Measuring Fiber Fineness by Horizontal Air-Flow
Part 1: Preliminary Experiment and Theory.

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Abstract

The horizontal air-flow method in which fibers fall freely in a horizontal air-flow is studied and their horizontal displacement is used to obtain the fineness distribution. This article describes a preliminary experiment and the theory of the descent of fibers. The preliminary experiment in which fibers were let to fall in still air shows that the higher the fineness, the higher the velocity of falling. The experiment shows also that fibers of the same fineness vary only slightly in the velocity of their falling even if they differ in fiber length.

A theoretical treatment of fibers falling in a horizontal air-flow has shown clearly that the fibers initially fall with a terminal velocity, its horizontal displacement being proportional to \((n-1)/4\) power of fiber fineness, where \(n\) is a constant depending upon the posture of the fiber falling \(n\) is equal to -1.840 if fibers fall with their axes perfectly horizontal; equal to -1.625 in case of a partially horizontal descent. These results show that it is possible to measure the fiber fineness distribution by the horizontal air-flow method.

1. Introduction

The fineness of cotton fibers is defined either as a weight fineness expressed in microgram per inch or as a geometrical fineness determined from the area of the cross section of a fiber. It is common knowledge that fiber fineness is an important factor in evaluating the spinnability of fibers and has a bearing on the quality of yarns and fabrics.[1-3] Much interest has been evinced in the subject and many researches have been made to measure fiber fineness.

The conventional method of measuring fiber fineness are:

1. The gravimetric method[4] which cuts fibers in a prescribed length and weighs them to obtain their weight and number.

2. The optical method[5-11] which uses the microscope or the birefringency of fibers.

3. The air-flow method[12-14] such as the Micronaire or Arealometer method, which measures the resistance of air-flow through an assemblage of fibers.

4. The vibroscopic method[15-19] which directly measures fineness by the transverse vibrations of a fiber.

Methods (2) and (4) take much time. Methods (1) and (3) fail to show the fineness distribution of fibers but give the average fineness.

In our experiment, single fibers were allowed to fall freely in a horizontal air-flow and their fineness distribution was obtained from the various points on which they landed, depending upon the degrees of their fineness.

This article describes a preliminary experiment made to see whether the velocity of the falling of fibers is varied by fiber characters, such as fineness and staple length, and find a theoretical approach to the mechanism of the falling.

2. Preliminary Experiment

1. Method of Experiment

Before the experiment, we looked into certain questions, such as:
(i) Whether the falling velocity varies with differences in fiber fineness.
(ii) Whether the falling velocity is the same for different fiber lengths.
(iii) Whether fiber crimps influence the falling velocity.

To obtain answers to these questions, the following experiments were made: Each fiber was allowed to fall freely from the open top of a glass cylinder, 120 cm high and 13 cm in inside diameter and closed at the bottom. A stopwatch was used to measure the time needed for a fiber to fall 100 cm with its axis kept horizontal.

The specimens used are shown in Table 1. To test fineness correctly, fibers picked from man-made, continuous-filament yarns were initially used. Specimen Nos. 1-4 were varied in length during the experiment to see what bearing fiber length would have on the falling velocity. Specimens Nos. 5-12 were varied in fiber fineness for the same purpose.

To estimate the fineness of cotton fibers, raw cotton stock dyed in the color shown in Table 1 by the differential dyeing method\[20\] was used for maturity test. About 50 measurements each were made for specimens Nos. 5-12 and 15 measurements were picked out per specimen as representing fibers which fell with an almost perfect horizontal posture.

(2) Experimental Results

Experimental results are shown in Figs. 1-3. The falling time (time needed for falling) in Fig. 1 is almost constant, irrespective of the fiber length, if fibers are the same in fineness. This is very useful for measuring fiber fineness by the air-flow method under review in this article.

Fig. 2 shows what bearing fineness has on the falling time and gives results estimated from Higuchi's article\[21\]. The velocity of falling is faster for the

<table>
<thead>
<tr>
<th>Table 1 Fibers used</th>
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<tbody>
<tr>
<td>Specimen No.</td>
</tr>
<tr>
<td>Material</td>
</tr>
<tr>
<td>Density (ρ₀)</td>
</tr>
<tr>
<td>As it is*</td>
</tr>
<tr>
<td>Denier of single fiber</td>
</tr>
<tr>
<td>Sample Size</td>
</tr>
<tr>
<td>Fiber length (in)</td>
</tr>
</tbody>
</table>

* shown by yarn denier/no. of filament.
** From 50 specimens tested, only those fibers were selected which fell with their axes horizontal.
coarser fibers in Fig. 2. The solid line in it shows results relating to fibers falling with their axes completely horizontal. The dotted line refers to average values, including those of fibers falling with their axes nearly horizontal.

Values obtained from Higuchi's are lower than our results, because the test cylinder is larger in diameter (30 or 65 cm) than that used in our experiment and may presumably contain also values of fibers falling not vertically but obliquely. The cylinder used in our experiment being small in diameter (13 cm), fibers falling even slightly obliquely touched the wall of the cylinder soon, and they were excluded from our data.

During sorting by blown air, which will be described later, some fibers take differing postures while falling. Therefore, the dotted rather the solid line, had better be used. The solid line is considered an ideal case.

Empirical equation for the two curves in Fig 2 are:

\[ T = 32.1D^{-0.74} \text{ (for solid line)} \]  
\[ T = 23.3D^{0.65} \text{ (for dotted line)} \]

where \( T \) = time (sec) needed for fibers to fall 1m, and \( D \) = fineness.

Higuchi[21] reports that the crimps of fibers have little to do with the falling time. We do not believe crimps, unless so large that the boundary layers of air flow on the fiber surface interfere with each other, have no bearing on air resistance. Thus, we shall assume that fiber crimps have no bearing on the falling time.

3. Theory

The preliminary experiment just described shows that differences in falling time depending on fiber fineness can be used to measure fineness. Now to consider theoretically the phenomenon of fibers falling in a horizontal air-flow.

Generally, when a relatively small substance, such as a single fiber, falls freely in still air, the velocity of its falling is comparatively small and reaches the terminal velocity after a short time, because its weight and air drag are balanced. Various attempts have, therefore, been made to separate materials by blowing them in a horizontal air-flow and using the differences in the velocity of their falling. Higuchi[22] applied this method to a picking-blending machine. Tanazawa[23] applied it to the measurement of the distribution of spray sizes. We study this subject under two headings:

(1) Terminal velocity

When a single fiber falls freely in still air with the fiber axis kept horizontal, the equation of motion is given as follows, assuming that the buoyancy of the fiber may be ignored; and that only gravity and force proportional to the square of the velocity of velocity act on the fiber:

\[ m\ddot{y} = mg - c_1 \frac{1}{2} \rho d l y^2 \]  
\[ y = \frac{\sqrt{2m g/c_1 d l}}{\rho} \]  

where

- \( m \) : mass of fiber
- \( \dot{y} \) : falling velocity
- \( g \) : acceleration of gravity
- \( \rho \) : air density
- \( l \) : fiber length
- \( d \) : fiber diameter
- \( c_1 \) : air-drag coefficient of fiber

All the forces acting on the fiber are in equilibrium at terminal velocity \( \ddot{y}_m \). Hence:

\[ mg = c_1 \frac{1}{2} \rho d l y_m^2 \]

or

\[ \ddot{y}_m = \frac{\sqrt{2m g/c_1 d l}}{\rho} \]

Substituting eq. 4 for eq. 3 and using the notation \( y/\ddot{y}_m = u \) give us:

\[ \frac{1}{2} \log \frac{1+u}{1-u} = -\frac{g}{\ddot{y}_m} t + c_1 \]  

Fig. 3 shows the frequency polygon of the falling time of cotton fibers. Mature fibers having thick walls (dyed red by the differential dyeing method) fall faster than immature fibers with thin walls (dyed green). This result agrees with that obtained in the above-mentioned experiment. Since individual fibers cannot be accurately distinguished by color, the cotton fibers used for this experiment were pulled out one by one from red- and green-dyed fiber stocks. Therefore, it was difficult to tell whether each fiber was mature or whether its fineness was high. Furthermore, the fibers took various postures of falling, thus making a wide distribution of falling time. These facts presumably explain why the frequency curves overlap at the center of Fig. 3.

Assuming that the initial condition is \( y = y_0 \), i.e. \( u = u_0 \), then \( c_1 \) is determinable. Therefore:

\[
\log \left( \frac{1 + u}{1 - u_0} \right) = 2g t \quad \cdots (5)
\]

Assume, for the sake of simplicity, \( 1 - u_0 = a, 1 + u_0 = b \), \( e^{zt} y_0 = z \). Then we have:

\[
y = y_0 + \frac{y_0^2}{2g} \log \left( \frac{(a + bz)^2}{z} \right) + c_1 \quad \cdots (6)
\]

Assuming the initial condition to be \( y = 0 \) at \( t = 0 \), then \( c_1 \) is determinable, and

\[
y = y_0 + \frac{y_0^2}{2g} \log \left( \frac{(a + bz)^2}{4z} \right) \quad \cdots (7)
\]

If a fiber is dropped at zero speed from zero height in still air, then:

\[
a = b = 1 \
\therefore y = y_0 + \frac{y_0^2}{2g} \log \left( \frac{1 + z}{4} \right) \quad \cdots (8)
\]

It is shown in Table 2 from which we conclude that 1 for values of \( z \) 0.05 second after the beginning of a falling in a normal falling test may be ignored.

\[
y = y_0 + \frac{y_0^2}{2g} \log \left( \frac{1 + z}{4} \right) \quad \cdots (8)
\]

Values calculated by eq. 8 are shown in Table 2, from which we can conclude that for values of \( z \) 0.05 second after the beginning of a falling, in a normal falling test may be ignored.

\[
y = y_0 + \frac{y_0^2}{2g} \log \left( \frac{1 + z}{4} \right) \quad \cdots (8)
\]

Assuming \( \gamma_0 = 24.5 \) cm/sec and height \( y \) of falling to be 100 cm, then:

\[
\frac{1.39 \gamma_0^2}{2gy} = \frac{1.39 \times 24.5^2}{2g \times 2980} = 4.26 \times 10^{-3}
\]

Therefore, the second term in the bracket of eq. 9, compared with 1, becomes small enough to be ignored. In other words, if a falling is considerably high even if the initial speed is zero, we have:

\[
y = \frac{y_0}{t} \quad \cdots (10)
\]

This means that the terminal velocity can be calculated correctly as (height of falling)/(falling time), namely, that a fiber falls at the terminal speed from the beginning of its falling.

(2) Horizontal displacement

Now to consider horizontal distance \( L \) travelled by a fiber when it falls in horizontal air-flow with constant velocity \( V \).

The time \( t \), required for a fiber to fall vertical distance \( H \) with falling velocity \( \gamma_0 \) is

\[
t = H/\gamma_0 = H/\sqrt{c_\rho d/2mg}
\]

\[
L = VL = VH/\sqrt{c_\rho d/2mg}
\]

Assuming that the fiber has a round cross section, then fiber fineness \( F \) is

\[
F = \frac{m}{l} = \frac{\pi d^2 l \rho_s}{4l}
\]

where \( \rho_s \) is apparent linear density of fiber,
\( d \) = fiber diameter,
\( l \) = fiber length,

then

\[
L = VH \sqrt{\frac{\rho}{\pi \gamma}} \sqrt{\frac{c_\rho}{\pi \rho_s F}}
\]

Here, as eq. 12 includes \( \rho_s \), which varies with the kinds of fibers, \( c_\rho \), should be calculated individually for each kind of fiber. However, with the relation of eq. 13, \( c_\rho \) is obtainable as follows, irrespective of the kind of fiber.

\[
\frac{c_\rho^3}{\rho_s} = k \gamma^2 F^2
\]

\[
L = kV \sqrt{\frac{\rho}{\pi \gamma}} \sqrt{\frac{c_\rho^2}{\pi \rho_s F}} \gamma^2
\]

In other words, horizontal distance \( L \) is obtained as proportional to \((n-1)/4\) power of a given fiber fineness \( F \).

4. Relation between \( C_y \), \( \rho_s \), and \( F \)

To see whether eq. 12 is established or not, \( c_\rho \) is obtained from the falling test above mentioned in which the fiber is fallen through 1 m height. The result is shown in Table 3. The fineness of cotton fibers was calculated from eq. 2 and their falling time, because the fiber denier was not accurately known.

### Table 3: Relation between Times Required to Fall through 1 m and \( c_\rho \)

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Falling time (sec)</td>
<td>11.3</td>
<td>9.7</td>
<td>10.2</td>
<td>4.0</td>
</tr>
<tr>
<td>( c_\rho )</td>
<td>38.3</td>
<td>31.5</td>
<td>31.1</td>
<td>9.7</td>
</tr>
<tr>
<td>( c_\rho^2/\rho_s )</td>
<td>1111</td>
<td>752</td>
<td>848</td>
<td>82</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Falling time (sec)</td>
<td>18.4</td>
<td>12.7</td>
<td>6.2</td>
<td>4.6</td>
</tr>
<tr>
<td>( c_\rho )</td>
<td>80.8</td>
<td>46.6</td>
<td>19.6</td>
<td>13.2</td>
</tr>
<tr>
<td>( c_\rho^2/\rho_s )</td>
<td>5727</td>
<td>1905</td>
<td>337</td>
<td>153</td>
</tr>
<tr>
<td>( c_\rho^2/\rho_s )</td>
<td>3434</td>
<td>1472</td>
<td>692</td>
<td>156</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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<th>9</th>
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<th>12</th>
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<tbody>
<tr>
<td>Falling time (sec)</td>
<td>15.2</td>
<td>11.2</td>
<td>8.0</td>
<td>4.7</td>
</tr>
<tr>
<td>( c_\rho )</td>
<td>55.9</td>
<td>36.6</td>
<td>25.1</td>
<td>11.9</td>
</tr>
<tr>
<td>( c_\rho^2/\rho_s )</td>
<td>3434</td>
<td>1472</td>
<td>692</td>
<td>156</td>
</tr>
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</table>

<table>
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<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Falling time (sec)</td>
<td>17.6</td>
<td>23.6</td>
<td>15.4</td>
<td>21.9</td>
</tr>
<tr>
<td>( c_\rho )</td>
<td>(70.9)</td>
<td>(102)</td>
<td>(60)</td>
<td>(91.1)</td>
</tr>
<tr>
<td>( c_\rho^2/\rho_s )</td>
<td>(3351)</td>
<td>(6936)</td>
<td>(2400)</td>
<td>(5533)</td>
</tr>
</tbody>
</table>
Eq. 15 is valid for the solid line in Fig. 4 (based on Table 3) denoting fibers falling with their axes completely horizontal. Eq. 16 is valid for the dotted line denoting other fibers, including those falling slightly obliquely.

\[
\frac{c_y^2}{\rho_s} = 157200 \quad F^{-1.510} \quad \ldots \ldots (15)
\]

\[
\frac{c_y^2}{\rho_s} = 36050 \quad F^{-1.425} \quad \ldots \ldots (16)
\]

Eq. 13 is, therefore, believed to hold good for fibers used in our experiment. Since the dotted line is more reasonable for the actual falling of fibers, eq. 16 is adopted and substituted for eq. 14, assuming \( g = 980 \text{ cm/sec}^2 \) and \( \rho = 1.225 \times 10^3 \text{ g/cm}^3 \). The result:

\[
L = 0.463 \quad \text{VHF}^{-0.656} \quad \ldots \ldots (17)
\]

where \( F = \) fiber fineness in \( \mu \text{g/in} \) or

\[
L = 0.235 \quad \text{VHD}^{-0.656} \quad \ldots \ldots (18)
\]

where \( D = \) fineness in denier.

The higher the fineness \( F \) of cotton fibers, the thicker their wall and the smaller their lumen, resulting in larger \( \rho_s \). The more immature a fiber, the larger its lumen and the smaller \( \rho_s \).

Therefore, the variation of the right term of eq. 12 is larger for cotton fibers than for blown-off man-made fibers which have the same values of \( \rho_s \) but differ in fineness. Accordingly, cotton fibers can be separated more easily from the distance blown off.

The foregoing equations assume that each fiber falls with its axis kept horizontal. In practice, however, fibers may fall with their axes kept vertical, producing a higher falling velocity. Still our preliminary experiments proved that almost all fibers fall with their axes kept horizontal; and that those falling with vertical axes take a horizontal posture in time, because of their natural crimps.

The assumption of round cross section should be modified to flat cross sections for cotton fibers, while the air drag coefficient and the surface area vertical to the air flow can differ somewhat from those mentioned above.

However, a cotton fiber has a natural convolution, and each local section shows different posture to the air flow during a fall. Eq. 3, therefore, is generally applicable if the average values of these factors are used.

5. Summary

(1) In an experiment preliminary to building an apparatus to measuring fiber fineness, we separated fibers into singles and let them fall freely.

The results on man-made fibers are:

(i) The higher the fineness, the faster the velocity.

(ii) If fineness is the same, the falling velocity varies slightly despite differences in fiber length.

(iii) Empirical equations between falling time \( T \) (sec) and fineness \( D \) (den) are obtained thus:

\[
T = 32.1 \quad D^{-0.714} \quad \text{for fibers falling with their axes kept almost horizontal)}
\]

\[
T = 23.3 \quad D^{-0.651} \quad \text{for other fibers, including those falling with their axes kept nearly horizontal)}
\]

Similar results were obtained for cotton fibers, too, and the classifying fibers by their fineness is possible.

(2) The falling of a fiber has been studied theoretically on the assumption that the air-drag is proportional to the square of the falling velocity when the fiber falls freely with its axis kept horizontal. The study has shown that:

(i) It is all right to think that the fiber falls with the terminal velocity from the beginning.

(ii) When a fiber falls freely from height \( H \) into a horizontal air-flow with velocity \( V \), the horizontal distance \( L \) reached by the fiber is:

\[
L = VH \sqrt{\frac{\rho}{g}} \sqrt{\frac{c_y^2}{\pi \rho_s F}}
\]

where

- \( \rho = \) density of air,
- \( \rho_s = \) apparent fiber density,
- \( g = \) acceleration of gravity,
- \( c_y = \) air-drag coefficient of fiber,
- \( F = \) fiber fineness.

(iii) The following relation has been generally established for fibers tested in this experiment.

\[
\frac{c_y^2}{\rho_s} = k F^x
\]
Therefore, \( L \) can be expressed as:

\[
L = kVH \sqrt{\frac{\nu}{\pi g}} F^{\frac{1}{2}}
\]

Accordingly, horizontal distance \( L \) can be determined unitarily from fineness, irrespective of the kind of fiber.

(iv) It is, therefore, possible to measure the distribution of fiber fineness by the horizontal air-flow method.

The authors thank Mr. T. Okita and S. Fukuda for their help in the laboratory work.

References:

[18] K. Wakayama, E. Jinen; ibid., 17, 126 (1964)
[19] K. Wakayama, E. Jinen; ibid., 17, 284 (1964)
[22] K. Higuchi; ibid., 11, 452 and 533 (1958)

Errata

Stability of Fluctuations in Alkali-Cellulose Concentration in Continuous Steeping, Pressing and Disintegration Process

<table>
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<td>(\cdots g_1(s)) to the left of...</td>
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