On Generalized Belt Tension Formulas

By Kumeo Nakajima*, Kenji Matsuoka** and Taku Inoue***

I. Introduction

The frictional force of a yarn moving around and in contact with a cylinder is calculated by using the belt tension formula. However, if the yarn moves not in a plane perpendicular to the cylinder axis, but obliquely to the cylinder, the belt tension formula is not to be used.

The authors have analyzed mathematically general cases where both yarn and cylinder move with similar assumptions to those adopted for the derivation of the belt tension formula. As for the law of friction, new theories are being developed for inter-fiber friction,(1,2,3) but here the conventional Amonton’s law is used for its mathematical simplicity. In some cases the effect of centrifugal force is neglected.

Thus, the quantity of air inflow calculated by the equation (12) corresponds approximately to the value calculated by reading the orifice meter, and we believe that the equation (12) can be used safely.

4. Conclusion

The foregoing may be summarized as follows:

A small multi-stage type unit cyclone about 30 mm. in diameter, whose resistance coefficient is known beforehand, should be used as a suction type, and connected with a high speed electric fan. When it sucks in the air stream together with cotton dust at the inflowing velocity of more than 30 m/s, it shows that the dust collecting efficiency of the cyclone is 98-99%. Therefore, the quantity of cotton dust contained in the air flow, i.e., the density of floating cotton dust will be found out by weighing the quantity of the cotton dust collected by the cyclone in a certain length of time, and by calculating the quantity of the air flowing into the cyclone due to the substitution of the value of static pressure difference measured at the inlet and outlet of the cyclone into the equation (12).

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\[ d = 16/1000 \text{ (m.)} \quad \zeta = 1.25 \quad \text{kg/m}^2 \]
\[ \zeta = 9.81 \text{ m/s}^2 \]
\[ \zeta = 2.1 \]

then,

\[ Q = 90 \times 10^{-6} \sqrt{1.25} \left( \frac{4 \times 6 \times 15 \times 10^{-6}}{3.14} \right)^{1/2} \]
\[ = 90 \times 10^{-6} \sqrt{1085} = 0.00298 \text{ m}^3/\text{s}. \]

(Hereupon, the value of the velocity of air inflow into the cyclone is \( V = 0.00298/6 \times 15 \times 10^{-6} \text{ m/s} \), and that the value as above calculated will be found to be correct compared with the \( H-T \) chart in Fig. 5).

On the other hand, by reading the orifice meter we can calculate the inflowing air quantity \( Q \) as follows:

\[ Q = 0.62 \pi \left( \frac{20}{1000} \right) \sqrt{2 \times 9.81 \times 15} \]
\[ = 0.00299 \text{ m}^3/\text{s}. \]

Hence, the value of the air flowing coefficient of the orifice meter (DIN Standard Type, 20 mm. diam.), which is set on the 2 in. gas pipe, is taken as 0.62.

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Fig. 2

The equations of the forces acting on a small increment of the yarn are:

\( (T_x + dT_x) - T_x - F_x = 0, \)
\( (T_\theta + dT_\theta) \cos \frac{d\theta}{2} - T_\theta \cos \frac{d\theta}{2} - F_\theta = 0, \)
\( (T_\theta + dT_\theta) \sin \frac{d\theta}{2} - T_\theta \sin \frac{d\theta}{2} - N - C = 0. \)

The equations for the equilibrium of the moments in Rd-\( dz \) plane are:

\( T_\theta d\theta - (T_x + dT_x) \frac{dz}{2} - T_x \frac{R d\theta}{2} = 0. \)  

Here

\( T_x = T \sin \phi, \quad T_\theta = T \cos \phi, \)
\( F_x = F \sin \beta, \quad F_\theta = F \cos \beta. \)

and

\( \tan \beta = \frac{\sin \alpha + \gamma \sin \phi}{\cos \alpha + \gamma \cos \phi}, \quad \gamma = \frac{V_c}{V}. \)

Assuming that the frictional force is proportional to the reaction as previously mentioned, then

\( F = fN. \)

The centrifugal force is proportional to the mass of the yarn increment, the square of the yarn velocity projected to the plane perpendicularly to the axis of the cylinder, and the reciprocal of the radius of the cylinder \( R \). Thus

\( C = \frac{\rho}{g} \frac{R d\theta}{\cos (\phi + \frac{d\phi}{2})} \cdot V_c^2 \cos^2 (\phi + \frac{d\phi}{2}) \cdot \frac{1}{R}, \)

and neglecting terms of higher order,

\( C = \frac{\rho}{g} V_c^2 \cos \phi d\theta. \)

3. General Cases

Solving Eq. (2.1) with Eqs. (2.3), (2.4), (2.5), and (2.6) and neglecting the small quantities of higher order,

\( \frac{T}{T_\theta} = \left( \frac{\sin (\phi - \alpha)}{\sin (\alpha - \phi)} \right)^2 \left( \frac{\tan (\phi - \alpha)}{\tan (\alpha - \phi)} \right)^2 \)  

\( \frac{d\phi}{f} = f \cos \phi \sin (\beta - \phi) d\theta. \)

where the quantities with suffix "0" are related to the point where contact begins, and those without suffix to the point where contact ends.

The dependence of the tension of the yarn upon the inclination of the yarn is shown by Eq. (3.1). Eq. (3.2) explains the relationship between the yarn inclination and the angle of contact, but is difficult to solve when the tension of the yarn and the centrifugal force are of the same order. Numerical integration will be needed. But, if the centrifugal force is small compared with the tension of the yarn, it is reduced to

\( d\phi = f \cos \phi \sin (\beta - \phi) d\theta. \)

Substituting Eq. (2.4),

\( f d\theta = \frac{\sqrt{1 + \gamma^2 \sin^2 (\alpha - \phi)}}{\cos \phi \sin (\alpha - \phi)} \sqrt{1 + \frac{2\gamma}{1 + \gamma} \cos (\alpha - \phi)} \)  

Ignoring the small quantities of higher order in Eq. (2.2) and substituting Eq. (3.3),

\( f \frac{dz}{R} = \frac{\sqrt{1 + \gamma^2 \sin^2 \phi}}{\cos ^2 \phi \sin (\alpha - \phi)} \sqrt{1 + \frac{2\gamma}{1 + \gamma} \cos (\alpha - \phi)} \)  

Integration of Eqs. (3.3) and (3.4) will be performed when \( \gamma \) is much smaller than one and when it is very large by expanding the radical on the right-hand side of the Eqs. (3.3) and (3.4). Then,
Eq. (3.5) indicates the relationship between the angle of contact and the inclination of the yarn. How the projected length of contact varies with the inclination of the yarn is shown by Eq. (3.7). These equations will be valid on the assumption that the centrifugal force is small in comparison with the tension of the yarn and that one of the yarn velocity and the cylinder velocity is much larger than the other. These assumptions will be allowed in many practical applications.

Integrating Eq. (3.6),

\[ I_i = \left. \frac{\cos^i(a - \varphi)}{\cos^i \varphi \sin (a - \varphi)} \right|_{\varphi = \varphi_0}^{\varphi}, \quad (i = 0, 1, 2, \ldots) \quad (3.8) \]

where

\[ J_i = \int_{\varphi_0}^{\varphi} \frac{\sin \varphi \cos^i(a - \varphi)}{\sin (a - \varphi)} d\varphi (i = 0, 1, 2, \ldots) \quad (3.3) \]

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When the cylinder moves in the direction of its axis, \( a = \pm \frac{\pi}{2} \), and

\[ I_0 = \left. \left[ \tan \varphi \right] \right|_{\varphi_0}^{\varphi}, \quad I_1 = \left. \frac{1}{\cos \varphi \varphi_0} \right|_{\varphi_0}^{\varphi}, \quad I_2 = \left. \left[ \tan \varphi - \varphi \right] \right|_{\varphi_0}^{\varphi}, \quad I_3 = \left. \frac{1}{\cos \varphi + \cos \varphi \varphi_0} \right|_{\varphi_0}^{\varphi}, \quad (3.9) \]

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Since the series in Eqs. (3.5) and (3.7) converge rather rapidly, higher terms than \( n \) are not needed for practical purposes.

4. Special Cases.

When yarn moves and cylinder is stationary, the relations are established. Substituting these in Eqs. (2.1), (2.2), (2.3), (2.4), and (2.5), the following results are obtained:

\[
\begin{align*}
I_0 &= \frac{1}{\cos \phi} - 2 \cos \phi - \frac{1}{3} \cos^3 \phi, \\
I_1 &= \frac{1}{\cos \phi} + 2 \cos \phi - \frac{1}{3} \cos^3 \phi, \\
I_2 &= \frac{1}{\cos \phi} + 2 \cos \phi - \frac{1}{3} \cos^3 \phi.
\end{align*}
\]

When the cylinder rotates, \( a = 0 \) or \( a = \pi \), and

\[
\begin{align*}
I_0 &= \pm \left[ \ln \left( \tan \phi \right) \right] \frac{\tan \phi}{\tan \phi}, \\
I_1 &= \left[ \ln \left( \tan \phi \right) \right] \frac{\tan \phi}{\tan \phi}, \\
I_2 &= \pm \left[ \ln \left( \tan \phi \right) \right] \frac{\tan \phi}{\tan \phi},
\end{align*}
\]

\[
\begin{align*}
I_0 &= \frac{1}{4} \left( \cos 2\phi - 1 \right) + \ln \left( \sin \phi \right) \frac{\tan \phi}{\tan \phi}, \\
I_1 &= \left[ \ln \left( \tan \phi \right) \right] \frac{\tan \phi}{\tan \phi}, \\
I_2 &= \left[ \ln \left( \tan \phi \right) \right] \frac{\tan \phi}{\tan \phi}.
\end{align*}
\]

\[
f \left( \theta - \theta_0 \right) = - \frac{1}{\cos \alpha} \ln \left( \tan \phi \right) - \frac{\tan \phi}{\tan \phi} \left( a = \pm \frac{\pi}{2} \right),
\]

\[
= \pm \left( \tan \phi - \tan \phi_0 \right) \left( a = \pm \frac{\pi}{2} \right),
\]

\[
= \pm \left( \tan \phi - \tan \phi_0 \right) \left( a = \pm \frac{\pi}{2} \right).
\]

As is seen from these equations, the angle of inclination of the yarn is not a constant. The tension of the yarn and the angle of contact are functions of the angle of inclination of the yarn. They are connected through the angle of inclination of the yarn as they do in the general cases discussed in the previous section.

Some of these results have already been used by several authors for calculating spinning tension of the ballooning yarn.(4,5)

5. Numerical Results

The analytical results derived in the previous sections are very elaborate to calculate. To save the labor of tedious calculations, charts are made connecting yarn tension, angle of contact, and angle of yarn inclination.

Figs. 3, 4, and 5 are diagrams showing Eqs. (3.1) and (3.5) when \( \alpha = \pi/2 \), in other words, when the cylinder moves in the direction of its axis. Fig. 3 is the diagram when yarn does not move, namely \( \gamma = 0 \). Figs. 4 and 5 are the curves when \( \gamma = 0.08 \) and 0.16, respectively. The angle of inclination of the yarn at the point where the contact begins, \( \phi_0 \), is taken as abscissa and that at the point where contact ends, \( \phi_e \), as ordinate. Curves are drawn on which the ratio of the yarn tension \( T/T_0 \) and the angle of contact times friction coefficient \( f (\theta - \theta_0) \) are constant.

When using these charts the cylinder should move downwards and the yarn should move from left to right as shown in Fig. 6(a). As the case where the yarn moves from right to left (Fig. 6b) is the image of the case where the yarn moves from left to right, the former is discarded. Three attitudes of the yarn going round the cylinder are
possible as shown in Fig. 6a, and for all of them \( \varphi_0 < \varphi \). Thus in Figs. 3, 4, and 5 only one half of the whole \( \varphi_0 - \varphi \) plane which corresponds to \( \varphi_0 < \varphi \) is shown. If any two of four quantities, \( \varphi_0, \varphi, T/T_0, f = (\theta - \theta_0) \), are given, then the remaining two quantities will be determined from the chart.