Yarn Tension in Winding Mechanism.

Part 1: Behavior of Tension in a Running Yarn.

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Abstract

The responses of tension in a running yarn given stepwise friction at two points and drafted stepwise at two other points in the winding system have been studied with the following results:

Assuming the yarn to be perfectly elastic, the tension is readily calculable and agrees well with experimental results on nylon and cotton yarn. To make a more exact estimate, the effects of tension "Creep" and creep of the yarn must be taken into consideration. "Creep" means the amount of tension a sample received being used in our experiments.

1. Introduction

Control of the tension of a running yarn in the winding, warping, sizing, weaving and other process is highly important. It is a difficult task, however, because a yarn, a complex fiber assembly, is neither perfectly elastic nor simply visco-elastic. With friction, draft and other factors acting on a running yarn tension shows a complex behavior and cannot be kept constant. Accordingly, any research on tension control must begin with an investigation of tension changes when draft or friction acts on a yarn.

There are a number of published reports in Japan on tension control. They include Maeda's theory [1, 2, 3, 4] which treats rheologically of the tension of viscose and nylon yarns in running state; and Sakai's [5, 6, 7] analysis which assumes a yarn to be elastic. Both studies dealt with the tension of a yarn running between two cylinders.

The present article is based on experiments on a winding system described in what follows. It gives friction to a yarn at two points and drafts it in two zones between the rollers. The article looks into the setp response and the steady-state response of tension in a running yarn.

2. Method of Experiment

Fig. 1 experimental apparatus

One thread delivered from a yarn package (e.g., a bobbin or cone) was wound on bobbin R_d through the winding system, as shown in Fig.1. The details follows:

2-1 Yarn

Three kinds of yarns, described by the following symbols, were used in our experiments:

N_1 : Nylon multifilament yarn (100 denier-24-20) delivered from a bobbin on which it had been wound with a constant winding tension.

N_2 : The same nylon as N_1 but used after being left in tensionless state for 72 hours.

C : Cotton combed yarn ($\frac{1}{16}$ S) delivered from a cone on which it had been wound with a constant winding tension.

2-2. Experimental Apparatus

The winding system used for this experiments is illustrated in Fig.1.

R_s in Fig.1 is the area from which the yarn was delivered and where a yarn package was set up. P_0 is a tension device of disc type. P_1 , P_2 and P_4 are rollers driven positively by a three-phase induction motor through the medium of a variable-speed gear of PIV type. P_3 is a guide roller, driven passively by frictional force between the running yarn and the guide roller.

R_d is the winding part where a running yarn was wound passively with the surface velocity of roller P_4 .

T_1 , T_2 , T_3 and T_4 express the degrees of yarn tension in the zones between, respectively, P_0 and P_1 , P_1 and P_2 , P_2 and P_3 and P_3 and P_4 . These tension degrees were measured with tensionmeters with strain gages attached. The tensionmeters are represented by G_1 , G_2 , G_3 and G_4 .

We investigated experimentally the effects of the following five factors on tension in our system:

f_0 : Frictional force acting on a running yarn when a weight was put on P_0 .

$\delta_{1.2}$: Draft estimated from the difference in surface velocity between P_1 and P_2 . The draft can be varied by the variable speed gear of PIV type placed between P_1 and P_2 .

$\delta_{2.4}$: Draft estimated from the difference in surface velocity between roller P_2 and P_4 .

f_3 : Frictional force acting on a running yarn when a brake was applied to guide roller P_3 .

V : Yarn velocity defined by the surface velocity of roller P_4 .

A yarn was delivered from R_s , ran through P_0 , P_1 , P_2 , P_3 and P_4 in the winding system, and was wound in winding part R_d . Yarn tension between R_s and P_0 was kept tensionless (nil). Then the yarn was

kept in standard state. When the value of only one of these factors was varied stepwise, the responses of the various degrees of tension were measured.

The four other factors were kept in standard state, a state in which the values of factors f_0 , $\delta_{1.2}$, $\delta_{2.4}$, f_3 and V were, respectively, 1, 0%, 1%, 0 (zero) and 20 centimeters per second. $f_0=1$ means a unit of weight added to tension device P_0 ; $f=2$ means two units of weight added. Draft $\delta_{1.2}$ and draft $\delta_{2.4}$ are expressed as values $(v_2-v_1)/v_1$ and $(v_4-v_3)/v_2$, respectively, where v_1 = surface velocity of P_1 , v_2 = surface velocity of P_2 , and v_4 (= V) = surface velocity of P_4 . The value of f_3 shows the number of the guide rollers to which a brake was applied. Yarn velocity V in standard state was 20 centimeters per second.

The transient response of tension was measured at intervals of three minutes when yarn velocity was $V=20$ cm/sec. The reason is that it took yarn tension three minutes to enter steady state after the variation of the value of the various factors stepwise for $V=20$ cm/sec. Therefore, the value after three minutes for $V=20$ cm/sec was used as the steady-state value of yarn tension.

3. Theoretical Analysis

3-1. Transfer Function

Assuming a yarn to be perfectly elastic, the tension of a running yarn is calculable theoretically thus, the following symbols being used:

l_i : Distance between P_{i-2} and P_i where $i=1, 2, 3, 4$.

ρ_i : Linear density of yarn between P_{i-1} and P_i .

τ : Elastic modulus of yarn

v_1, v_2, v_4 : Surface velocities of rollers P_1 , P_2 and P_4 , respectively.

[T_1] Assuming that the friction and the inertia in delivery part R_s is negligible, we obtain the following approximate equation:

$$T_1 = f_0 \quad \dots\dots\dots(1)$$

where n_j : frictional force given to the yarn at n_j

[T_2] Equation of continuity between P_1 and P_2 is

$$l_2 \frac{d\rho_2}{dt} = v_1\rho_1 - v_2\rho_2 \quad \dots\dots\dots(2)$$

Applying Hook's Law, we obtain:

$$\rho_0/\rho_i = 1 + T_i\gamma \quad (i=1, 2, 3, 4) \quad \dots\dots\dots(3)$$

where ρ_0 : value of the linear density of yarn between R_s and P_0 .

when $T_i \rightarrow T_i + \Delta T_i$ and $\rho_i \rightarrow \rho_i + \Delta \rho_i$, from eq.(3) emerges eq.(4):

$$\frac{\rho_i}{\rho_i + \Delta \rho_i} = \frac{1 + T_i/\gamma + \Delta T_i/\gamma}{1 + T_i/\gamma} \quad \dots\dots\dots(4)$$

Here, ΔT_i and $\Delta \rho_i$ are much smaller than T_i and ρ_i . Assuming $T_i \ll \tau$, the following relation is deducible from eq.(4):

$$\Delta T_i \doteq -\gamma \frac{\Delta \rho_i}{\rho_i} \quad \dots\dots\dots(5)$$

Assuming $\bar{v}_1, \bar{v}_2, \bar{\rho}_1$ and assuming $\bar{\rho}_2$ to be constant and $\Delta v_1, \Delta v_2, \Delta \rho_1$ and $\Delta \rho_2$ variable, we obtain $v_1 = \bar{v}_1 + \Delta v_1, v_2 = \bar{v}_2 + \Delta v_2, \rho_1 = \bar{\rho}_1 + \Delta \rho_1, \rho_2 = \bar{\rho}_2 + \Delta \rho_2$.

Therefore:

$$\frac{d\bar{\rho}_2}{dt} = \frac{d(\rho_2 + \Delta \rho_2)}{dt} = \frac{d\Delta \rho_2}{dt}$$

$$\text{and } \bar{v}_1 \bar{\rho}_1 - \bar{v}_2 \bar{\rho}_2 = 0.$$

If we ignore the second order infinitesimal, then:

$$\begin{aligned} v_1 \rho_1 - v_2 \rho_2 &= (\bar{v}_1 + \Delta v_1)(\bar{\rho}_1 + \Delta \rho_1) \\ &\quad - (\bar{v}_2 + \Delta v_2)(\bar{\rho}_2 + \Delta \rho_2) \\ &= \bar{v}_1 \Delta \rho_1 + \bar{\rho}_1 \Delta v_1 - \bar{v}_2 \Delta \rho_2 - \bar{\rho}_2 \Delta v_2 \end{aligned}$$

Accordingly, eq.(2) transforms as follows:

$$l_2 \frac{d\Delta \rho_2}{dt} = \bar{v}_1 \Delta \rho_1 + \bar{\rho}_1 \Delta v_1 - \bar{v}_2 \Delta \rho_2 - \bar{\rho}_2 \Delta v_2 \quad \dots\dots\dots(6)$$

Rewrite eq.(5) thus: $\rho_i = \bar{\rho}_i$. Then from eqs.(5) and (6) emerges this other equation:

$$\frac{l_2}{v_2} \cdot \frac{d\Delta T_2}{dt} + \Delta T_2 = \Delta T_1 + \gamma \left(\frac{\Delta v_2}{v_2} - \frac{\Delta v_1}{v_1} \right)$$

which, by Laplace Transform, is expressible as follows:

$$L[\Delta T_2] = \frac{1}{1 + sl_2/v_2} \left\{ L[\Delta T_1] + \gamma L \left[\frac{\Delta v_2}{v_2} - \frac{\Delta v_1}{v_1} \right] \right\} \quad \dots\dots\dots(7)$$

$$\text{Since } \Delta \delta_{1,2} = \Delta \left(\frac{v_2 - v_1}{v_1} \right),$$

$$\begin{aligned} \Delta \delta_{1,2} &= \frac{\Delta v_2 - \Delta v_1}{v_1} - \frac{\bar{v}_2 - \bar{v}_1}{\bar{v}_1^2} \cdot \Delta v_1 \\ &= \frac{\Delta v_2 - \Delta v_1}{v_1} - \delta_{1,2} \frac{\Delta v_1}{\bar{v}_1} \end{aligned}$$

v_1 being approximately equal in value to $\bar{v}_2$ and $\delta_{1,2}$ being small in value, we obtain:

$$\Delta \delta_{1,2} \doteq \frac{\Delta v_2 - \Delta v_1}{v_1} \doteq \frac{\Delta v_2}{v_2} - \frac{\Delta v_1}{v_1}$$

Substituting this equation into eq.(7) gives us:

$$L[\Delta T_2] = \frac{1}{1 + sl_2} \left\{ L[\Delta T_1] + \gamma L[\Delta \delta_{1,2}] \right\} \quad \dots\dots\dots(8)$$

$$\text{where } l_2/v_2 = t_2 (\doteq l_2/V)$$

$[T_3, T_4]$ Equation of continuity between P_2 and P_4 is given by this equation:

$$l_3 \frac{d\rho_3}{dt} + l_4 \frac{d\rho_4}{dt} = v_2 \rho_2 - v_4 \rho_4$$

The relation between frictional force f_3 generated when a running yarn drives the guide roller and tension degrees T_3 and T_4 is expressible approximately as follows:

$$T_4 - f_3 = T_3$$

Therefore, in the same way in which eq.(8) is obtained, these two equations are obtainable from the foregoing two:

$$L(\Delta T_3) = \frac{1}{1+s(t_3+t_4)} \left\{ L(\Delta T_2) - (1+st_4)L(\Delta f_3) + \gamma L(\Delta \delta_{2,4}) \right\} \dots\dots\dots(9)$$

$$L(\Delta T_4) = \frac{1}{1+s(t_3+t_4)} \left\{ L(\Delta T_2) + st_3 L(\Delta f_3) + \gamma L(\Delta \delta_{2,4}) \right\} \dots\dots\dots(10)$$

$$\text{where } t_3 = l_3/v_3 (=l_3/V), t_4 = l_4/v_4 (=l_4/V)$$

from which we deduce the transfer function shown in Table 1.

Table 1 Theoretical Responses of Tension

out put in put		ΔT_1	ΔT_2	ΔT_3	$T\Delta_4$
Δf_0	Transfer function	1	$\frac{1}{1+st_2}$	$\frac{1}{(1+st_2)(1+s(t_3+t_4))}$	Same as ΔT_3
	Step response	1	$1-e^{-t/t_2}$	$1 - \frac{t_3+t_4}{t_3+t_4-t_2} e^{-\frac{t}{t_3+t_4}} + \frac{t_2}{t_3+t_4-t_2} e^{-t/t_2}$	Same as ΔT_3
	Steady-state response	1	1	1	1
$\Delta \delta_{1,2}$	Transfer function	—	$\frac{\gamma}{1+st_2}$	$\frac{\gamma}{(1+st_2)(1+s(t_3+t_4))}$	Same as ΔT_3
	Step response	—	$\gamma(1-e^{-t/t_2})$	$\gamma \left(1 - \frac{t_3+t_4}{t_3+t_4-t_2} e^{-\frac{t}{t_3+t_4}} + \frac{t_2}{t_3+t_4-t_2} e^{-t/t_2} \right)$	Same as ΔT_3
	Steady-state response	0	γ	γ	γ
$\Delta \delta_{2,4}$	Transfer function	—	—	$\frac{\gamma}{1+s(t_3+t_4)}$	Same as ΔT_3
	Step response	—	—	$\gamma(1-e^{-\frac{t}{t_3+t_4}})$	Same as ΔT_3
	Steady-state response	0	0	γ	γ
Δf_3	Transfer function	—	—	$-\frac{1+st_4}{1+s(t_3+t_4)}$	$\frac{st_3}{1+s(t_3+t_4)}$
	Step response	—	—	$-\frac{t_3}{t_3+t_4} e^{-\frac{t}{t_3+t_4}} - 1$	$\frac{t_3}{t_3+t_4} e^{-\frac{t}{t_3+t_4}}$
	Steady-state response	0	0	-1	0

Fig. 2 Theoretical step response

3-2. Transient Performance (step response)

The step responses of tension in a running yarn given friction or drafted stepwise are calculable by the above theory with the results shown in Fig.2 and Table 1. In other words, tension T_1 , when given frictional force Δf_0 stepwise, increases approximately stepwise by ΔT_1 , the step response of T_2 being the first order lag and the step responses of T_3 and T_4 the second order lag.

When a running yarn is drafted stepwise by $\Delta \delta_{1.2}$, the step response of T_2 is the first order lag; those of T_3 and T_4 , the second order lag. However, tension T_1 is not affected by draft. When a running yarn is drafted stepwise by $\Delta \delta_{2.4}$, the step responses of T_3 and T_4 are the first order lag. Neither T_1 nor T_2 is affected by draft $\delta_{2.4}$.

When a running yarn is given frictional force Δf_3 stepwise, tension T_4 increases by $(t_3/t_3+t_4)\Delta f_3$ instantaneously, and thereafter, decreases exponentially with time as shown in Fig.2. Tension T_3 decreases by $(t_4/t_3+t_4)\Delta f_3$ instantaneously and thereafter decreases exponentially with time.

3-3. Steady-state Performance (Steady-state response)

Tensions in steady state are expressible in value as follows:

$$[T_1] \quad T_1 = f_0 \quad \dots\dots\dots(11)$$

$[T_2]$ From eq.(3) emerges this equation:

$$\rho_1/\rho_2 = (\gamma + T_2)/(\gamma + T_1),$$

and as $\rho_1/\rho_2 = v_2/v_1 = 1 + \delta_{1.2}$, we obtain:

$$T_2 = T_1(1 + \delta_{1.2}) + \gamma\delta_{1.2} \quad \delta_{1.2} \ll 1 \\ = T_1 + \gamma\delta_{1.2}$$

Substituting eq.(11) into the preceding equation leads to:

$$T_2 = f_0 + \gamma\delta_{1.2} \quad \dots\dots\dots(12)$$

$[T_3, T_4]$ Similarly, we obtain:

$$T_4 = T_2(1 + \delta_{2.4}) + \gamma\delta_{2.4} \quad \gamma_{2.4} \ll 1 \\ \doteq T_2 + \gamma\delta_{2.4} \\ = f_0 + \gamma(\delta_{1.2} + \delta_{2.4}) \\ = f_0 + \gamma\delta_{1.4} \quad \dots\dots\dots(13)$$

$$\text{where} \quad \delta_{1.4} = \frac{v_4 - v_1}{v_1} = \frac{v_2 - v_1}{v_1} + \frac{v_4 - v_2}{v_2} = \delta_{1.2} + \delta_{2.4}$$

Substituting equation, $T_3 = T_4 - f_3$, into eq.(13) gives us:

$$T_3 = f_0 + \gamma\delta_{1.4} - f_3 \quad \dots\dots\dots(14)$$

4-1. Transient Response

Five factors (i.e., f_0 , $\delta_{1.2}$, $\delta_{2.4}$, f_3 and V) were varied stepwise one by one and their effects on tension in each zone were examined as shown in Fig.3.

Except one factor changed stepwise, the four other factors were kept in standard state as mentioned above, i.e., $f_0=1$, $\delta_{1.2}=0\%$, $\delta_{2.4}=1\%$, $f_3=0$ and $V=20$ cm/sec Fig.3a shows the step responses of various degrees of tension measured when the value of f_0 was varied stepwise from 0 to 2. Similarly, Figs.3b, 3c and 3d show the step responses of various degrees of tension measured when $\delta_{1.2}$, $\delta_{2.4}$ and f_3 were varied in value from 0.5% to 0.5%, from 0.5% to 1.5% and from zero to one

Fig. 3d Step response ($f_3=0 \rightarrow 1$)

Fig. 3e Behavior of tension ($V=10 \text{ cm/sec} \rightarrow 20 \text{ cm/sec}$)

Fig. 3 Experimental step response

Table 2 Time Constant

Factors	Experimental results												Theoretical values			
	N_1				N_2				C							
	T_1	T_2	T_3	T_4	T_1	T_2	T_3	T_4	T_1	T_2	T_3	T_4	T_1	T_2	T_3	T_4
f_0 $0 \rightarrow 2$	0	7	61	61	0	4	68	55	0	6	42	42	0	5.6	51.2	51.2
$\delta_{1.2}$ $-0.5 \rightarrow 0.5\%$		8	54	52		6	50	40		7	46	39		5.6	51.2	51.2
$\delta_{2.4}$ $0.5 \rightarrow 1.5\%$			51	49			28	25			43	42			45.2	45.2
f_3 $0 \rightarrow 1$			40	35			12	25			40	38			45.2	45.2

perimental results, especially for sample N_2 , as shown in Table 2.

In steady state, $T_2 - T_1 = \tau \delta_{1.2}$ and $T_4 - T_2 = \tau \delta_{2.4}$. Therefore, the relation between $T_2 - T_1$ and $\delta_{1.2}$ was linear as a result of the experiment like the relation between $T_4 - T_2$ and $\tau \delta_{2.4}$ as shown in Figs. 4b and 4c.

For these relations to accord more exactly to the theory, the slope in Fig. 4b must be the same as the slope in Fig. 4c. However, the elastic modulus estimated from the slope in Fig. 4b was larger than that estimated from Fig. 4c, as shown in Table 3. The values of l_2 and $l_3 + l_4$ (ie., distance between P_2 and P_4) were 1.11 and 9.04 meters. Accordingly, the time required for a yarn to run from P_1 to P_2 is shorter than the time needed for a run from P_2 to P_4 .

Maeda [1] defined the time which required for a yarn to run from a delivery roller to a winding roller as creep time and said the shorter this time, the heavier the yarn tension. The elastic modulus estimated from

Fig. 3a Step response ($f_0=0 \rightarrow 1$)

Fig. 3b Step response ($\delta_{1,2} = -0.5\% \rightarrow 0.5\%$)

Fig. 3c Step response ($\delta_{2,4} = 0.5\% \rightarrow 1.5\%$)

Fig. 3 Experimental step response

Table 2 gives a time constant estimated from the experimental results in Fig.3 and calculated on the theory stated above.

4-2. Steady-state Response

According to the theory in steady state, the following equations emerge from eqs. (11), (12), (13) and (14):

$$\begin{aligned} T_1 &= f_0 \\ T_4 - T_2 &= \tau \delta_{1,2} \\ T_4 - T_2 &= \tau \delta_{1,2} \\ T_4 - T_3 &= f_3 \end{aligned}$$

Tension degrees T_1 , T_2 , T_3 and T_4 , observed three minutes after these factors were varied stepwise, were used as steady-state values, the yarn velocity being fixed at 20 cm/sec. The values of T_1 , $(T_2 - T_1)$, $(T_4 - T_2)$ and $(T_4 - T_3)$ were plotted on the Y-axis, and the values of the factors were plotted on the X-axis. These plots are given in Fig.4.

5-1. Effects of the Draft and Frictional Force.

Assuming a yarn to be perfectly elastic, the step responses of various degrees of tension on a running yarn given friction or draft are readily calculable and agreed qualitatively with the experimental results on nylon and cotton yarn. This is clear from a comparison of Fig.3 with Fig.2 or Table 1. In steady state, too, the experimental relation between tension and the factors was approximately linear, as shown in Fig.4 and, therefore, agreed qualitatively with the theory.

Quantitatively speaking, the theoretical values of various degrees of tension agreed approximately with the experimental results as to both step response and steady-state response. There were, however, some differences in time constant between theoretical values and ex-

Fig. 4a Effect of f_0 on T_1

Fig. 4b Effect of $\delta_{1.2}$ on $(T_2 - T_1)$

Fig. 4 b is larger than that estimated from Fig. 4 c, the difference being presumably explainable by the Maeda theory.

5-2. Effect of Yarn Velocity

It is believed that when yarn velocity increases, yarn tension does, too, because friction or inertia in the winding system increases and because a yarn takes less time to run between the delivery roller and the winding roller.

However, as shown in Fig. 3 e, tension in a running yarn in our experiments by only 15%, at most, indicating only a slight effect of yarn velocity on yarn tension.

Fig. 4c Effect of $\delta_{2.4}$ on $(T_4 - T_2)$

Fig. 4d Effect of f_3 on $T_4 - T_3$

Fig. 4 Effects of factors on tension in steady state

Table 3 Elastic Modulus (γ)

Measuring method		N_1	N_2	C
Values measured by static stress-strain curves	20°C 65% RH	4.35	3.45	3.25
Values estimated from Figs. 5•b and 5•c	$\delta_{1,2} : (T_2 - T_1)$	3.70	2.25	3.00
	$\delta_{2,4} : (T_4 - T_2)$	3.55	1.85	2.35

(unit : kg/thread)

5-3. Effect to Tension Career

There were some differences in the responses of the tension between samples N_1 and N_2 as shown in Figs.3 and 4, presumably due to differences in the amount of tension these yarns had received prior to our experiments. Sample N_1 had been wound at a constant winding tension on a bobbin. Sample N_2 had been left in tension less state for 72 hours.

winding tension on a bobbin. Sample N_2 had been left in tension less state for 72 hours.

Therefore, N_2 was smaller in elastic modulus than N_1 while N_2 received a larger effect of creep than did N_1 . The elastic moduli of N_1 and N_2 estimated from static stress-strain curves were 4.35 and 3.45, respectively, as shown in Table 3. (The relation between elastic modulus and creep is dealt with in detail by E.Catsiff

[8]). It follows, then, that tension in a running yarn is considerably influenced by the career of tension and by creep.

Tension remaining in the cotton yarn wound on a corn being slight (as will be explained in part 3), we did not investigate the effect of the "career" of the tension ——— how long it had been in the yarn and how the yarn had been subjected to the tension before our experiments.

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