Biaxial Tensile Properties of Two Bar Warp-knit Fabrics Theoretically Studied

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Abstract

In previous paper, a biaxial tensile deformation theory was developed of a plain tricot fabric, a fundamental type for all two-bar tricot fabrics.

In the present paper, this theory is expanded and more generalized to calculate the properties of two-bar tricot fabrics. The structure of the model proposed for the plain tricot fabric is modified to make it adaptable to more complicated structure.

When the number of wale spacings crossed by underlapping becomes large or when spun yarn is used, the original theory becomes more error-prone. To make the theory more precise, the compressive property of yarn should be introduced for calculation in stretching-effective region. In the new theory, a simple model where yarn is wound around a cylinder is used for the structure of the crossover region.

Finally, some theoretical calculations have been done for various factors such as the tensile and the frictional properties of yarn, structural constants, and the number of wale spacings crossed by underlapping.

KEY WORDS: WARP KNITTED FABRICS, FABRIC STRUCTURE, TRICOT STITCHES, BENDING, STRETCH, COMPRESSION, TENSILE PROPERTIES

1. Introduction

In the previous papers[1-4], the relation between the tensile load and the tensile deformation of a warp-knit fabric in a biaxial-tensile field was analyzed by applying the method developed by Kawabata[5]. Namely, the biaxial-tensile properties of one-bar tricot fabric with close lap were theoretically analyzed, and the accuracy of the theory was experimentally ascertained[1,2]. Then, those with open lap were investigated by the same method, and some differences seen in biaxial-tensile properties of warp-knit fabrics with close lap and with open lap were compared from the point of this theory[3]. The properties of plain tricot fabric, a fundamental type for two-bar tricot fabrics, were calculated theoretically both from the structure constants of the fabric and from some of mechanical properties of yarn[4].

In this paper the theory above-mentioned is expanded to a more general form for calculating the properties of two-bar tricot fabric. In the original theory, the larger the number of wales crossed by underlapping becomes, the more error-prone the theory is. The same tendency is also found in fabrics knitted by spun yarn. To make the theory more precise, the compressive property of the yarn should be introduced for calculation in stretching-effective region. The biaxial-tensile properties thus have been calculated theoretically for various factors such as the tensile and the frictional properties of yarn, the structural constants of fabric and the number of wales crossed by underlapping.

2. Theory

Each type of tricot stitches has a common structure, only differentiating in the number of wales crossed by underlapping. This difference leads to many names of structure, for example, plain tricot, cord, half, satin back. It also brings about large discrepancy between the density and tensile properties of the fabric.

The structures of all kinds of two-bar tricot fabrics can be represented by a simple structural model where the length of sinker loop is different for each kind of structures. The tensile properties of those fabrics will be calculated by using this simple model after expanding the theory developed in the previous paper[4].

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2-1 Biaxial Tensile Theory

The unit structure is shown in Fig. 1 in a heavy line. Let us take $X_1$ axis in the wale direction, and $X_2$ axis in the course direction. The structural constants required for calculating the tensile properties are as follows, similar to the previous paper\textsuperscript{47}.

- $y_{01}$: course spacing before deformation (mm)
- $y_{02}$: wale spacing before deformation (mm)
- $l$: total yarn length in unit structure (mm)
- $D$: yarn diameter (mm)

In this analysis is used Kawabata’s method\textsuperscript{51}, where the stretching process is divided into two, i.e., the stretching-effective region and the stretching-effective region, taking the critical-tensile ratios $\lambda_{c1}$ and $\lambda_{c2}$ as their border. The tensile properties in those regions are calculated separately and the true properties of the fabric are synthesized with them.

2-1-1 Calculation of Critical Tensile Ratios $\lambda_{c1}$ and $\lambda_{c2}$

The critical tensile state\textsuperscript{51} of the unit model is shown in Fig. 2. Comparing this structure with that of plain tricot stitch, only the lengths of their sinker loops are different due to the difference of the number of wales of underlapping. In the critical state, following equations are introduced from geometrical relation.

\begin{align*}
L_{1f} &= (3.5\pi + 1)D - A_c + L_{1b} + 2L_{1o} \quad \text{for } f = 1, 2, \ldots, n_b \quad \text{and } y_{1o} = k_1 y_{01} (y_{2c} - y_{02}) + y_{01} \quad \text{for } \varepsilon_1 \geq \varepsilon_2, \\
L_{2o} &= \sqrt{(y_{1o} - D)^2 + \left(\frac{n_b y_{02}}{y_{1o}} - D\right)^2 + A_c^2} \\
L_{1o} &= \sqrt{(y_{1o} - D)^2 + 5D^2} \\
L_{1b} &= \sqrt{(y_{1b} - D)^2 + (n_a y_{01} - D)^2 + A_c^2} \\
L_{1o} &= \sqrt{(y_{1o} - D)^2 + 9D^2} \\
\lambda_c &= 2.5D^2 / (y_{1o} - D) \\
L &= L_{1f} + L_b \quad \text{for } \varepsilon_1 \geq \varepsilon_2 \quad \text{and } y_{1o} = k_2 y_{y02} (y_{1c} - y_{02}) + y_{02} \quad \text{for } \varepsilon_1 \geq \varepsilon_2,
\end{align*}

where $k_1$ and $k_2$ are constants defined for presenting the deformation mode such as $k_1 = \varepsilon_1 / \varepsilon_2$ and $k_2 = \varepsilon_2 / \varepsilon_1$, and $\varepsilon_1$ and $\varepsilon_2$ are tensile strains along $X_1$ and $X_2$ axes respectively.\textsuperscript{51}

Therefore, course spacing $y_{1c}$ and wale spacing $y_{2c}$ at the critical-tensile state are obtained by solving eqs. (1) to (4) for $y_{1c}$ and $y_{2c}$. The critical-tensile ratios $\lambda_{c1}$ and $\lambda_{c2}$ may be calculated by the following equation:

\begin{align*}
\lambda_{c1} &= y_{1c} / y_{01} \\
\lambda_{c2} &= y_{2c} / y_{02}
\end{align*}

As is seen in the previous paper, the graphical method is convenient for obtaining $y_{1c}$ and $y_{2c}$. That is, after putting $L = l/2 - (3.5\pi + 1)D$, $y_{1c}$, $y_{2c}$, and $D$ are made into
no dimension by dividing them by $L$. The crossing point of a curve, $y_1c/L$ versus $y_2c/L$ drawn from eqs. (1) and (2), where $D/L$ is taken as a parameter, and a straight line of eq. (4) gives the course spacing and the wale spacing of no dimension in critical-tensile state.

2-1-2 Properties in yarn stretching-effective region.

In the region stretched beyond the critical tensile ratios, the biaxial-tensile properties of the fabric are introduced similar to the previous paper[4], in which it is assumed that yarn is perfectly flexible and stretched, and that the biaxial-tensile properties of fabric are mainly governed by the tensile properties of yarn.

![Fig. 3 Forces acting on yarn in stretching-effective region](image)

In this region, course spacing $y_{lc}$ and wale spacing $y_{2c}$ in the critical-tensile state are stretched with tensile ratios $\lambda_{1i}$ ($=\lambda_1/\lambda_{2i}$) and $\lambda_{2i}$ ($=\lambda_2/\lambda_{2i}$). Forces acting on yarn is shown in Fig. 3. Let $\mu$ be the frictional coefficient between yarns. Then, the relation between the tension of sinker loop, $T_{1f}$, and that of needle loop, $T_{mf}$, of front bar yarn, and the relation between the tension of sinker loop, $T_{1b}$, and that of needle loop, $T_{mb}$, of back bar yarn must satisfy the following equations:

$$
\frac{T_{1f}}{T_{mf}} \leq e^{\mu} \quad \text{and} \quad \frac{T_{1f}}{T_{mf}} \leq e^{\mu^*}
$$

$$
\frac{T_{1b}}{T_{mb}} \leq e^{\mu} \quad \text{and} \quad \frac{T_{1b}}{T_{mb}} \leq e^{\mu^*}
$$

Considering the equilibrium of forces in $X_2$ direction, the component of tension $T_{1f}$ is equal to that of tension $T_{1b}$, because the component of tension $T_{mf}$ of needle loop of front bar yarn is assumed equal to that of tension $T_{mb}$ of back bar yarn.

Therefore,

$$
T_{1f} \cos \varphi_{1f}(\lambda_{1i}, \lambda_{2i}) = T_{1b} \cos \varphi_{1b}(\lambda_{1i}, \lambda_{2i})
$$

where $\varphi_{1f}$ is the angle between the direction of $T_{1f}$ and $X_2$ axis, $\varphi_{1b}$ being that between $T_{1b}$ and $X_2$ axis. Both are obtained from

$$
\cos \varphi_{1f}(\lambda_{1i}, \lambda_{2i}) = \lambda_{2i}(n_f y_2c - D)/L_{1f}(\lambda_{1i}, \lambda_{2i})
$$

$$
\cos \varphi_{1b}(\lambda_{1i}, \lambda_{2i}) = \lambda_{1i}(n_b y_{2c} - D)/L_{1b}(\lambda_{1i}, \lambda_{2i})
$$

$$
L_{1f}(\lambda_{1i}, \lambda_{2i}) = \sqrt{\lambda_{2i}^2(n_f y_2c - D)^2 + (\lambda_{1i} y_2c - D)^2 + A^2(\lambda_{1i})}
$$

$$
L_{1b}(\lambda_{1i}, \lambda_{2i}) = \sqrt{\lambda_{1i}^2(n_b y_{2c} - D)^2 + (\lambda_{2i} y_{2c} - D)^2 + A^2(\lambda_{2i})}
$$

$$
A(\lambda_{1i}) = 2.5D^2/(\lambda_{1i} y_{2c} - D)
$$

If $n_f$ is not equal to $n_b$, it generally follows that $T_{1b} \neq T_{1f}$ and $\varphi_{1f} \neq \varphi_{1b}$. Therefore, it is usually impossible to obtain values of $T_{1f}$, $T_{mf}$, $T_{mb}$ and $T_{mb}$ which satisfy eqs. (6) and (7) at the same time. Actually, it is presumed that the needle loops of both front and back bar yarns are inclined a little in the same direction so as to balance the force components in course direction. But this inclined angle is neglected here for brevity because it is very small.

The tensile force per unit structure in course direction, $[F_{2s}]$, is assumed to be a mean value of components of $T_{1f}$ and $T_{1b}$ in $X_2$ axis direction, namely,

$$
[F_{2s}] = (T_{1f} \cos \varphi_{1f}(\lambda_{1i}, \lambda_{2i}) + T_{1b} \cos \varphi_{1b}(\lambda_{1i}, \lambda_{2i}))/2
$$

here, $T_{1f}$, $T_{mf}$, $T_{1b}$ and $T_{mb}$ are determined from eq. (6) by the method described later.

![Fig. 4 Layer construction of unit structure](image)

The fabric can be considered to be the layer of $(n_f + n_b)$ sheets of unit structure piled up as shown in Fig. 4. It is assumed that there are no mechanical interactions between each of layers. Then, the tensile force per course, $F_{s2}$, is obtained after multiplying $[F_{2s}]$ by $(n_f + n_b)$.

$$
F_{s2} = [F_{2s}] \times (n_f + n_b) = (T_{1f} \cos \varphi_{1f}(\lambda_{1i}, \lambda_{2i}) + T_{1b} \cos \varphi_{1b}(\lambda_{1i}, \lambda_{2i}))(n_f + n_b)/2
$$

Tensile force per wale, $F_{s1}$, is equal to the total of components of yarn tension in unit structure in $X_1$ direction.

$$
F_{s1} = (\lambda_{1i} y_1c - D)(T_{1f}/L_{1f}(\lambda_{1i}, \lambda_{2i}) + T_{1b}/L_{1b}(\lambda_{1i}, \lambda_{2i}) + 2T_{mf}/L_{1f}(\lambda_{1i}, \lambda_{2i}) + 2T_{mb}/L_{1b}(\lambda_{1i}, \lambda_{2i}))
$$

where

$$
L_{1f}(\lambda_{1i}, \lambda_{2i}) = \sqrt{(\lambda_{1i} y_1c - D)^2 + (0.5\lambda_{2i}^2 + \lambda_{2i})D^2 + 3.5D^2}
$$

$$
L_{1b}(\lambda_{1i}, \lambda_{2i}) = \sqrt{(\lambda_{1i} y_1c - D)^2 + (0.5\lambda_{2i}^2 + \lambda_{2i})D^2 + 7.5D^2}
$$

The yarn tensions $T_{1f}$, $T_{mf}$, $T_{1b}$ and $T_{mb}$ are calculated by the following procedure.

Yarn tension $T$ is given as a function of the tensile ratio of yarn, $\lambda_y$, that is,
Let the tensile ratios corresponding to $T_{1f}$, $T_{mf}$, $T_{1b}$ and $T_{mb}$ be $\lambda_{1f}$, $\lambda_{mf}$, $\lambda_{1b}$ and $\lambda_{mb}$, respectively. For given tensile ratios $\lambda_{1}$ and $\lambda_{2}$, are obtained $\lambda_{1f}$ and $\lambda_{1b}$ by

$$
\lambda_{1f} = \frac{L_{1f}(\lambda_{1}, \lambda_{2}) + (\pi - 1)D - A(\lambda_{1})}{L_{1f} + (\pi - 1)D - A_{i}} \quad \text{and} \quad \lambda_{1b} = \frac{L_{1b}(\lambda_{1}, \lambda_{2}) + (\pi - 1)D - A(\lambda_{1})}{L_{1b} + (\pi - 1)D - A_{i}}
$$

Also, $\lambda_{mf}$ and $\lambda_{mb}$ are

$$
\lambda_{mf} = \frac{L_{mf}(\lambda_{1}, \lambda_{2}) + \pi \lambda_{D}D + (2.5\pi + 1)D}{L_{mf} + (2.5\pi + 2)D} \quad \text{and} \quad \lambda_{mb} = \frac{L_{mb}(\lambda_{1}, \lambda_{2}) + \pi \lambda_{D}D + (2.5\pi + 2)D}{L_{mb} + (2.5\pi + 2)D}
$$

Then, the yarn tensions $T_{1f}, T_{mf}, T_{1b}$ and $T_{mb}$ are obtained by substituting these yarn-tensile ratios into eq. (14) respectively.

In the equilibrium condition of forces, however, the values of $T_{1f}$ and $T_{mf}$ must satisfy the following relation:

$$
T_{1f}/T_{mf} \leq e_{Fm} \quad \text{and} \quad T_{mf}/T_{1f} \leq e_{Fm} \quad \ldots \ldots \quad (17)
$$

Unless eq. (17) is satisfied, the slippage of front-bar yarn will occur at the contact point. In this case, the next procedure is required to get $T_{1f}$ and $T_{mf}$. First, $L_{1f}$ and $L_{2f}$ are calculated.

$$
T_{1f}/T_{mf} = e_{Fm} \quad \text{or} \quad T_{mf}/T_{1f} = e_{Fm} \quad \ldots \ldots \quad (18)
$$

Then, $T_{1f}$ and $T_{mf}$ corresponding to $L_{1f}$ and $L_{2f}$ give the values which are wanted.

Same procedures are repeated for back-bar yarn, and $T_{1b}$ and $T_{mb}$ can be obtained.

Since the function $T = f(\lambda_{2})$ is non-linear in general, the analytical solution for the these values cannot easily be obtained. The graphical method adopted in the previous paper is useful to determine these values.

### 2-1-3 Properties in bending-effective region

In the bending-effective region, from $\lambda_{i1} = 1$ to $\lambda_{i} = \lambda_{ci}$ $(i = 1,2)$, it is assumed that the arc of elastic yarn is stretched as a built-in beam during the deformation and the yarn itself is not elongated.

The length of the arc in undeformed state is assumed to be the same as that of the linear part in the structure in the critical state where $k = 1$. Thus, the length of the arc, $L'_{of}$, and the distance between two ends of the arc of sinker loop of front-bar yarn, $d'_{of}$, are obtained by

$$
L'_{of} = L_{of} - 2D
$$

$$
d'_{of} = \sqrt{(y_{01} - D)^2 + (n_{f}y_{02} - D)^2 + A_{0}^2} \quad \ldots \ldots \quad (19)
$$

where

$$
A_{0} = 2.5D^2/(y_{01} - D)
$$

When the fabric is stretched by $\lambda_{1}$ and $\lambda_{2}$, the distance $d'_{of}$ becomes $d'_{f}$ such that

$$
d'_{f}(\lambda_{1}, \lambda_{2}) = \sqrt{(\lambda_{1}y_{01} - D)^2 + (\lambda_{2}y_{02} - D)^2 + A^2(\lambda_{1})} \quad \ldots \ldots \quad (20)
$$

The lengths $L'_{of}, d'_{of}$ and $d'_{f}$ for needle loop of front-bar yarn have already been obtained in the previous paper. Therefore, forces applied at the arc-ends, $P'_{f}$ and $P_{f}$, for the sinker loop and the needle loop of front-bar yarn respectively are calculable by using eqs. (26) and (31) in the previous paper where $d', d_{o}$ and $L_{o}$ have to be replaced by $d'_{f}, d'_{of}$ and $L'_{of}$ in this paper.

For back-bar yarn, we have,

$$
d'_{b}(\lambda_{1}, \lambda_{2}) = \sqrt{(\lambda_{1}y_{01} - D)^2 + (\lambda_{2}y_{02} - D)^2 + A_{b}^2(\lambda_{1})} \quad \ldots \ldots \quad (21)
$$

$$
d''_{b}(\lambda_{2}) = \sqrt{(\lambda_{2}y_{02} - D)^2 + 9D^2}
$$

$$
d'_{b} = [d'_{(\lambda_{1}, \lambda_{2})}]_{1 \rightarrow 2} \quad \ldots \ldots \quad (22)
$$

$$
L'_{ab} = L_{ab} - 2D
$$

$$
L''_{ab} = L_{ab}
$$

Forces $P'_{b}$ and $P_{b}$ applied on the arc-ends of back-bar yarn can be obtained by substituting the above obtained values from $d'_{b}$ to $L''_{ab}$ into eqs. (29) and (30) in the previous paper.

In case of

$$
P'_{f}/P'_{b} > e_{Fn} \quad \text{or} \quad P''_{f}/P'_f > e_{Fn}
$$

$$
P'_{f}/P''_{f} > e_{Fn} \quad \text{or} \quad P''_{f}/P'_f > e_{Fn}
$$

they are to move through the crossing point. This is the same as the case in stretching-effective region. In this case, $P_{f}, P''_{f}, P_{b}$ and $P''_{b}$ are determined by the same method as already described in obtaining $T_{1f}, T_{mf}, T_{1b}$ and $T_{mb}$.

The tensile load per wale in wale direction, $F_{1}$, and the tensile load per course in course direction, $F_{2}$, are obtained by following equations, similar to the case for calculating stretching-effective region.

$$
F_{1} = (\lambda_{y_{01}} - D)[P''_{f}/d'_{f}(\lambda_{1}, \lambda_{2}) + 2D + 2P''_{f}/d'_{f}(\lambda_{1}, \lambda_{2})] + 2P''_{f}/d'_{f}(\lambda_{1}, \lambda_{2}) + 2D]
$$

$$
F_{2} = (\lambda_{y_{02}} - D)[m + n_{c}][P''_{f}/d'_{f}(\lambda_{1}, \lambda_{2}) + 2D + 2P''_{f}/d'_{f}(\lambda_{1}, \lambda_{2}) + 2D]
$$

### 2-2 Effect of Compressive Property of Yarn

When $n_{f}$ or $n_{b}$ or both become large, the fabric's extensibility decreases. And its tensile properties tend to be greatly affected by small change in yarn diameter. As shown in the previous paper, for high-density fabrics, it is observed that the values of the tensile properties theoretically obtained become larger than those measured experimentally at high tensile ratio. This difference between theoretical and measured values may be derived from the effect of yarn thickness. From this consideration, the theory can be improved to be more precise by introducing the compressive property of yarn in the calculation for stretching-

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effective region where the compressive force is larger than that in the bending effective region.

The structure around the crossover point of yarns at high tensile ratios is comparatively simple as shown in Fig. 5. If yarn is compressed in every direction the flatteness of the yarn will be small. On the other hand, under uni-directional force, the flattening will occur at the place where the yarn is compressed. The places are shown in Fig. 6 as hatched areas a, b, c, d, e and f.

The amount of deformation of compressed yarn in these places is not exactly equal, because the yarn tension in each place is generally different. But, for simplicity, the amount of compressive deformation is calculated on the assumption that all tension is equal to the mean value.

The model of compressed yarn is shown in Fig. 7, where the yarn under tension \( T \) is wound around a rod having a diameter \( D \). Let \( D_{pf} \) and \( D_{pb} \) be the thickness of the flattened yarn of front-bar and of back-bar respectively, at given tensile ratios \( \lambda_{s1} \) and \( \lambda_{s2} \). Then, the lengths of front-bar yarn, \( S_f \), and of back-bar yarn, \( S_b \), in unit structure, are given by

\[
\begin{align*}
S_f(\lambda_{s1}, \lambda_{s2}, D_{pf}) &= B_f(\lambda_{s1}, D_{pf}) + C_f(D_{pf}) \\
& - A(\lambda_{s1}, D_{pf}) + L_f(\lambda_{s1}, \lambda_{s2}, D_{pf}) \\
& + 2L_f(\lambda_{s1}, \lambda_{s2}, D_{pf}) \\
S_b(\lambda_{s1}, \lambda_{s2}, D_{pb}) &= B_b(\lambda_{s1}, D_{pb}) + C_b(D_{pb}) \\
& - A(\lambda_{s1}, D_{pb}) + L_b(\lambda_{s1}, \lambda_{s2}, D_{pb}) \\
& + 2L_b(\lambda_{s1}, \lambda_{s2}, D_{pb})
\end{align*}
\]

where

\[
\begin{align*}
B_f(\lambda_{s1}, D_{pf}) &= (\lambda_{s1} + 1)D + (D + 1.5D_{pf})
\pi \\
B_b(\lambda_{s1}, D_{pb}) &= (\lambda_{s1} + 1)D + (D + 1.5D_{pb})
\pi \\
C_f(D_{pf}) &= 0.5\pi(D + D_{pf}) - D \\
C_b(D_{pb}) &= 0.5\pi(D + D_{pb}) - D
\end{align*}
\]

\[
\begin{align*}
L_f(\lambda_{s1}, \lambda_{s2}, D_{pf}) &= \sqrt{(\lambda_{s1}y_{pf} - D)^2 + \lambda_{s2}^2(n_{pf}y_{pf} - D)^2 + A^2(\lambda_{s1}, D_{pf})} \\
L_b(\lambda_{s1}, \lambda_{s2}, D_{pb}) &= \sqrt{(\lambda_{s1}y_{pb} - D)^2 + \lambda_{s2}^2(n_{pb}y_{pb} - D)^2 + A^2(\lambda_{s1}, D_{pb})}
\end{align*}
\]

\[
\begin{align*}
A(\lambda_{s1}, D_{pf}) &= (1.5D + D_{pf})D/(\lambda_{s1}y_{pf} - D) \\
A(\lambda_{s1}, D_{pb}) &= (1.5D + D_{pb})D/(\lambda_{s1}y_{pb} - D)
\end{align*}
\]

As \((\lambda_{s1}y_{pf} - D)^2\) in the root sign of the second or the fourth formula in eq. (29) is much larger than the other terms, we can use eq. (13) instead of the above equations for calculating \( L_{sf} \) and \( L_{sb} \). That is, \( L_{sf}(\lambda_{s1}, \lambda_{s2}, D_{pf}) \approx L_f(\lambda_{s1}, \lambda_{s2}) \) and \( L_{sb}(\lambda_{s1}, \lambda_{s2}, D_{pb}) \approx L_b(\lambda_{s1}, \lambda_{s2}) \), and also \( L_{1f}(\lambda_{s1}, \lambda_{s2}, D_{pf}) \approx L_1(\lambda_{s1}, \lambda_{s2}) \) and \( L_{1b}(\lambda_{s1}, \lambda_{s2}, D_{pb}) \approx L_1(\lambda_{s1}, \lambda_{s2}) \).

Therefore, mean tensile ratios \( \lambda_{s1f} \) and \( \lambda_{s1b} \) and mean tensions \( T_f \) and \( T_b \) of front and back bar yarn are given respectively by the following equations:

\[
\begin{align*}
\lambda_{s1f} &= S_f(\lambda_{s1}, D_{pf})/l_f \\
\lambda_{s1b} &= S_b(\lambda_{s1}, D_{pb})/l_b
\end{align*}
\]

\[
\begin{align*}
T_f &= f(\lambda_{s1f}) \\
T_b &= f(\lambda_{s1b})
\end{align*}
\]

When the yarn with tensions \( \bar{T}_f \) and \( \bar{T}_b \) is wound around a rod having a diameter \( D \), the compressive pressures \( Q_f \) and \( Q_b \), acting on the contact area of the yarn is obtained respectively by

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\[ Q_f = \frac{2T_f}{D}, \quad Q_b = \frac{2T_b}{D} \]

On the other hand, thickness of compressed yarn, \( D_p \), can be measured as a function of compressive pressure \( Q \).

\[ D_{pf} = g(Q_f) \]
\[ D_{pb} = g(Q_b) \]

Therefore, \( D_{pf} \) and \( D_{pb} \) can be calculated by solving eqs. (33) and (34). But, as eqs. (32) and (34) are generally non-linear, it is difficult to obtain the analytical solution. For this reason, it is convenient to use the graphical method as follows.

Tensile ratios \( \lambda_{1s} \) and \( \lambda_{1b} \) are given and yarn thickness \( D_p \) is measured as a function of pressure \( Q \). At first, the arbitrary yarn thickness \( D_p \) is taken, and \( Q \) corresponding to \( D_p \) is determined from the graph for the compressive properties of yarn. On the other hand, \( Q \) is calculated by substituting \( D_p \) into eqs. (31), (32) and (33). Then, two values of \( Q \) are obtained, one from eq. (34), the other from eqs. (31), (32) and (33). The similar calculation is repeated for different values of \( D_p \), and a graph with two curves is drawn by taking \( D_p \) as abscissa and \( Q \) as ordinate. Thus the required values of \( D_p \) and \( Q \) are obtained from a crossing point of these two curves. This procedure is separately done with regard to both front-bar and back-bar yarn.

By using \( D_{pf} \) and \( D_{pb} \) obtained in this way, the tensile ratios of yarn are calculated from the following equations instead of eqs. (15) and (16) in which the compressive properties of yarn are out of consideration.

\[
\begin{align*}
\lambda_{1f} &= (L_{1f}(\lambda_{1s}, \lambda_{1b}, D_{pf}) + C_f(D_{pf})) - A(\lambda_{1s}, D_{pf}) / (L_{1f} + (\pi - 1)D - A) \\
\lambda_{1b} &= (2L_{1f}(\lambda_{1s}, \lambda_{1b}, D_{pb}) + B_f(\lambda_{1s}, D_{pf})) / (2L_{1f} + (2.5\pi + 2)D) \\
\lambda_{2f} &= (L_{2f}(\lambda_{2s}, \lambda_{2b}, D_{pf}) + C_f(D_{pf})) - A(\lambda_{2s}, D_{pf}) / (L_{2f} + (\pi - 1)D - A) \\
\lambda_{2b} &= (2L_{2f}(\lambda_{2s}, \lambda_{2b}, D_{pb}) + B_f(\lambda_{2s}, D_{pf})) / (2L_{2f} + (2.5\pi + 2)D)
\end{align*}
\]

Thus the yarn tension of each loop corresponding to these tensile ratios gives the biaxial tensile properties of the fabrics.

3. Some Calculations

By using this theory, let us calculate some examples and estimate the effects of various factors on the biaxial tensile properties of two-bar warp knitted fabrics.

Before the theoretical calculation, it is required to fix the structure of the fabric and the mechanical properties of yarn.

Grosberg\(^{[6]}\) presented the following equation for the structural constants.

\[ l = 4.08y_{y1} \sec \theta : A \left( \pi y_{y1} + 2D - 1.07y_{y1} \sec \theta \right)^2 + y_{y1}^{1/4} \]

where

\[ A = \left( \pi / 2 \theta / 2 \right) / \cos \left( \theta / 2 \right) \]
\[ \theta = 2 \tan^{-1} \left( y_{y1} / (y_{y2} + 2D - 1.07y_{y1} \sec \theta) \right) \]

and \( \theta \) is calculated by solving the following formula.

\[ \sin \theta + (D/y_{y1}) \cos \theta = 0.535 \]

<table>
<thead>
<tr>
<th>( y_{y1} )</th>
<th>( y_{y2} )</th>
<th>( D )</th>
<th>( n_f )</th>
<th>( n_b )</th>
<th>( l )</th>
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</table>

Table 1 Structural constants used for calculation

![Fig. 8 Tensile properties of yarn used for calculation](image)

![Fig. 9 Compressive properties of yarn used for calculation](image)

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Fig. 10 Effect of tensile property of yarn (Results in bending-effective region and in stretching-effective region)

Fig. 11 Effect of tensile property of yarn (Synthesized results of properties in two regions)

Fig. 12 Effect of compressive property of yarn ($n_f = 2$, $n_b = 2$, tensile property of yarn used is $W$ in Fig. 8)
are used for examining their effect on the final result. Also, three types of yarn compressive-property are given in Fig. 9.

3-1 Effect of Tensile Property of Yarn

Calculations of the biaxial tensile property of 2 x 2 double cord fabric are shown in Figs. 10 and 11, where four types of tensile property of yarn are used for the calculation. Fig. 10 shows the results in bending-effective region and in stretching-effective region separately, Fig. 11 showing the synthesized results of these two.

These results demonstrate that the properties in stretching-effective are influenced strongly by the tensile property of yarn, and that the properties of the fabric at relatively high tensile ratios depends consequently on the yarn property.

3-2 Effect of Compressive Property of Yarn

Fig. 12 shows the fabric properties corresponding to three types of compressive property of yarn, A, B and C in Fig. 9 where the quantity of compressive deformation of B is only one half of that of A, and the deformation of C is constant.

3-3 Effect of $n_f$ and $n_b$

To make clear the effect of the number of wales crossed by underlapping, $n_f$ or $n_b$, on biaxial tensile properties of the fabric, some theoretical calculation was done by using the values in Table 1. The results are shown in Fig. 13.

Fig. 13 Effect of $n_f$ and $n_b$ (Tensile property of yarn used is X in Fig. 8)

Fig. 14 Effect of frictional coefficient between yarns ($n_f=2$, $n_b=1$)
3-4 Effect of Frictional Coefficient between Yarn

Frictional coefficient of yarn, $\mu$, is assumed to be 0.1, 0.3 and 0.5, and the dependence of tensile property on these frictional property are examined as shown in Fig. 14. For this theoretical calculation, the following constants are used.

$\gamma_0 = 0.2 \text{ mm}$
$\gamma_2 = 0.2 \text{ mm}$
$L = l/2 - (3.5\pi + 1)D = 1 \text{ mm}$
$D = 0.02 \text{ mm}$
$EL = 0.06 \text{ g.mm}^2$

Yarn thickness is assumed constant through the entire stretching process of fabric, and the tensile properties of yarn used in this calculation are shown in the same figure.

From Fig. 14, it is observed that the frictional coefficient increases force $F_2$ but it decreases force $F_1$.

4. Conclusion

The theory developed in the previous paper was expanded to a more general form to calculate the biaxial tensile properties of two bar tricot fabrics.

When the number of wales crossed by underlapping becomes large, or when spun yarn is used, the original theory becomes error-prone. To make the original theory more precise, the compressive property of yarn should be introduced for the calculation in stretching-effective region. In this paper, a simple model where a yarn is wound around a rod is used for the structure of cross-over region.

Finally, some theoretical calculation has been done for various factors such as the tensile and the frictional properties of yarn, the number of wales crossed by underlapping.

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**Literature cited**