A Statistical Analysis of Yarn Breakage on Spinning Frame

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Abstract

The problem of yarn breakage during spinning is quite serious, but it is difficult to predict the number of yarn breakages, because the tension and strength of the yarn always vary. The author has used simple mathematical models of the variation of the tension and the strength of the yarn, and calculated the probability of yarn breakages. The result has shown that the variation of the yarn strength influence yarn breakages most. Thus the evenness of the yarn is needed much more than the lower tension on spinning frames to lessen the number of yarn breakages.

1. Introduction

The problem of yarn breakage during spinning is serious but is not easy to solve because of the complexity of the breakage mechanisms. The yarn breakage phenomena are complicated because the causes are numerous, including the influences of humidity and temperature of the surrounding atmosphere, but the most influential is, no doubt, the fact that the tension on the yarn and the strength of the yarn always vary. This is why we can scarcely predict the number of yarn breakages, even if, for example, we ignore the rheological properties of yarn and merely assume that yarn breaks just at the moment the tension exceeds the strength. So long as yarn tension and strength vary, it makes hardly any sense to compare the average values of tension and strength as a basis for predicting the number of yarn breakages. For, in practice, the average tension cannot be higher than the average strength of yarn; and yarn breaks. Therefore, when we calculate the number of yarn breakages, we must investigate the characteristics of variations of yarn strength and tension, and must treat the problem by some statistical method.

2. Variations of Yarn Tension and Yarn Strength

Let us consider the variations of tension and strength of running yarn at a certain point, for example, at the exit of a front roller. (In practice, almost all yarn breakages occur at the exit of the front roller, probably because, at that point, yarn has not been twisted yet and is still weak.)

A) Variations of strength of yarn.

The variations of yarn strength may be, for example, shown by a curve such as that in Fig. 1, in which the ordinate shows yarn strength and the abscissa shows the time or the yarn length. But this curve is too complicated. So, in its place, we use a simpler, modified one (idealised model) which has the same basic characteristics as the curve in Fig. 1.

![Fig. 1. Variation of strength](image1)

![Fig. 2. Model for variation of strength](image2)
S2, S2, S2,... are the maximum and minimum values of strength, which are random variables and have a normal distribution with means mS1, mS2, and standard deviations sS1, sS2, respectively. The basic characteristics connecting this model with the actual variations of strength are Zs, mS, mS, sS, sS. In order that this substitution may be made, it is necessary to assume beforehand:

(a) That the number of maximum and minimum values of strength appearing at a certain interval of time, which is long enough, are always constant, i.e., variation of yarn strength, roughly speaking, is periodic in a sense.

(b) That maximum and minimum values of yarn strength are random variables having a normal distribution, and that the parameters of the distribution always remain constant.

These assumptions mean that the variation of yarn strength have the so-called 'steady randomness.'

B) Variation of tension on yarn.

The variation of yarn tension is also very complicated, but may be divided roughly into the following two categories:

(a) Variations of yarn tension owing to the rotation of travellers, vibrations of yarn itself, etc.

(b) Variation of yarn tension owing to the building motion of rings.

Usually, variation (b) is much slower than that of yarn strength and moreover it is a 'scheduled' variation, but variation (a) is very quick and, moreover, random.

First, let us consider the variation of tension (variation (a)). This may be also represented by a curve similar to the one in Fig. 1. In just the same way as in chapter 2, paragraph A, we replace this curve by step-function curve in Fig. 3 in which the symbols have the same meanings as in Fig. 2.

Next, let us consider the variation (b). This variation means the variation of \( \frac{1}{2}(mT1 + mT2), \) which is usually very slow. So, it is easy to take the effect of this variation into calculation.

3. Probability of Yarn Breakage

As already stated, variation of yarn tension is usually much quicker than the variation of yarn strength; so if we put \( 2r = Z / Zr \), where \( Zr \) and \( Zs \) are the period of variation of tension and strength, respectively, \( r > 1 \). And a number \( n = Cr \) of maximums of tension corresponds to every maximum or minimum of strength (see Fig. 4, where the relation between variations of strength and tension is shown) where C is a correction factor.

First, we calculate the probability \( p \) that a yarn breakage occurs during the first \( Zs \) of time, which is the sum of the probabilities \( P1 \) of a yarn breakage during the first \( Zs/Zr \) and the second \( Zs/Zr \) respectively. In order to get the probabilities \( P1, P2 \), we must compare the yarn strength with the largest value of the randomly chosen \( n \) maximum values of tension.

Generally speaking, even if the population of the maximum values have a normal distribution, the largest values of the samples of \( n \) from it does not have any normal distribution. The probability density function of the largest value of a sample of \( n \) is shown in Fig. 5. But, for the sake of simplicity, we assume that the largest value of the sample has a normal distribution:

\[
T_m \cdot \frac{1}{\sigma_{TIV} \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{T_{IV} - m_{TIV}}{\sigma_{TIV}} \right)^2} \ldots \ldots (1)
\]

where \( m_{TIV} \) and \( \sigma_{TIV} \) are the mean and the standard deviation, respectively.

On the other hand, maximums and mini-
mums of strength are assumed to have also normal distributions, as already stated:

\[ S_1: \frac{1}{\sigma_{S_1}\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{S_1 - m_{S_1}}{\sigma_{S_1}} \right)^2} \]  

\[ S_2: \frac{1}{\sigma_{S_2}\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{S_2 - m_{S_2}}{\sigma_{S_2}} \right)^2} \]

where \( m \) and \( \sigma \) are the mean and the standard deviation, respectively.

Then we can easily calculate \( p_1 \) and \( p_2 \), which are the probabilities that \( S_1 - T_{IV} < 0 \) and \( S_2 - T_{IV} < 0 \), respectively. Since the strength \( S_1 \), \( S_2 \) and tension \( T_{IV} \) have normal distributions in this case, owing to a well-known theorem in the statistics, both \( S_1 - T_{IV} \) and \( S_2 - T_{IV} \) have also normal distributions:

\[ x_1 = S_1 - T_{IV} : \frac{1}{\sigma_{S_1}\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x_1 - m_{S_1}}{\sigma_{S_1}} \right)^2} \]  

\[ x_2 = S_2 - T_{IV} : \frac{1}{\sigma_{S_2}\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x_2 - m_{S_2}}{\sigma_{S_2}} \right)^2} \]

where \( m_1 = m_{S_1} - m_{T_{IV}}, \sigma_1 = \sqrt{\sigma_{S_1}^2 + \sigma_{T_{IV}}^2} \),  

\[ x_2 = S_2 - T_{IV} : \frac{1}{\sigma_{S_2}\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x_2 - m_{S_2}}{\sigma_{S_2}} \right)^2} \]

where \( m_2 = m_{S_2} - m_{T_{IV}}, \sigma_2 = \sqrt{\sigma_{S_2}^2 + \sigma_{T_{IV}}^2} \).

And if probability density function of \( x_1 = S_1 - T_{IV} \) is as shown by Fig. 6, for example, the probability \( p_1 \) that \( S_1 - T_{IV} < 0 \) is given by the area of the shaded part. This area is shown by the curve in Fig. 7, which has been drawn in the light of the table of normal distribution. \( p_1 \) will be found in the same way.

Next, when \( N \) spindles are running and if we tie the yarn \( Z_s \) after each breakage, the number of yarn breakages during \( Z \) will be given by

\[ K = N \left( \frac{p_1 + p_2}{Z_s} \right) Z \]  

under usual condition of running.

4. Some Factors that Influence on the Number of Yarn Breakages

It is easily seen from the previous chapter that the following four factors influence the number of yarn breakage:

a) \( m_1/\sigma_1 \)  
b) \( m_2/\sigma_2 \)  
c) \( Z_s \)  
d) \( Z_T \)

Of these four factors the first two are the most influential. That is to say, if these values increase, the number of yarn breakages decreases very rapidly. This will be explained by some numerical examples in the next chapter. The last two factors are not so influential.

5. Numerical Examples

In order to make clear what has been stated above, let us calculate the number of yarn breakages, assuming some values of strength and tension, etc. By doing this, we can see how the difference of mean values of strength and tension and the variation of standard deviation influence yarn breakages.
Therefore, 10 maximums of tension correspond to each maximum and minimum of strength.

Then, we take a random sample of 10 from the population having a normal distribution with mean \( m_{p1} = 15g \) and standard deviation \( \sigma_{p1} = 3.3g \), and we find that the largest values of the sample have a normal distribution with mean \( m_{p12} = 20g \) and standard deviation \( \sigma_{p12} = 2g \). (see Fig. 5). For comparison with the following examples, we state here the strengths and the rewritten tensions of yarn.

Therefore, according to (6), the number of yarn breakage per 1 hr., 400 spindles is

\[
K = 400 \times \frac{3 \times 10^{-7} + 4 \times 10^{-6}}{0.2} \times 3600 \approx 31.
\]

**c) Example II**

Strength : \( \{m_{s1} = 115g, \sigma_{s1} = 15g\} \)

Tension : \( m_{p12} = 20g, \sigma_{p12} = 2g \)

In order to lessen the number of breakages, this time, the standard deviation of the strength has been reduced by 5 g.

In this case, we get

\[
m_1/\sigma_1 = 6.3, \quad m_2/\sigma_2 = 5.6
\]

and so

\[
P_1 \approx 10^{-10}, \quad P_2 \approx 10^{-8}
\]

Therefore, according to (6), the numbers of yarn breakages per 1 hr., 400 spindle is

\[
K = 400 \times \frac{10^{-10} + 10^{-8}}{0.2} \times 3600 \approx 0.1
\]

It is easily seen from the above examples that the standard deviation of variation of strength has quite big effects on yarn breakages. That is to say, the most important thing for the reduction of the number of breakages is to lessen the variation of strength. Of course, we can reduce the number of breakages by reducing average tension. But 50g reduction of average tension is needed in place of the 5g reduction of the deviation of variation of strength in Example II. This is practically impossible.

### 6. Conclusion

In this article, we have used a simple mathematical model as a basis for calculating the variation of the tension and the strength of the yarn, and calculated the probability of the yarn breakages, with the following results:

1) The variations of yarn strength and tension and the average strength and tension influence the numbers of the yarn breakages.

2) Above all, the variation of yarn strength is most influential on the yarn breakages under usual conditions. Thus the evenness of the yarn is needed much more than the lower tension on spinning frames to reduce yarn breakages.

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