Automatic Control of Sliver Thickness by High-Frequency Capacitance Measuring Principle.

Part 3: Measuring of Control System Property and Designing of Controller.

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Abstract

We measured the dynamic property of each component part of our control system with the following results:

- (1) Transfer function $G_d(s)$ of the detector is:

$$G_d(s) = \frac{k_d(1 + T_e s) e^{-L_d s}}{(1 + T_d s)(1 + T_c s)}$$

- (2) Transfer function $G_f(s)$ of the final control element is:

$$G_f(s) = \frac{k_f \omega_{na}^2 \cdot G_N(h/|a|)}{(1 + T_{1v} s)(1 + T_{2v} s)(s^2 + 2z_a \omega_{na} s + \omega_{na}^2)}$$

- (3) Open-loop transfer function $G_l(s)$ of the control system, except the controller, is:

$$G_l(s) = G_p(s) \cdot G_d(s) \cdot G_f(s)$$

- (4) Specifications of a PID controller which we designed by the Ziegler-Nichols method from the open loop transfer function are:

P : 0.3-20 Continuous variable
 T_I : 0.3-10 sec Steplessly variable
 T_D : 0.1-5 sec "

1. Introduction

The configuration of the control system dealt with in this article was explained in Part 1. [1] The present instalment gives the results of measurement of the characteristics of both individual and whole elements that make up our control system. We measured, first, the dynamic property of each individual component part element of the control system, then the whole open-loop control system by the frequency response method. With these results as a basis, we express their dynamic properties in terms of transfer functions.

From the open-loop transfer function we calculated the stability limit and the periodic time of sustained oscillations. We measured the practical periodic time on the closed-loop control system and compared it with a value calculated under the same conditions.

Finally, with the aid of the measured stability limit and periodic time, we designed an electronic PID controller suited for use on this control system.

2. Detector

As we said in Part 1, electric capacitance-type equipment on the small-capacitance measuring principle was intended for use as a detecting element for the control system, but it could not be used as such without a modification, because of violent variations in the zero point (the so-called "drift") with changes in the temperature of the measuring electrode.

We, therefore, designed a device to compensate for the "drift" automatically. The compensator, an accessory to the detecting part, has two input terminals, as shown in Fig. 1: one connected to the output of the small-capacitance measuring equipment; the other to the output of the measuring roller which detects the thickness of delivered slivers by measuring its mechanical displacement. The two inputs combine to make the output of the detector after the former input passes through a high-pass filter and the latter through a low-pass filter.

Fig. 1 Principle-diagram of "drift" compensator

Generally, the “drift” is a low-frequency signal and does not show up in the output of the detector, even if it does in the small-capacitance measuring equipment. The detector, designed on this principle, has been proved by experiment to compensate for the “drift” adequately.

It has been made clear by the frequency response method that the dynamic property of the whole detector, including the accessory element, is expressible in the following transfer function:

$$G_d(s) = \frac{k_d (1 + T_c s) e^{-L_d s}}{(1 + T_d s)(1 + T_e s)} \quad \dots\dots (1)$$

where

- k_d : Gain of the detector
- T_d : Time constant of the small-capacitance measuring device
- T_c and T_e : Time constants of the phase-advance
- L_d : Delay in detection (= 5.5 cm/ V_f)
- V_f : Surface velocity of the front roller

3. Final Control Element

The final control element that varies the surface velocity of the front roller, as described in Part 1 [1], consists mainly of an electro-hydraulic power actor and a PIV speed variable device. Its make-up is shown in Fig. 2 and its outside view in Fig. 3.

In the light of the measured results obtained by the frequency response method, the property of the final control element is expressible in the following transfer function:

$$\begin{aligned} G_f(s) &= G_a(s) \cdot G_n(h/a) \cdot G_v(s) \quad \dots\dots\dots (2) \\ &= \frac{k_a \cdot k_v \omega_{na}^2 \cdot G_n(h/a)}{(s^2 + 2\zeta_a \omega_{na} s + \omega_{na}^2)(1 + T_{1v}s)(1 + T_{2v}s)} \quad \dots\dots\dots (3) \end{aligned}$$

where

- $G_f(s)$: Transfer function of final control element
- $G_a(s)$: Transfer function of electro-hydraulic power actor
- $G_v(s)$: Transfer function of PIV speed variable device
- $G_n(h/a)$: Description function of back-lash present in coupling part between power actor and speed variable device [3]
- h : Magnitude of the back-lash, about 1.6% of the full scale of power actor output
- $|a|$: Magnitude of input signal of the back-lash

Fig. 4 shows a block-diagram of the final control system.

4. Designing of Controller

4-1. Calculating Open-loop Transfer Function, Stability Limit and Periodic Time of Sustained Oscillations.

Fig. 2 Constituted diagram of final control element.

Fig. 3 Outside view of final control element

It is clear from the measured results obtained by the frequency response method that the transfer function of each component part of the control system, if it is in a frequency range of less than 20 rad/sec, is expressible as follows:

Controlled process [2]

Process for control

$$G_p(s) = \frac{k_p \omega_{np}^2 (1 + T_p s)}{s^2 + 2\zeta_p \omega_{np} s + \omega_{np}^2}$$

Detecting part

$$G_d(s) = \frac{k_d e^{-L_d s} (1 + T_c s)}{(1 + T_d s)(1 + T_e s)}$$

Final control element

$$G_f(s) = \frac{k_f \omega_{na}^2 \cdot G_n(h/a)}{(1 + T_{1v}s)(1 + T_{2v}s)(s^2 + 2\zeta_a \omega_{na} s + \omega_{na}^2)}$$

Disturbance element [3]

$$G_n(s) = \frac{k_n \omega_{np}^2 (1 - T_n s)}{s^2 + 2\xi_p \omega_{np} s + \omega_{np}^2}$$

Accordingly, the block-diagram of a constituted control system is as shown in Fig. 5, while open-loop transfer function $G_t(s)$ of the control system without a controller is expressible as follows:

$$G_t(s) = G_p(s) \cdot G_d(s) \cdot G_f(s) \quad \dots\dots\dots (8)$$

$$= \frac{k_t \omega_{np}^2 (1 + T_p s) e^{-L_d s} (1 + T_c s)}{(s^2 + 2\xi_p \omega_{np} s + \omega_{np}^2) (1 + T_a s) (1 + T_c s)}$$

$$\cdot \frac{\omega_{na}^2 \cdot G_n(h/|a|)}{(1 + T_{1v} s) (1 + T_{2v} s) (s^2 + 2\xi_a \omega_{na} s + \omega_{na}^2)} \quad \dots\dots\dots (9)$$

where k_t is the open-loop gain of the control system.

The measured values of the constants in eq. (9) are given by Tables 1 and 2. Table 1 shows the values measured at "high temperature and humidity"; Table 2, at "low temperature and humidity." [2]

On the basis of the measured values, we calculated the gain and phase delay of $G_t(s)$ and express them in a Bode diagram in Figs. 6 and 7.

Fig. 6 Bode-diagram of open-loop transfer function (High temperature and humidity).

Fig. 4 Block-diagram of whole final control element.

Fig. 5 Block-diagram of control system.

Fig. 7 Bode-diagram of open-loop transfer function (Low temperature and humidity)

Table 1 Measured Values of Constants of $G_t(s)$ (at "High Temperature and Humidity")

Surface velocity of front roller (cm/sec)	Surface velocity of back roller (cm/sec)	ω_{na} (rad/sec)	ξ_p	T_p (sec)	L_d (sec)	Others
40	13.2	13.6	0.50	0.06	0.14	$k_t = 1$
60	..	14.7	0.43	0.08	0.09	$T_c = 0.25$ sec
80	..	15.0	0.33	0.10	0.07	$T_d = 0.20$ sec
100	..	15.3	0.27	0.12	0.055	$T_{1v} = 0.33$ sec $T_{2v} = 0.11$ sec $\omega_{na} = 42.0$ rad/sec $\xi_a = 0.33$

Table 2 Measured Values of Constants of $G_t(s)$ (at "Low Temperature and Humidity")

Surface velocity of front roller (cm/sec)	ω_{np} (rad/sec)	ξ_p	T_p (sec)
40	7.5	0.37	0.20
60	9.8	0.28	0.18
80	13.0	0.27	0.14
100	14.7	0.24	0.14

The upper curves in these figures show phase delay and the lower curves, the gains of open-loop transfer function $G_t(s)$. The figures added near the curves express the surface velocity of the front roller (mean value). We excluded the back-lash from calculation.

With the aid of the Bode-diagram we investigated the stability limit and the periodic time of oscillations, with the results given Tables 3 and 4.

4-2. Measuring Periodic Time of Sustained Oscillation

We actually measured the periodic time of sustained oscillations when fed slivers were being drafted with the control system at work, and compared it with the value

Table 3 Stability Limit and Periodic Time of Sustained Oscillation (at "High Temperature and Humidity")

Surface velocity of front roller (cm/sec)	Draft ratio (calculated)	Stability Limit Gain	dB	Sustained Oscillations Frequency (rad/sec)	Period (sec)
40	3.02	2.01	6.6	6.0	1.05
60	4.57	2.40	7.6	7.6	0.83
80	6.05	2.46	7.8	9.0	0.70
100	7.56	2.21	6.9	9.8	0.61

Table 4. Stability Limit and Periodic Time of Sustained Oscillation (at "Low Temperature and Humidity")

Surface velocity of front roller (cm/sec)	Stability limit Gain	dB	Sustained oscillation Frequency (rad/sec)	Period (sec)
40	1.21	1.6	6.0	1.05
60	1.45	3.2	7.6	0.83
80	1.72	4.7	9.0	0.70
100	1.78	5.0	10.3	0.61

calculated from the Bode-diagram under the same conditions.

a. Conditions and Method of Measurement

We drafted a fed sliver of the least possible unevenness with a drafting frame equipped with the automatic control device set in proportional motion. We increased the gain of the controller gradually, so that sustained oscillations occurred on the control system and the drafted sliver had periodic unevenness. We measured the periodic time by using the autocorrelation function of the drafted sliver. We used the same kind of sliver and the same conditions as in Tables 1 and 2 in Part 2. [2]

Before making the experiments, we investigated whether there were limit cycles on the control system, because we were not sure whether the Ziegler-Nichols method was applicable even if there were limit cycles. The investigation was made as in Fig. 8, in which are drawn both a Nyquist

Fig. 8 Nyquist plot of open-loop frequency transfer function and locus of negative reversed description function of back-lash.

plot of open-loop frequency transfer function $G_L(s)_{s=j\omega}$ and the locus of a negative, reversed description function of the back-lash, $-1/[G_n(h/|a|)]$. We were satisfied that there were no limit cycles on the control system.

b. Measured Results

A recorded example of the unevenness of the drafted sliver with periodic unevenness is given in Fig. 9. A recorded example of the autocorrelation function is given in Fig. 10.

The measured values of the periodic time are shown in Fig. 11. The circles in it indicate the measured values; the solid and broken lines express values calculated from the transfer function of the control system. These results show that the measured values almost agree with the calculated values.

4-3. Designing of Controller

We obtained Table 5 by calculating the optimal gain, integral and differential time. The "Optimal gain" columns contain sets of two figures: the upper figures are values at "high temperature and humidity"; the lower figures,

Fig. 9 Recorded sample of drafted sliver having sustained oscillated unevenness.

Fig. 10 Recorded sample of autocorrelation function of drafted sliver having oscillated unevenness

(a) General PID Controller

(b) Ideal PID Controller

Fig. 12 Block-diagram of PID controller

values at "low temperature and humidity."

Using these figures in Table 5 as a basis, we designed an electric PID controller. Its block-diagram is as in Fig. 12 (b).

Fig. 11 Relation between surface velocity of front roller and periodic time of sustained oscillations

Table 5. Optimal Condition Calculated by Ziegler-Nichols Method

Surface velocity of front roller	P Action Optimal Gain	PI Action		PID Action		
		Optimal Gain	T _I (sec)	Optimal Gain	T _I (sec)	T _D (sec)
40	1.01	0.90	0.87	1.21	0.53	0.13
	0.61	0.54		0.73		
60	1.20	1.08	0.69	1.44	0.41	0.10
	0.73	0.65		0.87		
80	1.23	1.11	0.58	1.48	0.35	0.09
	0.86	0.77		1.03		
100	1.11	0.99	0.53	1.33	0.32	0.08
	0.89	0.80		1.07		

Its specifications are:

- P : 0.3–20 (Proportional band 300–5%)
 T_I : 0.3–10 sec
 T_D : 0.1–5 sec

5. Summing up

(1) We have measured the dynamic property of each component part of our control system with the following results:

(a) Transfer function $G_d(s)$ of the detector is:

$$G_d(s) = \frac{k_d(1 + T_e s)e^{-L_d s}}{(1 + T_d s)(1 + T_e s)}$$

(b) Transfer function $G_f(s)$ of the final control element is:

$$G_f(s) = G_a(s) \cdot G_n(h/a) \cdot G_v(s) \\ = \frac{k_p \cdot k_a \cdot \omega_{na}^2 \cdot G_N(h/a)}{(s^2 + 2\zeta_a \omega_{na} s + \omega_{na}^2)(1 + T_{1r} s)(1 + T_{2r} s)}$$

(c) The open-loop transfer function of the control system without a controller is:

$$G_t(s) = G_p(s) G_d(s) G_f(s) \\ = \frac{k_t \omega_{np}^2 (1 + T_p s) e^{-L_d s} (1 + T_r s)}{(s^2 + 2\zeta_p \omega_{np} s + \omega_{np}^2)(1 + T_d s)(1 + T_e s)}$$

$$\cdot \frac{\omega_{na}^2 \cdot G_N(h/a)}{(1 + T_{1r} s)(1 + T_{2r} s)(s^2 + 2\zeta_a \omega_{na} s + \omega_{na}^2)}$$

(2) The measured values of the periodic time of sustained oscillations almost agree with the values calculated from open-loop transfer function $G_t(s)$.

(3) We have designed an electric PID controller which fits this system. Its specifications are:

- P : 0.3–20 (Proportional band 300–5%) Continuous variable
 T_I : 0.3–10 sec steplessly variable
 T_D : 0.1–5 sec "

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