Pneumatic Conveyance of Fiber Assemblies

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Abstract

The authors have looked into pressure drops in horizontal and vertical straight conveyor pipes and horizontal bent conveyor pipes to obtain basic data for use in designing a pneumatic conveyor system for fiber assemblies which will help forward the modernization of the spinning process.

The concept of friction velocity u_f has been applied to the pneumatic conveyance of fiber assemblies to show that the ratio u/u_f of mean air velocity u to u_f is linearly correlated to the mixing ratio (weight) m ; and that frictional coefficients A and A' of fluid for mixed-phase flow (air and fiber assembly) can be obtained.

The friction coefficient for a horizontal straight conveyor pipe is as given in eq.(15)

$$A = \frac{8}{\left(\sqrt{\frac{8}{\lambda}} - 5.10 \text{ m}\right)^2} \dots\dots\dots (15)$$

The friction coefficient for a vertical straight pipe is as given in eq.(29)

$$A' = \frac{8}{\left(0.95 \sqrt{\frac{8}{\lambda}} - 12.5 \text{ m}\right)^2} \dots\dots\dots (29)$$

The coefficient of the pressure drop in a horizontal bent pipe has been calculated by dividing it into ξ for air flow and ζ which increases as a fiber assembly is added to, and the following equation has been obtained:

$$\zeta = 0.11 \frac{2R}{D} (m - 0.10) \dots\dots\dots (43)$$

KEY WORDS: FIBER ASSEMBLIES, COEFFICIENT OF FRICTION, PIPES, TRANSFERRING (MATERIAL)

1. Introduction

Planning a pneumatic conveyor system requires a full grasp of the relation between the amount of material transported and the horse-power needed for transportation. This relation resolves itself into the relation between the mixing ratio and the pressure drop. The energy consumed by a pneumatic conveyor is supplied from the pressure energy of the air, which supply constitutes a pressure drop.

The relations between the mixing ratio and the pressure drop as they concern pneumatic conveyors for grains have been looked into, in most cases experimentally, by a host of research workers. Each research worker has his own way of expressing the pressure drop, while each material conveyed by air has its own equation on the pressure drop.

It will not do to design a pneumatic conveyor system for fiber assemblies with the aid of equations on the pressure drop for grain transportation. We must design the conveyor system for fiber assemblies by drawing on equations

on the pressure drop for fiber assembly transportation. A conveyor system for fiber assemblies being generally composed of combinations of horizontal and straight conveyor pipes and bent conveyor pipes, we must calculate the total pressure drop for this system in designing it. The present article tries to clarify the relations between the mixing ratio and the pressure drop concerning horizontal and vertical straight conveyor pipes and bent conveyor pipes.

Symbols

- A_1, A_2 : Cross section area at the entrance to and exit of bent pipe (m^2)
- C : Acoustic velocity in fluid flowing in bent pipe (m/s)
- D : $4 \times D_m$ (m)
- D_m : Hydraulic mean depth (m)
- F_r : Froude's number
- g : Gravitational acceleration (m/s^2)
- H : A side of cross section of duct (m)

H/W :	Aspect ratio of duct
h :	Fluid friction loss of head or length of vertical pipe (m)
L_d :	Downstream length of bent pipe (m)
L_u :	Upper stream length of bent pipe (m)
l :	Length of pipe (m)
l' :	Length of center line of bent pipe (m)
M :	Mach's number
m :	Mixing ratio (weight mixing ratio)
Δp :	Pressure loss (kg/m ²)
Δp_b :	Pressure loss in bent pipe (kg/m ²)
Δp_{ba} :	Pressure loss in bent pipe when air is flowing (kg/m ²)
Δp_{bf} :	Additional pressure loss in bent pipe caused by fiber assembly (kg/m ²)
Δp_d :	Pressure loss in lower stream uninfluenced by bend (kg/m ²)
Δp_u :	Pressure loss in upper stream uninfluenced by bend (kg/m ²)
R :	Radius of curvature of center line of bent pipe (m)
Re :	Reynolds' number
R/W :	Radius ratio
u :	Mean velocity in pipe (m/s)
u_f :	Friction velocity (m/s)
W :	A side of cross section of pipe (m)
γ :	Specific weight of air (kg/m ³)
ε :	Roughness of surface (m)
ε/D :	Relative roughness
ξ :	Coefficient of additional pressure loss in bent pipe caused by fiber assembly
θ :	Angle of bend
Λ :	Coefficient of fluid friction on mixed-phase flow of air and fiber assembly in horizontal pipe
Λ' :	Coefficient of fluid friction on mixed-phase flow of air and fiber assembly in vertical pipe
λ :	Coefficient of fluid friction
λ_f :	Additional coefficient of fluid friction by fiber assembly in horizontal pipe
λ'_f :	Additional coefficient of fluid friction coming from presence of fiber assembly in vertical pipe
μ :	Coefficient of viscosity of fluid (kg·s/m ²)
ξ :	Coefficient of pressure loss in bent pipe
ρ :	Specific mass of air (kg·s ² /m ⁴)
τ_0 :	Frictional stress on wall surface (kg/m ²)

2. Experimental Apparatus

An outline of our experimental apparatus is as shown in Fig.1. In our experiment we used the vacuum pneumatic conveyor system. We first weighed an assembly of fibers obtained from cotton lap and put it on belt-conveyor (1). We then broke the fiber assembly into pieces about 10mm to 40mm in diameter with a small special scratcher. The pieces were drawn into duct (3) which had a 50mm square

Fig. 1 Experimental apparatus

cross section made of sheet zinc. The duct had holes (1mm in diameter) at intervals of 500mm on every side for the measurement of pressure.

After passing through duct (3), the fiber assembly was separated from the air in box (4). (5) was an orifice for the measurement of the air flow rate. The mean velocity of the air flowing through the duct was measured by this orifice. (6) was a turbo-blower. The air flow rate was controlled by (7).

3. Pressure Loss in Horizontal Straight Pipes

In dealing with the pressure loss in flow through a duct, we should not ignore the roughness of the inner surface of the duct. If the roughness of the inner surface of a duct varies, the pressure loss, when we let the air flow, varies and so also does the additional pressure loss coming from the presence of a fiber assembly. Therefore, to grasp the pressure loss in a pneumatic conveyor for fiber assemblies, we had to clarify the coefficient of fluid friction when only the air flowed through the pipes used in our experiment.

Those pipes had square cross sections and the flow of the air belonged to the domain of turbulence. Accordingly, to calculate the frictional loss of head, we used $4Dm$ as the representative scale showing the size of the cross section, rather than the diameter of a circular pipe. Frictional loss h of head can be expressed thus:

$$h = \lambda \cdot \frac{l}{4Dm} \cdot \frac{u^2}{2g} \dots \dots \dots (1)$$

the hydraulic mean depth^[2] Dm being defined by

$$Dm = \frac{\text{Cross section area of a pipe}}{\text{Peripheral length of cross section of pipe}}$$

When only the air flows, λ is obtainable, as in Fig.2, from the experimental data using eq.(1). Fig.2 also shows Blasius' equation^[2] as applied within the range of Reynolds' number $3 \times 10^3 - 10^5$ for smooth circular pipes.

$$\lambda = \frac{0.3164}{Re^{1/4}} \dots \dots \dots (2)$$

With the aid of pressure loss Δp , we can obtain the frictional stress on the surface of a wall. From this frictional stress we can obtain the friction velocity^[1] u_f during conveyance of a fiber assembly.

The flow in horizontal straight pipes within the limits of this experiment belongs to the domain of turbulence. Therefore, by using the concept of equilibrium of force, we can get the relation between the frictional stress τ_0 and

Fig. 2 Renolds' number and coefficient of fluid friction

coefficient λ of fluid friction, as follows:

$$\frac{\pi}{4} D^2 \Delta p = \pi D l \tau_0 \quad (3)$$

where $D = 4D_m$.

From the equation on energy, the following equation is obtainable:

$$\Delta p = \lambda \frac{l}{D} \frac{\rho u^2}{2} \quad (4)$$

Eqs. (3) and (4) lead to:

$$\sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{\lambda}{8}} \cdot u \quad (5)$$

$\sqrt{\tau_0/\rho}$ has a dimension of velocity, which we will call the friction velocity and denote by u_f :

$$\sqrt{\frac{\tau_0}{\rho}} = u_f \quad (6)$$

Substitute for $\sqrt{\tau_0/\rho}$ in eq. (5):

$$\frac{u}{u_f} = \sqrt{\frac{8}{\lambda}} \quad (7)$$

and eq. (3) reduces itself to

$$\tau_0 = \frac{\Delta p}{l} \frac{D}{4} \quad (8)$$

Therefore, with values of Δp experimentally obtained, values of τ_0 are obtainable by eq. (8). Using this τ_0 , we get values of friction velocity u_f from eq. (6). The relation between u/u_f and m on the basis of our experimental data

Fig. 3 Relation between u/u_f and m in horizontal straight pipe

is as shown in Fig.3. The relation between u/u_f and m , then, is expressible by a straight line. When no fiber assembly is in the pipe, the mixing ratio m is zero and u/u_f is $\sqrt{8/\lambda}$ at $m=0$, in the light of eq. (7). Hence:

$$\frac{u}{u_f} = \sqrt{\frac{8}{\lambda}} - 5.10m \quad (9)$$

Assuming that λ_f is the additional coefficient of fluid friction coming from the presence of a fiber assembly, then

$$\Delta p = (\lambda + \lambda_f) \frac{l}{D} \frac{\rho u^2}{2} \quad (10)$$

From Eqs. (8) and (10), the following equation is obtainable:

$$\tau_0 = \frac{\lambda}{8} \rho u^2 \left(1 + \frac{\lambda_f}{\lambda}\right) \quad (11)$$

Therefore,

$$\frac{u_f}{u} = \sqrt{\frac{\lambda}{8} \left(1 + \frac{\lambda_f}{\lambda}\right)} \quad (12)$$

Eqs. (9) and (12) lead to:

$$\lambda_f = \frac{8}{\left(\sqrt{\frac{8}{\lambda}} - 5.10m\right)^2} - \lambda \quad (13)$$

this being the equation on additional pressure loss coming from conveying a fiber assembly. Hence, λ_f is obtainable from coefficient λ of fluid friction and mixing ratio m .

Assume that Λ is the coefficient of fluid friction for mixed-phase flow:

$$\Lambda = \lambda + \lambda_f \quad (14)$$

Therefore, eqs. (13) and (14) lead to:

$$\Lambda = \frac{8}{\left(\sqrt{\frac{8}{\lambda}} - 5.10m\right)^2} \quad (15)$$

4. Pressure Loss in Vertical Straight Pipes

The pressure loss in vertical straight pipes is composed of the pressure loss in horizontal straight pipes and the pressure loss caused by gravity. The relation between u/u_f and m is as shown in Fig.4 on the basis of experimental data.

Fig. 4 Relation between u/u_f and m in vertical straight pipe

Let the air flow in vertical pipes and assume that Δp is the difference in static pressure between points A and B which are spaced apart by height h . Then:

$$\frac{\Delta p}{\gamma} = h + \lambda \frac{h}{D} \frac{u^2}{2g} \quad (16)$$

Assuming that the pressure loss due to the height is included in frictional stress τ_0 , then:

$$\tau_0 = \frac{\Delta p}{h} \frac{D}{4} \quad (17)$$

The friction velocity is, therefore, expressible as follows:

$$u_f = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{\Delta p}{h} \frac{D}{4} \frac{g}{\gamma}} \quad (18)$$

Hence:

$$\tau_0 = \frac{D}{4} \gamma \left(1 + \lambda \frac{1}{D} \frac{u^2}{2g} \right) = \frac{u_f^2 \gamma}{g} \quad (19)$$

$$\frac{u}{u_f} = \sqrt{\frac{8}{\lambda} \left(1 - g \frac{D}{4} \frac{1}{u_f^2} \right)} \quad (20)$$

When only the air flows, pressure loss Δp is 10 mmAq at $u=15.1\text{m/s}$, so that u_f^2 is 1.05 in the light of eq. (18). Eq.(20), then, transforms into:

$$\frac{u}{u_f} = \sqrt{\frac{8}{\lambda}} \times \sqrt{1 - 0.116} = 0.9597 \sqrt{\frac{8}{\lambda}} \quad (21)$$

which is applicable where only the air flows.

The relation between u/u_f and m in mixed-phase flow made up of the air and a fiber assembly is expressible by a straight line as shown in Fig.4. Hence the following equation forms:

$$\frac{u}{u_f} = 0.95 \sqrt{\frac{8}{\lambda}} - 12.5m \quad (22)$$

When only the air flows, the influence of height, i.e., factors $\sqrt{8/\lambda}$ in eqs. (21) and (22), agree with each other, to wit:

$$\Delta p = (\lambda + \lambda'_f) \frac{h}{D} \frac{\rho}{2} u^2 \quad (23)$$

$$\tau_0 = \frac{\Delta p}{h} \frac{D}{4} \quad (24)$$

Accordingly:

$$\tau_0 = \frac{\lambda}{8} \rho u^2 \left(1 + \frac{\lambda'_f}{\lambda} \right) \quad (25)$$

$$\frac{u_f}{u} = \sqrt{\frac{\lambda}{8} \left(1 + \frac{\lambda'_f}{\lambda} \right)} \quad (26)$$

From eqs.(22) and (25) the following equation is deducible:

$$\lambda'_f = \frac{8}{(0.95 \sqrt{\frac{8}{\lambda}} - 12.5m)^2} - \lambda \quad (27)$$

We have divided the pressure loss in vertical straight pipes for the pneumatic conveyance of fiber assemblies into the pressure loss only in the air in horizontal straight pipes and the pressure loss added by fiber assemblies in vertical

straight pipes. We have found that the latter is expressible by eq.(27), the λ'_f being calculated from the coefficient of fluid friction of the air and mixing ratio m .

Assuming Λ' to be coefficient of fluid friction of the mixed-phase flow of the air and fiber assemblies in vertical straight pipes, then:

$$\Lambda' = \lambda'_f + \lambda \quad (28)$$

so that

$$\Lambda' = \frac{8}{(0.95 \sqrt{\frac{8}{\lambda}} - 12.5m)^2} \quad (29)$$

5. Pressure Loss in Bent Pipes

(1) Defining pressure loss and coefficient of loss in bent pipes

Generally, the pressure loss in bent pipes is expressible as follows^[8]:

$$\Delta p = F(u, c, \rho, \mu, g, D, R, H, W, \varepsilon, l', A_1, A_2) \quad (30)$$

Express this equation in dimensionless numbers by the method of dimensional analysis. Since u and c , D , R , H , W , ε and l' , A_1 and A_2 all have the same dimension, and in view of

$$\Delta p = F(u, \rho, \mu, g, D) \quad (31)$$

the following dimensional matrix is obtainable:

$$\begin{matrix} & u & \rho & \mu & \Delta p & g & D \\ L & \begin{pmatrix} 1 & -4 & -2 & -2 & 1 & 1 \end{pmatrix} \\ F & \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix} \\ T & \begin{pmatrix} -1 & 2 & 1 & 0 & -2 & 0 \end{pmatrix} \end{matrix}$$

As this matrix has rank 3, dimensionless products are 3 in number:

$$\left. \begin{aligned} \pi_1 &= u^x \rho^y \mu^z \Delta p \\ \pi_2 &= u^{x'} \rho^{y'} \mu^{z'} g \\ \pi_3 &= u^{x''} \rho^{y''} \mu^{z''} D \end{aligned} \right\} \quad (32)$$

The values of $x, y, z, x', y', z', x'', y'', z''$ and z'' calculated with the aid of the foregoing equation are:

$$\begin{aligned} x &= -2, & x' &= -3, & x'' &= 1 \\ y &= -1, & y' &= -1, & y'' &= 1 \\ z &= 0, & z' &= 1, & z'' &= -1 \end{aligned}$$

Hence:

$$\left. \begin{aligned} \pi_1 &= \frac{\Delta p}{\rho u^2} \\ \pi_2 &= \frac{g \mu}{u^3 \rho} \\ \pi_3 &= \frac{D u \rho}{\mu} \end{aligned} \right\} \quad (33)$$

Using dimensionless product F_r^{-1} in place of π_2 results in

$$\begin{aligned} \frac{\Delta p}{\rho u^2} &= f \left(\frac{\rho u D}{\mu}, \frac{u}{\sqrt{g D}}, \frac{u}{c}, \frac{R}{W}, \frac{H}{W}, \frac{\varepsilon}{D}, \right. \\ &\quad \left. \frac{l'}{R}, \frac{A_1}{A_2} \right) \quad (34) \end{aligned}$$

where

$$\frac{g\mu}{u^3\rho} \frac{Du\rho}{\mu} = \frac{gD}{u^2} = Fr^{-1} \dots\dots\dots (35)$$

or

$$\xi = f(Re, Fr, M, \frac{R}{W}, \frac{H}{W}, \frac{\varepsilon}{D}, \theta, \frac{A_1}{A_2}) \dots\dots\dots (36)$$

Since we are dealing here only with bent pipes having a constant and square cross section area and a bent angle of 90°, we may omit the terms of A_1/A_2 , θ , and H/W . Since we may ignore compressibility at the velocity of a pneumatic conveyor, we may also omit Mach's number M .

Froude's number Fr is a number showing the influence of gravity. Since only horizontal bent pipes are considered here, we may ignore the influence of gravity and omit Froude's number. ε/D and Re are replaceable with coefficient λ of fluid friction in straight pipes. Hence, the coefficient of the pressure loss in bent pipes is a function of λ and R/W , i.e., λ and R/D .

$$\xi = f(\lambda, \frac{R}{W}) \dots\dots\dots (37)$$

The R/D of the bent pipes used in this experiment are as given in the following table:

Table 1 Measurements of Bent Pipes Used in Our Experiment

Angle of bend	D mm	R mm	R/D
90	50	500	10
90	50	400	8
90	50	300	6
90	50	200	4

The flow of air being deformed noticeably at the exit of bent pipes, a fairly long length is needed for the gradual recovery of the velocity distribution, which is deformed, and for the full re-development of flow. It is difficult to measure the pressure drop near the inlet or outlet of pipes exactly, because it is affected by the curve of the pipes. We, therefore, measured the pressure drop at the point of upper flow $L_u/D=40$ and lower flow $L_d/D=30$.

Fig.5 denotes the length of upper flow AB in the straight section of a bent pipe by L_u and that of lower flow CD in the bent section by L_d . Pressure losses in straight sections AB and CD without the influence of the bent section BC are denoted by ΔP_u and ΔP_d ; the pressure loss in AD, by Δp . The pressure loss in bent section BC is definable as follows:

$$\Delta p_b = \Delta p - \Delta p_u - \Delta p_d \dots\dots\dots (38)$$

Denote the pressure loss when only the air flows in pipes by the suffix a ; and the additional pressure loss caused by

Fig. 5 Pressure loss in bent pipe

a fiber assembly by the suffix f . Δp_b , then, takes this form:

$$\Delta p_b = \Delta p_{ba} + \Delta p_{bf} \dots\dots\dots (39)$$

Assuming Δp to be the pressure loss between points A and B at mean air velocity u , then coefficient ξ of the pressure loss in bent pipes and additional coefficient ζ of the pressure loss in bent pipes caused by a fiber assembly are definable by the following equations:

$$\Delta p_{ba} = \xi \frac{\rho u^2}{2} \dots\dots\dots (40)$$

$$\Delta p_{bf} = \zeta \frac{\rho u^2}{2} \dots\dots\dots (41)$$

(2) Pressure loss in bent pipes

With Δp_u , Δp_b and Δp_d defined as in Fig.5, Δp_b can be given by eqs.(39), (40) and (41).

The relations between coefficient ξ of pressure loss and Reynolds' number Re are as shown in Figs. 6 and 7.

Assume that Δp_b is the pressure loss in mixed-phase flow in bent pipes. Divide it into the pressure loss Δp_{ba} only in air flow and additional pressure loss Δp_{bf} caused by a fiber assembly. By using this Δp_{bf} , we obtain additional coefficient ζ of pressure loss caused by a fiber assembly as in eq.(41). Examples of the relation between this coefficient ζ and the mixed ratio m are as shown in Figs.8 and 9, from which is deducible the following equation:

$$\left. \begin{aligned} \zeta &= 20(0.104m - 0.0108) \text{ for } 2R/D=20 \\ \zeta &= 16(0.114m - 0.0113) \text{ for } 2R/D=16 \\ \zeta &= 12(0.112m - 0.0112) \text{ for } 2R/D=12 \\ \zeta &= 8(0.109m - 0.0107) \text{ for } 2R/D=8 \end{aligned} \right\} \dots\dots\dots (42)$$

which, consolidated, transforms into:

$$\zeta = 0.11 \frac{2R}{D} (m - 0.10) \dots\dots\dots (43)$$

R being the radius of curvature of the center line of a bent pipe; D the hydraulic mean depth of a square pipe; and m

Fig. 6 Reynolds' number R_e and coefficient ξ of pressure loss ($2R/D=20$, $\theta=90^\circ$)

Fig. 7 Reynolds' number R_e and coefficient ξ of pressure loss ($2R/D=12$, $\theta=90^\circ$)

Fig. 8 Mixing ratio m and coefficient ζ of pressure loss ($2R/D=20$, $\theta=90^\circ$)

the mixing ratio.

The pressure loss in bent pipes needed in designing a pneumatic conveyor for fiber assemblies is obtainable

Fig. 9 Mixing ratio m and coefficient ζ of pressure loss ($2R/D=12$, $\theta=90^\circ$)

by eq. (44):

$$\Delta p_b = \left(\xi + \zeta \right) \frac{\rho u^2}{2} \dots \dots \dots (44)$$

where ζ is given as in eq.(43).

6. Conclusions

The authors have explained the relations between the pressure drop and the mixing ratio in horizontal and vertical straight pipes and bent pipes as a problem bearing on the pressure drop in the pneumatic conveyance of fiber assemblies.

We have obtained coefficient Λ of fluid friction for mixed-phase flow composed of the air and a fiber assembly in a horizontal straight pipe from eq.(15), and coefficient Λ' of fluid friction for mixed-phase flow composed of the air and a fiber assembly in a vertical straight pipe from eq.(29).

We have divided the coefficient of pressure loss in bent pipes into coefficient ξ of only the air and additional coefficient ζ caused by a fiber assembly, ζ having been obtained from eq.(43). Hence pressure loss Δp_b in bent pipes is as given by eq.(44).

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