Characteristics of Dice Checked Sateen

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Abstract

Many of the published works on how to interlace dice checked sateen seem incomplete to the present author, who holds that the characteristics of the interlacing of perfect dice checked sateen are obtainable by multiplying matrices.

If a matrix on perfect checked sateen, with the elements of the matrix indicated by the interlacing marks of warp sateen, is multiplied on the right by a transposed matrix of the matrix whose elements are indicated by the interlacing marks of weft sateen, then the elements of the matrix of the product \( A_{ik}(i+k=n+1) \) are all zero, if the marks of weft up are represented by zero. The interlacing marks of dice checked sateen make a symmetric point in relation to the center of one repeat of a design.

1. Introduction

Typical textile fabrics made of checked sateen include damask and rinzu (figured sateen), but this article will deal with dice checked sateen.

The principle of the arrangement of interlacing marks of dice checked sateen is described in many reports. In his work, Kinji Ohta \[1\] defines dice checked sateen as a sateen in which the warp and weft sateens are arranged in a checked pattern. He shows the following conditions for the arrangement of interlacing marks of dice checked sateen:

1. The boundary marks of the warp and weft sateens should be reversed.
2. The going direction of the counter should be reversed.
3. No interlacing point of the weft sateen should be put in a corner of a design.

The author agrees with these three principles, but if we put the interlacing marks of the weft sateen in a corner of a design, we shall get the dice checked sateen shown in Fig. 1, even if condition 3 is not met.

William Watson in his work \[2\] says on the interlacing marks of dice checked sateen: “The marks of the base weave should be arranged in such a manner that the first and last picks are alike, and also the first and last ends, when followed in opposite directions”.

Similarly, Franz Donat says in his work \[3\]: “Man wird deshalb bei Atlassen und versetzten Köpfern nicht mit dem ersten Tupfen wie in Fig. 376 [note: on the Fig. 376 the first interlacing mark is put on the corner of the design] beginnen, sondern

![Fig. 1](image1.png)

**Fig. 1** 5-harness dice checked sateen with an interlacing mark of weft sateen in the corner of a design.

![Fig. 2](image2.png)

**Fig. 2** Imperfect eight-harness dice checked sateen with no interlacing mark of the weft sateen placed in a corner of a design and the direction of counter reversed as shown by arrows.
den Einsatz so richten, dass der letzte Kettenfaden
des Quadrates von oben nach unten genau so bindet
wie der erste von unten nach oben”.

Checked sateen which satisfies the conditions
described by Watson or Donat is perfect dice checked
sateen, but there are many difficulties in getting the
interlacing marks which satisfy these conditions.
Take, for example, the eight-harness sateen shown
in Fig. 3. If we take a point of the 7th row and
and the 1st column as a standard point and take a
counter of three, then the marks on the first and
last ends are certainly alike, as also the first and
last picks, when followed in opposite directions. If
we take a counter of five, then the marks fail to
satisfy Watson’s or Donat’s conditions, as shown in
Fig. 4, and do not make perfect dice checked sateen.

![Fig. 3 Eight-harness weft sateen whose cardinal point is
in the 7th row and the 1st column and the counter
is three. This sateen is the base weave of perfect
dice checked sateen.](image)

![Fig. 4 Eight-harness weft sateen whose cardinal point is
in the 7th row and the 1st column and counter is
five. This weft sateen fails to satisfy Watson’s or
Donat’s conditions and does not make dice checked
sateen.](image)

We may say, then, that Watson and Donat
give one condition for dice checked sateen but
that, depending on the way the first mark and the
counter are taken, it is difficult to obtain perfect
sateen which satisfies their conditions. Moreover,
there is the kind of dice checked sateen shown in
Fig. 5 which does not satisfy their conditions. In
other words, the arrangement of the interlacing
marks of this figure do not follow Watson’s or
Donat’s conditions.

It may be argued that the interlacing marks of
the dice checked sateen arranged as in Fig. 1 or 5
are not put in harmony with each other. What,
then, are the characteristics of dice checked sateen
with a harmonic distribution of the marks? How
can checked sateen which satisfies Watson’s and
Donat’s conditions be obtained?

2. Characteristics of Perfect Dice Checked
Sateen

The preceding chapter has suggested the charac-
teristics of dice checked sateen. Generally, if a
matrix on perfect dice checked sateen, with the
elements of the matrix indicated by the interlacing
marks of warp sateen, is multiplied on the right by
a transposed matrix of the matrix whose elements
are indicated by the interlacing marks of weft
sateen, and the marks of weft up are indicated by
zero, then the elements of the matrix of the product
$A_{ik}(i+k=n+1)$ are all zero. If $a$ on such warp
and weft sateen represents the warp up mark of
warp sateen, $b$ represents the warp up mark of
weft sateen, and weft up mark is indicated by zero,
then the warp sateen shown in Fig. 6 can be
represented by a matrix such as (1).

![Fig. 6 Five-harness warp sateen with counter of
two.](image)

$$
\begin{align*}
\begin{array}{cccc}
   a_{11} & a_{12} & a_{13} & 0 & a_{16} \\
   0 & a_{22} & a_{23} & a_{24} & a_{15} \\
   a_{31} & a_{32} & 0 & a_{34} & a_{35} \\
   a_{41} & a_{42} & a_{43} & a_{44} & 0 \\
   a_{51} & 0 & a_{53} & a_{54} & a_{55}
\end{array}
\end{align*}
\quad \ldots \ldots (1)
$$

Also, the weft sateen as shown in Fig. 7 can be
represented by a matrix such as (2).

![Fig. 7 Five-harness weft sateen with counter of three.](image)
If the matrix (1) is multiplied on the right by a transposed matrix of matrix (2), the product is as follows:  

\[
\begin{pmatrix}
0 & b_{12} & 0 & 0 & 0 \\
0 & 0 & b_{23} & 0 & 0 \\
b_{11} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & b_{41} & 0 \\
0 & 0 & b_{33} & 0 & 0 \\
0 & 0 & 0 & 0 & b_{54} \\
0 & 0 & b_{12} & 0 & 0 \\
0 & 0 & b_{25} & 0 & 0 \\
0 & 0 & b_{25} & 0 & 0 \\
0 & 0 & b_{25} & 0 & 0 \\
0 & 0 & b_{25} & 0 & 0 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 & b_{12} & 0 & 0 & 0 \\
0 & 0 & b_{23} & 0 & 0 \\
b_{11} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & b_{41} & 0 \\
0 & 0 & b_{33} & 0 & 0 \\
0 & 0 & 0 & 0 & b_{54} \\
0 & 0 & b_{12} & 0 & 0 \\
0 & 0 & b_{25} & 0 & 0 \\
0 & 0 & b_{25} & 0 & 0 \\
0 & 0 & b_{25} & 0 & 0 \\
0 & 0 & b_{25} & 0 & 0 \\
\end{pmatrix}^T
\]

(2)

On matrix (3), the elements 0 which exist in the position \(A_{ik}(i + k = n + 1)\) correspond to \(a_{14} b_{54}, a_{23} b_{63}, a_{33} b_{43}, a_{45} b_{35}, a_{52} b_{12}\) from the right top to left down. With these elements denoted by \(a_{ij} b_{kj}\), then sub-indices \(i\) are 1, 2, 3, 4 and 5, \(k\) are 5, 4, 3, 2 and 1, and \(j\) are 4, 1, 3, 5 and 2.

In this case, the number of threads in one repeat being five, the order of 4, 1, 3, 5 and 2 means the order 9-5, 1, 3, 5 and 7-5, respectively. In other words, these orders may be called an arithmetical progression with a common difference 2.

This common difference 2 represents the counter of the direction of the row, i.e., horizontal counter. Hence, the element \(a_{i+1,j}\) in which \(i\) is equal to \(j\) is, in this case, \(a_{55}\). Accordingly, with \(a_{55}\) taken as a center and the elements of \(i \pm 1, j \pm 2\) (e.g. \(a_{33} a_{33}\)) as zero, this matrix represents one part of the required warp sateen.

In this example, the elements \(a_{14}, a_{23}, a_{33}, a_{45}\) and \(a_{52}\), being taken as zero, they are weft up marks of warp sateen, and \(b_{12}, b_{12}, b_{25}, b_{25}\) and \(b_{12}\) are the warp up marks of weft sateen.

\(a_{44}\) and \(b_{44}\) on the seven-harness sateen are cardinal points of warp and weft sateen, respectively, and the former is weft up and the latter is warp up mark.

\(a_{55}\) and \(b_{55}\) on the nine-harness sateen are cardinal points.

Generally, if the threads of one repeat make an odd number, the cardinal points are \(a_{n+1,n+1}\) and \(b_{n+1,n+1}\).

\(A_{ik}(i + k = n + 1)\) on the eight-harness sateen should be zero, as are \(a_{12} b_{65}, a_{23} b_{74}, a_{33} b_{85}, a_{45} b_{35}, a_{52} b_{12}\) and \(a_{52} b_{12}\). In this case, there is no element of \(i = k = j\). Therefore, if we take two elements \(a_{45} b_{35}\) and \(a_{52} b_{12}\) which are near the center, and make them a symmetric point in relation to the center and bring the difference of \(j\) to the common difference, then we get a required dice checked sateen.

For example, if we take a counter of three, then \(a_{44} b_{54}\) and \(a_{53} b_{43}\) are a cardinal point and the points of \(j \pm 3\) are the required marks. That is to say, \(a_{44} b_{54}, a_{33} b_{63}, a_{24} b_{74}, a_{15} b_{85}\) and \(a_{53} b_{43}, a_{68} b_{38}, a_{75} b_{25}, a_{82} b_{12}\) are marks of zero. Fig. 8 is eight-harness dice checked sateen obtained by this method.

\(a_{55}\) and \(b_{55}\) of mark \(\Box\) each in Fig. 8 (1), make a symmetric point in relation to the center+ and represent the weft up marks of warp sateen. \(b_{55}\) and \(b_{85}\) on weft sateen each make a symmetric point in relation to the center+ and they are warp up marks represented with \(\Box\). Similarly, \(a_{31}\) and \(b_{61}\) in Fig. 8 (2) are obtained on warp and weft sateens, respectively, and also \(a_{68}, b_{38}\) are obtained. \(a_{44} b_{54}, a_{34} b_{64}\) in Fig. 8 (3), and \(a_{53}\) and \(a_{68}, b_{12}\) and \(b_{87}\) in Fig. 8 (4) are obtained as the required marks. Fig. 8 (5), thus obtained, is perfect eight-harness dice checked sateen.

3. Deduction

On one of the characteristics of perfect dice checked sateen the author deduces that if a matrix on perfect dice checked sateen with the elements of the matrix indicated by the interlacing marks of warp sateen is multiplied on the right by a transposed matrix of the matrix whose elements are indicated by the interlacing marks of weft sateen, and the marks of weft up are indicated by zero, then the elements of the matrix of the product \(A_{ik}(i = j)\)
(n+1) are all zero.

Since there can be only warp up or weft up in a spot of a woven design, the calculation of a matrix is simplified if 1 represents warp up and 0 weft up. In the case of eight-harness warp sateen, it is represented by matrix (4):

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}
\]

Eight-harness weft sateen is represented as follows:

\[
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

and the transposed matrix of (5) is

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

By multiplying matrix (4) on the right by a transposed matrix (6) of the matrix (5), we get

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}
\]

These calculations are shown in Fig. 9.

In other words, such left side warp sateen is one part of perfect dice checked sateen. If the middle weft sateen is transposed, it is one part of perfect dice checked sateen and the interlacing marks of such dice checked sateen make a symmetric point in relation to the center of one repeat of a design.

**Literature cited**


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*Journal of The Textile Machinery Society of Japan*