h}{2} \right) \sinh \left[\alpha_i \left(y - \frac{h}{2} \right) \right] \quad \cdots \cdots (11a)$$

$$f_{2i}(y) = c_{3i} \cosh \left[\beta_i \left(x - \frac{L}{2} \right) \right] + c_{4i} \beta_i \left(x - \frac{L}{2} \right) \sinh \left[\beta_i \left(x - \frac{L}{2} \right) \right] \quad \cdots \cdots (11b)$$

c_{1i} , c_{2i} , c_{3i} and c_{4i} are coefficients. Using the equation (9), σ_x , σ_y and τ_{xy} are expressed as in (12 a), (12 b) and (12 c)³⁾⁷⁾.

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} = \sum_{j=1,2,3,\dots} \left[\alpha_j^2 \sin(\alpha_j x) \left[c_{1j} \cosh \left[\alpha_j \left(y - \frac{h}{2} \right) \right] + c_{2j} (2 \cosh \left[\alpha_j \left(y - \frac{h}{2} \right) \right] + \alpha_j \left(y - \frac{h}{2} \right) \sinh \left[\alpha_j \left(y - \frac{h}{2} \right) \right]) \right] - \beta_j^2 \sin(\beta_j y) \left[c_{3j} \cosh \left[\beta_j \left(x - \frac{L}{2} \right) \right] + c_{4j} \beta_j \left(x - \frac{L}{2} \right) \cosh \left[\beta_j \left(y - \frac{L}{2} \right) \right] \right] \right] \quad \cdots \cdots (12a)$$

$$\sigma_y = \frac{\partial^2 F}{\partial x^2} = \sum_{j=1,2,3,\dots} \left[-\alpha_j^2 \sin(\alpha_j x) \left[c_{1j} \cosh \left[\alpha_j \left(y - \frac{h}{2} \right) \right] \right] \right]$$

$$\begin{aligned}
& + c_{2i} \left(y - \frac{h}{2} \right) \sinh \left[\alpha_i \left(y - \frac{h}{2} \right) \right] \\
& + \beta_i^2 \sin(\beta_i y) \left[c_{3i} \cosh \left[\beta_i \left(x - \frac{L}{2} \right) \right] \right. \\
& + c_{4i} \left(2 \cosh \left[\beta_i \left(x - \frac{L}{2} \right) \right] + \beta_i \left(x - \frac{L}{2} \right) \right. \\
& \left. \left. \sinh \left[\beta_i \left(x - \frac{L}{2} \right) \right] \right) \right] + p_{y0} \dots \dots (12 \text{ b})
\end{aligned}$$

$$\begin{aligned}
\tau_{xy} = \frac{\partial^2 F}{\partial x \partial y} = - \sum_{j=1,2,3,\dots} \left[\alpha_i^2 \cos(\alpha_i x) \left[c_{1i} \sinh \right. \right. \\
\left. \left[\alpha_i \left(y - \frac{h}{2} \right) \right] + c_{2i} \left(\sinh \left[\alpha_i \left(y - \frac{h}{2} \right) \right] \right. \right. \\
\left. + \alpha_i \left(y - \frac{h}{2} \right) \cosh \left[\alpha_i \left(y - \frac{h}{2} \right) \right] \right] \\
+ \beta_i^2 \cos(\beta_i y) \left[c_{3i} \sinh \left[\beta_i \left(x - \frac{L}{2} \right) \right] \right. \\
+ c_{4i} \left(\sinh \left[\beta_i \left(x - \frac{L}{2} \right) \right] + \beta_i \left(x - \frac{L}{2} \right) \right. \\
\left. \left. \cosh \left[\beta_i \left(x - \frac{L}{2} \right) \right] \right) \right] \dots \dots (12 \text{ c})
\end{aligned}$$

c_{1i} and c_{2i} are expressed as in the equations (13), (14) and (15)⁹⁾.

$$\begin{aligned}
c_{1i} = \frac{2\nu p_{y0} K_{2i}}{\pi \alpha_i^2 (2i+1) K_{1i}^2} \\
(i=0, 1, 2, 3, \dots, i_0) \dots \dots (13)
\end{aligned}$$

$$\begin{aligned}
c_{2i} = \frac{K_{1i}}{K_{2i}} c_{1i} \\
(i=0, 1, 2, 3, \dots, i_0) \dots \dots (14)
\end{aligned}$$

$$\begin{aligned}
K_{1i} = \cosh \left(\frac{\alpha_i h}{2} \right), K_{2i} = \left(\frac{\alpha_i h}{2} \right) \sinh \left(\frac{\alpha_i h}{2} \right) \\
(i=0, 1, 2, 3, \dots, i_0) \dots \dots (15)
\end{aligned}$$

c_{3i} and c_{4i} are expressed as in (16), (17 a), (17 b), (17 c) and (18).

$$\begin{aligned}
c_{3i} = \frac{\alpha_i^2 [(c_{1i} + c_{2i}) K_{3i} + c_{2i} K_{4i}] K_{6i}}{\beta_i^2 [K_{6i} K_{7i} - K_{5i} (K_{7i} + K_{8i})]} \\
(i=0, 1, 2, 3, \dots, i_0) \dots \dots (16)
\end{aligned}$$

$$K_{3i} = \cosh \left(\frac{\beta_i L}{2} \right), K_{4i} = \left(\frac{\beta_i L}{2} \right) \sinh \left(\frac{\beta_i L}{2} \right) \dots \dots (17 \text{ a})$$

$$K_{5i} = \sinh \left(\frac{\alpha_i h}{2} \right), K_{6i} = \left(\frac{\alpha_i h}{2} \right) \sinh \left(\frac{\alpha_i h}{2} \right) \dots \dots (17 \text{ b})$$

$$K_{7i} = \sinh \left(\frac{\beta_i L}{2} \right), K_{8i} = \left(\frac{\beta_i L}{2} \right) \sinh \left(\frac{\beta_i L}{2} \right) \dots \dots (17 \text{ c})$$

$$\begin{aligned}
c_{4i} = - (K_{5i}/K_{6i}) c_{3i} \\
(i=0, 1, 2, 3, \dots, i_0) \dots \dots (18)
\end{aligned}$$

2.3 Bending and bending stress

Deflection w and bending stresses σ_{bx} , σ_{by} and τ_{bxy}

are going to be discussed. Due to the requirement for deformation originating from a box shape, these stresses in this paper will be more complicated than those in the preceding paper¹⁰⁾. Therefore, the equation expressing bending will be more complicated.

According to the equation (7), bending force is produced by force $T \tau_{xy}$ in a neighboring side plate.

As in Fig. 2, the bending moment should be continuous from the side plate to its neighboring plate. So, the bending moment M_x and M_y which are produced by $T \tau_{xy}$ of the neighboring plate are required to be as in the equations (19 a), (19 b) and (20) at its peripheral edges.

$$M_x(x=0 \text{ or } L) = 0 \dots \dots (19 \text{ a})$$

$$M_x(y=0 \text{ or } h) = 0 \dots \dots (19 \text{ b})$$

$$M_y(y=0 \text{ or } h) = 0 \dots \dots (20)$$

The deflection w should also satisfy deformation requirement as in the equation (21).

$$w(y=0 \text{ or } h) = 0 \dots \dots (21)$$

As Q_y at the side edges ($x=0$ or L) should satisfy the equation (7) due to the balance of forces, the equations (22), (23 a) and (23 b) are deducted¹⁴⁾.

$$Q_y = -D \left(\frac{\partial^3 w}{\partial y^3} + \frac{\partial^3 w}{\partial x \partial y^2} \right) \dots \dots (22)$$

$$D = ET^3/[12(1-\nu)]$$

$$\begin{aligned}
Q_y(x=0 \text{ or } L) = T \tau_{xy}(x=0 \text{ or } L) = \sum_{i=1,2,3,\dots} T \tau_{xyi} \\
\dots \dots (23 \text{ a})
\end{aligned}$$

$$\tau_{xyi} = \tau_{xy1i} + \tau_{xy2i} \dots \dots (23 \text{ b})$$

From the equations (7) and (12 c), τ_{xy1i} and τ_{xy2i} are expressed as in the equations (24 a) and (24 b) respectively.

$$\tau_{xy1i} = [\alpha_i^2 (c_{1i} + c_{2i}) K_{3i} + c_{2i} K_{4i}] \dots \dots (24 \text{ a})$$

$$\tau_{xy2i} = [\beta_i^2 (c_{3i} + c_{4i}) K_{7i} + c_{4i} K_{8i}] \dots \dots (24 \text{ b})$$

Taking into account the relation between τ_{xy1i} , which is a main part of working force in the equation (23) and the working force itself, Fourier function using $\cos(\gamma_n y)$ is more reasonable, which is different from one used in the preceding paper¹⁰⁾. Using integration equations bellow, Fourier function is expressed as in the equations (25 a), (25 b), (26 a) and

$$\begin{aligned}
\int \exp(\alpha_i y) \sin(\beta_i y) dy = \frac{\exp(\alpha_i y)}{\alpha_i^2 + \beta_i^2} \\
[\alpha_i \cos(\beta_i y) + \beta_i \sin(\beta_i y)] \\
\int y \exp(\alpha_i y) \sin(\beta_i y) dy = \frac{y \exp(\alpha_i y)}{\alpha_i^2 + \beta_i^2}
\end{aligned}$$

$$[\alpha_i \cos(\beta_i y) + \beta_i \sin(\beta_i y)] \quad \cdots \cdots (31 \text{ a})$$

$$- \frac{\exp(\alpha_i y)}{(\alpha_i^2 + \beta_i^2)^2} [(\alpha_i^2 - \beta_i^2) \cos(\beta_i y) + 2\alpha_i \beta_i \sin(\beta_i y)] \quad \sum_{n=1,2,3,\dots} G_{\text{Ain}} = T \alpha_i^2 [(c_{1i} + c_{2i}) K_{3i} + c_{2i} K_{4i}] \quad \cdots \cdots (31 \text{ b})$$

$$T \tau_{xyi} = \sum_{n=1,2,3,\dots} G_{\text{Ain}} \cos(\gamma_n y) \quad \cdots \cdots (25 \text{ a})$$

$$\gamma_n = \pi n / h \quad \cdots \cdots (25 \text{ b})$$

$$G_{\text{Ain}} = \frac{4}{\pi} \left[\frac{\alpha_i^3}{(\alpha_i^2 + \gamma_n^2)} \left[(c_{1i} + c_{2i}) L_{1i} + c_{2i} \left[\frac{\alpha_i^3}{(\alpha_i^2 + \gamma_n^2)} L_{2i} - \frac{\alpha_i^3 (\alpha_i^2 - \gamma_n^2)}{(\alpha_i^2 + \gamma_n^2)^2} L_{1n} \right] \right] \right] \quad \cdots \cdots (26 \text{ a})$$

$$L_{1n} = \cosh\left(\frac{\gamma_n L}{2}\right), \quad L_{2n} = \left(\frac{\gamma_n L}{2}\right) \sinh\left(\frac{\gamma_n L}{2}\right) \quad \cdots \cdots (26 \text{ b})$$

$$(n = 1, 3, 5, \dots, n_0)$$

In the equation (25 a), $\sin(\gamma_n y)$ will be zero at each n value, and $\cos(\gamma_n y)$ will be zero at every even number.

Using the equation (12 c) which satisfies the requirement for deformation in the equation (7) and the equation (23 a), the relation between $G_{\text{Ain}} T$ of τ_{xyi} and τ_{xyi} is as in the equation (27).

$$T \gamma_{xyi} (x=0 \text{ or } L, y=0 \text{ or } h) = \sum_{n=1,2,3,\dots} G_{\text{Ain}} \quad \cdots \cdots (27)$$

$$(n = 1, 3, 5, \dots, n_0)$$

The bending in the side plate satisfied the equation (28)⁽¹⁴⁾.

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = 0 \quad \cdots \cdots (28)$$

As for bending w , more reasonable equations (29), (30 a), (30 b) and (30 c), which satisfy the symmetry of bending deformation, are used. They are different from ones used in the preceding paper⁽¹⁰⁾.

$$w = \sum_{i=1,2,3,\dots} [\sin(\gamma_i y) (C_{1i}^* f_{1i}^* + C_{2i}^* f_{2i}^*)] \quad \cdots \cdots (29)$$

$$f_{1i}^* = \sinh\left[\gamma_i \left(x - \frac{L}{2}\right)\right] \quad \cdots \cdots (30 \text{ a})$$

$$f_{2i}^* = \left[\gamma_i \left(x - \frac{L}{2}\right)\right] \sinh\left[\gamma_i \left(x - \frac{L}{2}\right)\right] \quad \cdots \cdots (30 \text{ b})$$

$$\gamma_i = \pi (2i+1) / h \quad \cdots \cdots (30 \text{ c})$$

Substituting the equation (29) into the equation (22), Q_y is expressed as followings.

$$Q_y = \sum_{i=1,2,3,\dots} \gamma_i^3 D [[(C_{1i}^* 2C_{2i}^*) f_{3i} + C_{2n} f_{4i}] \cos(\gamma_i y)]$$

$$= \sum_{i=1,2,3,\dots} \left[\left[\sum_{n=1,2,3,\dots} G_{\text{Ain}} \cos(\gamma_i y) \right] + G_{\text{Bi}} \cos(\gamma_i y) \right]$$

$$\sum_{n=1,2,3,\dots} G_{\text{Ain}} = T \alpha_i^2 [(c_{1i} + c_{2i}) K_{3i} + c_{2i} K_{4i}] \quad \cdots \cdots (31 \text{ b})$$

$$G_{\text{Bi}} = \beta_i^2 (c_{3i} + c_{4i}) K_{7i} + c_{4i} K_{8i} \quad \cdots \cdots (31 \text{ c})$$

$$\gamma_n = \beta_n = \pi (2n+1) / L \quad \cdots \cdots (31 \text{ d})$$

Exchanging i and n of G_{Ain} each other, and using resulting G_{Ani} , the summation of G_{Ain} is expressed as in the equation (31 e).

$$G_{\text{Ai}} = \sum_{n=1,2,3,\dots} G_{\text{Ani}} \quad \cdots \cdots (31 \text{ e})$$

Working force Q_y at the side edges ($x=0$ or L) has to be preserved. So, in the equation (12 c), τ_{xyi} at $y=0$ or h is expressed as in the equation (32).

$$T \tau_{xyi} = -\gamma_i^3 D [(C_{1i}^* + 2C_{2i}^*) K_{7i} + C_{2i}^* K_{8i}]$$

$$= (G_{\text{Ai}} + G_{\text{Bi}}) \quad \cdots \cdots (32)$$

$$M_x = -D \left[\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right]$$

$$= \sum_{i=1,2,3,\dots} \gamma_i^2 D [[(C_{1i}^* + 2C_{2i}^*) f_{3i} + C_{2i}^* f_{4i}]$$

$$+ \nu (C_{1i}^* f_{3i} + C_{2i}^* f_{4i})] \sin(\gamma_i y) \quad \cdots \cdots (33)$$

From the equation (29), M_x is expressed as in the equation (39)⁽¹⁵⁾. Introducing the equation (20) into the equation (33) gives the equation (34).

$$[(1-\nu) K_{7i} C_{1i}^* + [2K_{7i} + (1-\nu) K_{8i}] C_{2i}^*] = 0 \quad \cdots \cdots (34)$$

$$C_{1i}^* = -\frac{2K_{7i} + (1-\nu) K_{8i}}{(1-\nu) K_{7i}} C_{2i}^* \quad \cdots \cdots (35)$$

Then, from the equation (34), the equation (35) is obtained. From the equations (33) and (35), C_{2i} is expressed as in the equation (36).

$$C_{2i}^* = -\frac{(G_{\text{Ai}} + G_{\text{Bi}})}{D \gamma_i^3 [(1+A_{0i}) K_{7i} + A_{0i} K_{8i}]}$$

$$A_{0i} = -\frac{(1-\nu) K_{7i}}{2K_{7i} + (1-\nu) K_{8i}} \quad \cdots \cdots (36)$$

M_y and M_{xy} in the paper⁽¹⁵⁾ and the equation (29) give the equation (37).

$$M_y = -D \left[\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right]$$

$$= \sum_{i=1,2,3,\dots} \sum_{j=1,2,3,\dots} \gamma_n^2 D [(1-\nu) f_{1i}^* C_{1i}^* + [(1-\nu) f_{2i}^* + 2f_{1i}^*] C_{2i}^*] \sin(\gamma_i y) \quad \cdots \cdots (37)$$

$$M_{xy} = -\frac{GT^3}{6} \frac{\partial^2 w}{\partial x \partial y}$$

$$= \sum_{i=1,2,3,\dots} \left(\frac{GT^3 \gamma_i^2}{6} \right) \cos(\gamma_i y) [(C_{1i}^* + C_{2i}^*) f_{1i}^* + C_{2i}^* f_{2i}^*] \quad \dots\dots(38)$$

Bending normal stress in x direction σ_{bx} and that in y direction σ_{by} and bending shear stress τ_{bxy} at the outer surface of the side plate are expressed as follows¹⁵⁾

$$\sigma_{bx} = -\frac{TE}{2(1-\nu^2)} \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \quad \dots\dots(39)$$

$$\sigma_{by} = -\frac{TE}{2(1-\nu^2)} \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \quad \dots\dots(40)$$

$$\tau_{bxy} = -TG \frac{\partial^2 w}{\partial x \partial y} \quad \dots\dots(41)$$

Bending stress components at the outer surface of the side plate ($z = T/2$) are expressed as in the equation⁴²⁾, using the equations (35), (37)–(41).

$$\sigma_{bx} = \frac{6M_x}{T^2}, \quad \sigma_{by} = \frac{6M_y}{T^2}, \quad \tau_{bxy} = \frac{6M_{xy}}{T^2} \quad \dots\dots(42)$$

2.4 Principal bending stress and principal bending shear stress

The side plate collapses at the location where stress becomes its maximum. Maximum normal stress is regarded as principal stress, and maximum shear stress is regarded as principal shear stress. When the side plate collapses under load due to bending stress, it occurs not at the location where principal bending stress σ_{b0} becomes its maximum but at the location where principal shear stress becomes its maximum. It is where plastic deformation starts and elastic stress breakdown begins. Therefore, getting σ_{b0} and τ_{b0} is very important for designing box strength. σ_{b0} and τ_{b0} are in expressed as in the equations (43) and (44)¹⁷⁾.

$$\sigma_{b0} = \frac{1}{2} (\sigma_{bx} + \sigma_{by}) \pm \frac{1}{2} \sqrt{(\sigma_{bx} - \sigma_{by})^2 + 4\tau_{bxy}^2} \quad \dots\dots(43)$$

$$\tau_{b0} = \pm \frac{1}{2} \sqrt{(\sigma_{bx} - \sigma_{by})^2 + 4\tau_{bxy}^2} \quad \dots\dots(44)$$

3. Result and discussion

In this paper, a model box is under uniform compression load. To make a model box similar to one used in market, it is assumed as follows⁷⁾. $L = 350$ mm, $h = 300$ mm, $T = 5.44$ mm, $E = 2.91 \times 10^5$ N/mm², and $\nu = 0.10$

Though i in the equations (10 a) to (11 b) is an integral number up to infinite, it goes to 24 ($i_0, n_0 = 24$) where calculation error becomes less than 5%.

3.1 σ_{bx} , σ_{by} , τ_{bxy} and w in the side plate

σ_{bx} and σ_{by} from the equation⁴²⁾ and w are symmetrical to a center line of L (width) and are also symmetrical to a center line of h (height). On the other hand, τ_{bxy} from the equation⁴²⁾ is anti-symmetrical to those two center lines. So, an area where x is from zero to $L/2$ and y is from zero to $h/2$ is enough to show their distributions in the side plate. The behavior of every stress in that area is plotted in Fig. 3 to 6.

From Fig. 3, followings are concluded. At $y = 0$ and $y = h$, σ_{bx} is zero in the range where x is from zero to L . At a fixed y between $h/8$ to $7h/8$ and with x increasing from zero to L , σ_{bx} decreases and then increases, and produces a minimum. At $x = 0$ and $x = L$, σ_{bx} is zero in the range where y is from zero to h . At a fixed x between $L/8$ to $7L/8$ and with y increasing from zero to h , σ_{bx} decreases and then increases, and produces a minimum. At the location where x is $L/2$ and y is $h/2$, σ_{bxmax} , which is the maximum value of $|\sigma_{bx}|$ and is -0.0652 N/mm², appears. The fact that σ_{bx} is zero at the peripheral edges and is negative maximum at the location ($x = L/2$ and $y = h/2$) suggests that the distribution shape is concave. It is clear that σ_{bx} satisfies the boundary requirement for M_x , which is expressed in the equations (19 a) and (19 b).

From Fig. 4, followings are concluded. At $y = 0$ and $y = h$, σ_{by} is zero in the range where x is from zero to L . At a fixed y between $h/8$ to $7h/8$ and with x increasing from zero to L , σ_{by} decreases and then increases, and produces a minimum. At a fixed x between zero and L and with y increasing from zero to h , σ_{by} increases and then decreases, and produces a maximum. At the location where y is $h/2$ and x is zero or h , σ_{bymax} , which is the maximum value of σ_{by} and is 0.262 N/mm², appears. The σ_{by} is zero at the upper and lower edges and is positive maximum at the location ($y = h/2$ and $x = 0$ or L) suggests that its distribution is a saddle shape along y direction. It is clear that σ_{by} satisfies the boundary requirement for M_y , which is expressed in the equation (20).

From Fig. 5, followings are concluded. With x increasing from zero to L , τ_{bxy} is always zero at $y = h/2$. At a fixed y below $h/2$, τ_{bxy} increases along with x . At a fixed y over $h/2$, τ_{bxy} decreases along with x . With y increasing from zero to h , τ_{bxy} is always zero at $x = L/2$. At a fixed x below $L/2$, τ_{bxy} increases along with y .

At a fixed x over $L/2$, τ_{bxy} decreases along with y . At the four corners of the side plate ($x=0$ or L , $y=0$ or h), $\tau_{bxy\max}$ which is the maximum value of $|\tau_{bxy}|$ and is $\pm 0.163 \text{ N/mm}^2$ appears. τ_{bxy} is zero on a center line of the width and is also zero on a centerline of the height. At the two locations ($x=0$ and $y=0$) and ($x=L$ and $y=h$), τ_{bxy} becomes its positive maximum $\tau_{bxy\max}$. At the two locations ($x=0$ and $y=h$) and ($x=L$ and $y=0$), τ_{bxy} becomes its negative maximum. The distribution is a complicated bending shape.

From Fig. 6, followings are concluded. With x increasing from zero to L , w is always zero at $y=0$ and $y=h$. At a fixed y between $h/8$ and $7h/8$ and with x increasing from zero to L , w decreases and then increases, and produces a minimum. With y increasing from zero to h , w increases and then decreases, and produces a maximum. At the two locations ($x=0$ and $y=h/2$) and ($x=L$ and $y=h/2$), $|w|$ becomes its maximum $w_{\max}$ which is 3.36 mm . Its distribution shapes like a saddle as in σ_{bx} . It is clear that w satisfies the equation (21), which is the requirement for deformation on the upper and lower edges.

Ratios between stresses are calculated as followings.

$$\sigma_{bx\max}/\sigma_{x\max} = 6.5, \quad \sigma_{by\max}/\sigma_{y\max} = 2.6, \quad \tau_{bxy\max}/\tau_{xy\max} = 33.5$$

Bending stress is most influential. The reason why the bending stress is so large is that the side plate is thin and large enough and it is easily deformed by bending force $T \tau_{xy}$. It is analogous to the fact that a long rod can be bent more easily than be stretched or be compressed. The distribution of every stress and that of w in this paper are similar to those in the preceding paper¹⁰⁾. But, the value of every stress is about a half of the counterpart, and w is about a third of one in that paper.

3.2 σ_{b0} and τ_{b0} in the side plate

σ_{b0} which is deducted from the equations (43) and (44) is symmetrical to a center line of the width and also to a center line of the height. τ_{b0} is anti-symmetrical to those two center lines. σ_{b0} and τ_{b0} are plotted in Fig. 7 and Fig. 8.

From Fig. 7, followings are concluded. With x increasing from zero to L , σ_{b0} decreases and then increases, and produces a minimum. With y increasing from zero to h , σ_{b0} increases and decreases, and produces a maximum. At the center of the side plate,

$\sigma_{b0\max}$, which is the maximum value of $|\sigma_{b0}|$ and is 0.261 N/mm^2 , appears.

From Fig. 8, followings are concluded. At $y=0$, τ_{b0} decreases with x increasing from zero to L . At $y=h$, τ_{b0} increases with x increasing from zero to L . At a fixed y below $h/2$, τ_{b0} decreases along with x . At a fixed y over $h/2$, τ_{b0} increases along with x . With y increasing from zero to h and $x=0$, τ_{b0} decreases. At $x=L$, τ_{b0} increases along with y . At a fixed x below $L/2$, τ_{b0} increases along with y . At a fixed x over $L/2$, τ_{b0} decreases along with y . At the four corners of the side plate, $\tau_{b0\max}$, which is the maximum value of $|\tau_{b0}|$ and is $\pm 0.163 \text{ N/mm}^2$, appears. σ_{b0} keeps continuity on a center line of the width and on that of the height. But, a tangential line to them is not continuous. τ_{b0} is not continuous as well.

4. Conclusions

A model corrugated container of a box shape is under uniform compression load, and bending deformation of its side plate is calculated. A side plate is 350 mm in width, 300 mm in height and 5.44 mm in thickness. One corner of it is located at the origin. Its width is in x axis, and its height is in y axis. Uniform compression load p_{y0} is on the peripheral edges of the upper and lower faces. It is assumed that deformation in x direction of the upper and lower edges is negligibly zero, and normal stain of the side edges ϵ_x is also zero. Elastic modulus in y direction is assumed to be $2.91 \times 10^9 \text{ N/mm}^2$. Poisson's ratio ν is assumed to be 0.10 .

Every stress component of the side plate, bending w , principal bending stress σ_{b0} and principal bending shear stress τ_{b0} are calculated, and are plotted against L and h . Then, the maximum value of w is discussed.

1) At $y=0$ and $y=h$, σ_{bx} is zero in the range where x is from zero to L . At a fixed y between $h/8$ to $7h/8$ and with x increasing from zero to L , σ_{bx} decreases and then increases, and produces a minimum. At $x=0$ and $x=L$, σ_{bx} is zero in the range where y is from zero to h . At a fixed x between $L/8$ to $7L/8$ and with y increasing from zero to h , σ_{bx} decreases and then increases, and produces a minimum.

At $y=0$ and $y=h$, σ_{by} is zero in the range where x is from zero to L . At a fixed y between $h/8$ to $7h/8$ and with x increasing from zero to L , σ_{by} decreases and then increases, and produces a minimum. At a

fixed x between zero and L and with y increasing from zero to h , σ_{by} increases and then decreases, and produces a maximum.

With x increasing from zero to L , τ_{bxy} is always zero at $y=h/2$. At a fixed y below $h/2$, τ_{bxy} increases along with x . At a fixed y over $h/2$, τ_{bxy} decreases along with x . With y increasing from zero to h , τ_{bxy} is always zero at $x=L/2$. At a fixed x below $L/2$, τ_{bxy} increases along with y . At a fixed x over $L/2$, τ_{bxy} decreases along with y .

2) At the center of the upper and lower edges, the maximum value of $|\sigma_{bx}|$, which is -0.0652 N/mm^2 , and that of σ_{by} , which is 0.262 N/mm^2 , appear. At the four corners of the side plate, the maximum value of $|\tau_{bxy}|$, which is $\pm 0.163 \text{ N/mm}^2$, appears.

3) With x increasing from zero to L , w is always zero at $y=0$ and $y=h$. At a fixed y between $h/8$ and $7h/8$ and with x increasing from zero to L , w decreases and then increases, and produces a minimum. With y increasing from zero to h , w increases and then decreases, and produces a maximum. At the two locations ($x=0$ and $y=h/2$) and ($x=L$ and $y=h/2$), $|w|$ becomes its maximum $w_{\max}$ which is 3.36 mm .

4) With x increasing from zero to L , σ_{b0} decreases and then increases, and produces a minimum. With y increasing from zero to h , σ_{b0} increases and decreases, and produces a maximum. At $y=0$, τ_{b0} decreases

with x increasing from zero to L . At $y=h$, τ_{b0} increases with x increasing from zero to L . At a fixed y below $h/2$, τ_{b0} decreases along with x . At a fixed y over $h/2$, τ_{b0} increases along with x . With y increasing from zero to h and $x=0$, τ_{b0} decreases. At $x=L$, τ_{b0} increases along with y . At a fixed x below $L/2$, τ_{b0} increases along with y . At a fixed x over $L/2$, τ_{b0} decreases along with y .

5) At the center of the side plate, $\sigma_{b0\max}$, which is the maximum value of $|\sigma_{b0}|$ and is 0.261 N/mm^2 , appears. At the four corners of the side plate, $\tau_{b0\max}$, which is the maximum value of $|\tau_{b0}|$ and is $\pm 0.163 \text{ N/mm}^2$, appears. σ_{b0} keeps continuity on a center line of the width and on that of the height. But, a tangential line to them is not continuous. τ_{b0} is not continuous as well.

6) σ_{bx} , σ_{by} and w distribute symmetrical to each center line. τ_{bxy} is anti-symmetrical to those lines. The maximum value of every bending stress, which is $\sigma_{bx\max}$, $\sigma_{by\max}$ and $\tau_{bxy\max}$, is larger than that of every plane stress of counterpart, which is $\sigma_{x\max}$, $\sigma_{y\max}$ and $\tau_{xy\max}$. The ratio is 6.5, 5.2 and 33.5 respectively.

These results satisfy the criteria required, and are thought to be reasonable. They will be helpful for discussing the compression strength of corrugated container.