Flow Control of Duct System

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Abstract

The following article contains the results of experiments by the authors with two methods of obtaining uniform duct-system distribution of air flow from each branch pipe or of adjusting air flow to specific conditions—without changing the existing duct system.

Method No. 1 starts with the fundamental flow equation and proceeds by stages to finding out unknown factors to be controlled.

Method No. 2, which cannot be described in the scope of this preface, has been shown by the experiments to be simpler and more practical.

As to the duct system with parallel blowers, the authors have reached the conclusion that uniform flow from each branch pipe can be obtained more easily by adjusting the revolutions of blowers than by inserting the control resistance body.

Introduction

To obtain a uniform distribution of temperature and humidity in spinning mills, it is most essential that air flow from each branch pipe in the duct system be uniform. But in any given ducts of the conventional design, the flow is not always distributed uniformly.

We have made a series of practical studies on air flow from the duct system and have published^{(1)~(4)} our findings on previous occasions. In the present article, we shall confine ourselves to a discussion of methods of obtaining uniform distribution of air flow or adjusting it to desired conditions by means of the existing duct system without changing its form.

1. Fundamental Flow Equation by Duct System

Before proceeding to our discussion, we list the nomenclatures used in this article:

- Q, q : flow quantity of duct in main pipe or branch pipe kg/s
- W, w : velocity of duct in main pipe or branch pipe m/s
- P, P_0 : pressure in duct section or outside of pipe line kg/m²
- L, l : length of duct in main pipe or branch pipe m
- D, d : diameter of duct in main pipe or branch pipe m
- F, f : area of duct section in main pipe or branch pipe m²
- ζ_a : resistance coefficient of separate flow or confluent flow from main pipe to main pipe
- ζ_b : resistance coefficient of separate flow or confluent flow from main pipe to branch pipe

ζ : resistance coefficient

K, K' : synthetic resistance coefficient of main pipe

k : synthetic resistance coefficient of branch pipe

k', k'' : resistance coefficient of inserted resistance body to change the flow condition

λ : coefficient of fluid friction

γ : specific weight kg/m³

r : ratio of flow quantity—with the suffixes i meaning inlet; 0, outlet; s , suction or exhaust of pipe of blower.

Suffixes 1, 2, ..., n represent the main or branch pipes number and show the flow condition.

(A) In the Case of Blow Duct

In the typical duct system with branch pipe numbered n as shown in Figure 1, the flow from main pipe of blower to branch pipe n is, generally, as follows:

Fig. 1

$$P_n + \frac{\gamma}{2g} W_n^2 = \lambda \frac{L_n}{D_n} \frac{\gamma}{2g} W_n^2 + \zeta_{bn} \frac{\gamma}{2g} W_n^2 + \lambda \frac{L_n}{d_n} \frac{\gamma}{2g} W_n^2 + \zeta_{on} \frac{\gamma}{2g} W_n^2 + \frac{\gamma}{2g} W_s^2 + P_0 \quad (1)$$

$$P_0 = \zeta_s \frac{\gamma}{2g} W_s^2 + \lambda \frac{L_s}{D_s} \frac{\gamma}{2g} W_s^2 + P_s + \frac{\gamma}{2g} W_s^2 \quad (2)$$

$$P_n - P_s = \left\{ \lambda \frac{L_n}{D_n} + \zeta_{bn} + \left(1 + \lambda \frac{L_s}{D_s} + \zeta_s \right) \left(\frac{W_n}{W_s} \right)^2 - 1 \right\} \frac{\gamma}{2g} W_n^2 + \left(\lambda \frac{L_n}{d_n} + \zeta_{on} + 1 \right) \frac{\gamma}{2g} W_n^2 \dots \dots \dots (3)$$

However, from the equation of continuous flow, we have

$$W_n F_n \gamma = Q_n, \quad w_n f_n \gamma = q_n \dots \dots \dots (4)$$

$$\begin{aligned} W_s F_s \gamma = Q_n \quad \therefore \quad \left(\frac{W_s}{W_n} \right)^2 &= \left(\frac{F_n}{F_s} \right)^2 \\ \left\{ \lambda \frac{L_n}{D_n} + \zeta_{bn} + \left(1 + \lambda \frac{L_s}{D_s} + \zeta_s \right) \left(\frac{F_n}{F_s} \right)^2 - 1 \right\} \frac{1}{2g\gamma} \left(\frac{1}{F_n} \right)^2 &= K_n \\ \left(\lambda \frac{L_n}{d_n} + \zeta_{on} + 1 \right) \frac{1}{2g\gamma} \left(\frac{1}{f_n} \right)^2 &= k_n \end{aligned} \dots \dots \dots (5)$$

$$\therefore P_n - P_s = K_n Q_n^2 + k_n q_n^2 \dots \dots \dots (6)$$

With the flow of blow $b(n-1)$ blower, we have

$$\begin{aligned} P_n + \frac{\gamma}{2g} W_n^2 &= \lambda \frac{L_n}{D_n} \frac{\gamma}{2g} W_n^2 + \zeta_{dn} \frac{\gamma}{2g} W_n^2 + \lambda \frac{L_{(n-1)}}{D_{(n-1)}} \frac{\gamma}{2g} W_{(n-1)}^2 + \zeta_{b(n-1)} \frac{\gamma}{2g} W_{(n-1)}^2 + \lambda \frac{L_{(n-1)}}{d_{(n-1)}} \frac{\gamma}{2g} w_{(n-1)}^2 \\ &\quad + \frac{\gamma}{2g} w_{(n-1)}^2 + \zeta_{o(n-1)} \frac{\gamma}{2g} w_{(n-1)}^2 + \zeta_s \frac{\gamma}{2g} W_n^2 + \lambda \frac{L_s}{D_s} \frac{\gamma}{2g} W_s^2 + P_s + \frac{\gamma}{2g} W_s^2 \\ \therefore P_n - P_s &= \left\{ \lambda \frac{L_n}{D_n} + \zeta_{dn} + \left(1 + \lambda \frac{L_s}{D_s} + \zeta_s \right) \left(\frac{F_n}{F_s} \right)^2 - 1 \right\} \frac{\gamma}{2g} W_n^2 + \left(\lambda \frac{L_{(n-1)}}{D_{(n-1)}} + \zeta_{b(n-1)} \right) \frac{\gamma}{2g} W_{(n-1)}^2 \\ &\quad + \left(\lambda \frac{L_{(n-1)}}{d_{(n-1)}} + \zeta_{o(n-1)} + 1 \right) \frac{\gamma}{2g} w_{(n-1)}^2 \dots \dots \dots (7) \end{aligned}$$

$$\left\{ \begin{aligned} \left\{ \lambda \frac{L_n}{D_n} + \zeta_{dn} + \left(1 + \lambda \frac{L_s}{D_s} + \zeta_s \right) \left(\frac{F_n}{F_s} \right)^2 - 1 \right\} \frac{1}{2g\gamma} \left(\frac{1}{F_n} \right)^2 &= K'_n \\ \left(\lambda \frac{L_{(n-1)}}{D_{(n-1)}} + \zeta_{b(n-1)} \right) \frac{1}{2g\gamma} \left(\frac{1}{F_{(n-1)}} \right)^2 &= K_{(n-1)} \\ \left(\lambda \frac{L_{(n-1)}}{d_{(n-1)}} + \zeta_{o(n-1)} + 1 \right) \frac{1}{2g\gamma} \left(\frac{1}{f_{(n-1)}} \right)^2 &= k_{(n-1)} \end{aligned} \right\} \dots \dots \dots (8)$$

$$\therefore P_n - P_s = K'_n Q_n^2 + K_{(n-1)} Q_{(n-1)}^2 + k_{(n-1)} q_{(n-1)}^2 \dots \dots \dots (9)$$

Accordingly, if the relation $(P_n - P_s)$ is equal to the blower pressure P , general flow equation may be expressed as following:

$$P = \sum_{m=1}^n K'_m Q_m^2 + K_m Q_m^2 + k_m q_m^2 \dots \dots \dots (10)$$

where

$$\left\{ \begin{aligned} K_m &= \left(\lambda \frac{L_m}{D_m} + \zeta_{bm} \right) \frac{1}{2g\gamma} \left(\frac{1}{F_m} \right)^2 \\ K'_m &= \left(\lambda \frac{L_m}{D_m} + \zeta_{dm} \right) \frac{1}{2g\gamma} \left(\frac{1}{F_m} \right)^2, \quad m < n \\ k_m &= \left(\lambda \frac{L_m}{d_m} + \zeta_{om} + 1 \right) \frac{1}{2g\gamma} \left(\frac{1}{f_m} \right)^2 \end{aligned} \right.$$

(B) In the Case of Suction Duct.

Following the same manner as (A), we have the following relative equations:

$$\begin{aligned} P_n + \frac{\gamma}{2g} W_n^2 &= P_0 - \lambda \frac{L_n}{D_n} \frac{\gamma}{2g} W_n^2 - \zeta_{bn} \frac{\gamma}{2g} W_n^2 \\ &\quad - \lambda \frac{L_n}{d_n} \frac{\gamma}{2g} w_n^2 - \zeta_{in} \frac{\gamma}{2g} w_n^2 \dots \dots \dots (11) \end{aligned}$$

$$\begin{aligned} P_s + \frac{\gamma}{2g} W_s^2 &= \zeta_s \frac{\gamma}{2g} W_s^2 + \lambda \frac{L_s}{D_s} \frac{\gamma}{2g} W_s^2 \\ &\quad + P_0 + \frac{\gamma}{2g} W_s^2 \dots \dots \dots (12) \end{aligned}$$

$$\begin{aligned} \therefore P_s - P_n &= \left\{ \lambda \frac{L_n}{D_n} + \zeta_{bn} + \left(\lambda \frac{L_s}{D_s} + \zeta_s \right) \left(\frac{F_n}{F_s} \right)^2 \right. \\ &\quad \left. + 1 \right\} \frac{\gamma}{2g} W_n^2 + \left(\lambda \frac{L_n}{d_n} + \zeta_{in} \right) \frac{\gamma}{2g} w_n^2 \dots (13) \end{aligned}$$

and also

$$\begin{aligned} P_s - P_n &= \left\{ \lambda \frac{L_n}{D_n} + \zeta_{dn} + \left(\lambda \frac{L_s}{D_s} + \zeta_s \right) \left(\frac{F_n}{F_s} \right)^2 \right. \\ &\quad \left. + 1 \right\} \frac{\gamma}{2g} W_n^2 + \left\{ \lambda \frac{L_{(n-1)}}{D_{(n-1)}} + \zeta_{b(n-1)} \right\} \frac{\gamma}{2g} W_{(n-1)}^2 \\ &\quad + \left(\lambda \frac{L_{(n-1)}}{d_{(n-1)}} + \zeta_{o(n-1)} \right) \frac{\gamma}{2g} w_{(n-1)}^2 \dots \dots (14) \end{aligned}$$

$$\left\{ \begin{aligned} K_m &= \left(\lambda \frac{L_m}{D_m} + \zeta_{bm} \right) \frac{1}{2g\gamma} \left(\frac{1}{F_m} \right)^2 \\ K'_m &= \left(\lambda \frac{L_m}{D_m} + \zeta_{dm} \right) \frac{1}{2g\gamma} \left(\frac{1}{F_m} \right)^2, \quad m < n \\ k_m &= \left(\lambda \frac{L_m}{d_m} + \zeta_{in} \right) \frac{1}{2g\gamma} \left(\frac{1}{f_m} \right)^2 \end{aligned} \right\} \dots (15)$$

By using (15), the equations of (13), (14) are obtainable in the same form as equation (10).

2. Uniform Distribution of Branch Pipe Flow (which we will call Method No.1)

In any given duct system having unknown

pressure loss coefficient, we assume that the following are already known:

- (1) Flow rate of branch pipe, that is, K_m , K'_m are unknown and k_m , q_m known.
- (2) Performance of blower is known and the relation between P and Q_n is also known. And when each branch pipe flow is uniformly distributed, resistance values Δk to be inserted are as follows:

$$q_1 = q_2 = \dots = q_{(n-1)} = q \equiv q \dots\dots\dots (16)$$

$$\begin{aligned} P &= K_n Q_n^2 + k_n q^2 + \Delta k_n q^2 \\ &= K'_n Q_n^2 + K_{(n-1)} Q_{(n-1)}^2 + k_{(n-1)} q^2 \\ &\quad + \Delta k_{(n-1)} q^2 = \dots\dots\dots \\ &= \sum_{m=1}^n K'_m Q_m^2 + K_m Q_m^2 + k_m q^2 \\ &\quad + \Delta k_m q^2 = \dots\dots\dots \\ &= \sum_{m=1}^n K'_m Q_m^2 + K_1 Q_1^2 + k_1 q^2 + \Delta k_1 q^2 \end{aligned} \dots\dots\dots (17)$$

And we have following equations for the flow condition 1, 2 and 3.

$$\begin{aligned} P_1 &= \sum_{m=1}^n K'_{m1} Q_{m1}^2 + K_{m1} Q_{m1}^2 + k_{m1} q_{m1}^2 \\ P_2 &= \sum_{m=1}^n K'_{m2} Q_{m2}^2 + K_{m2} Q_{m2}^2 + k_{m2} q_{m2}^2 + k'_{m2} q_{m2}^2 \\ P_3 &= \sum_{m=1}^n K'_{m3} Q_{m3}^2 + K_{m3} Q_{m3}^2 + k_{m3} q_{m3}^2 + k'_{m3} q_{m3}^2 \end{aligned} \dots\dots\dots (18)$$

$$\begin{aligned} P_{m1} &= K_{m1} Q_{m1}^2 + k_{m1} q_{m1}^2 \\ P_{m2} &= K_{m2} Q_{m2}^2 + k_{m2} q_{m2}^2 + k'_{m2} q_{m2}^2 \\ P_{m3} &= K_{m3} Q_{m3}^2 + k_{m3} q_{m3}^2 + k'_{m3} q_{m3}^2 \end{aligned} \dots\dots\dots (19)$$

By expanding K_m and $\sum_{m=1}^n K'_m$ in the above equation (18) into Taylor series under condition 1 and neglecting terms higher than the second order, the following equation is derived:

$$\begin{aligned} K_m, \sum_{m=1}^n K'_m &= f(r_{m1}) + \left(\frac{df}{dr_m} \right)_{r_m=r_{m1}} (r_{m1} - r_{m1}) \\ &\quad + \frac{1}{2!} \left(\frac{d^2f}{dr_m^2} \right)_{r_m=r_{m1}} (r_m - r_{m1})^2 \dots\dots\dots (20) \end{aligned}$$

By using the above equations, $(df/dr_m)_{r_m=r_{m1}}$, $(d^2f/dr_m^2)_{r_m=r_{m1}}$ are calculated, so the value of K_m , $\sum_{m=1}^n K'_m$ in the uniform distribution is evaluated and controlling resistance value to be inserted by substitution of this value in equation (17) can be arrived at.

However, this method requires not only the value of static pressure in branch pipe, but also the value of static pressure in each main pipe section.

3. Uniform Distribution of Branch Pipe Flow (by which we will call Method No. 2)

$$\begin{aligned} P &= \sum_{m=1}^n K'_m Q_m^2 + K_m Q_m^2 + k_m q_m^2 = (K_m) Q_n^2 \\ &\quad + k_m q_m^2 \dots\dots\dots (21) \end{aligned}$$

As shown in the above relation, we assume that blower pressuer P consists of two parts: one, branch pipe term with constant factor; the other, total flow term of main pipe with variable factor.

By figuring this relation in terms of the uniform flow condition, we arrive at the following formula:

$$\begin{aligned} P &= (K_n) Q_n^2 + (k_n + \Delta k_n) q^2 \quad (a) \\ &= (K_{(n-1)}) Q_n^2 + (k_{(n-1)} + \Delta k_{(n-1)}) q^2 = \dots (b) \\ &= (K_m) Q_n^2 + (k_m + \Delta k_m) q^2 = \dots\dots\dots (m) \\ &= (K_2) Q_n^2 + (k_2 + \Delta k_2) q^2 \quad (s) \\ &= (K_1) Q_n^2 + (k_1 + \Delta k_1) q^2 \quad (t) \end{aligned} \dots\dots\dots (22)$$

If we place (a)=(b) together in the above equation

$$\begin{aligned} (K_n) Q_n^2 + (k_n + \Delta k_n) q^2 &= (K_{(n-1)}) Q_n^2 \\ &\quad + (k_{(n-1)} + \Delta k_{(n-1)}) q^2 \\ \therefore \{ (k_{(n-1)} - k_n) + (\Delta k_{(n-1)} - \Delta k_n) \} \\ &= \{ (K_n) - (K_{(n-1)}) \} \left(\frac{Q_n}{q} \right)^2 \end{aligned}$$

and also

$$Q_n = nq \quad \therefore \left(\frac{Q_n}{q} \right)^2 = n^2$$

Accordingly

$$\begin{aligned} \{ (k_n - k_{(n-1)}) + (\Delta k_n - \Delta k_{(n-1)}) \} \\ &= \{ (K_{(n-1)}) - (K_n) \} n^2 \\ \{ (k_{(n-1)} - k_{(n-2)}) + (\Delta k_{(n-1)} - \Delta k_{(n-2)}) \} \\ &= \{ (K_{(n-2)}) - (K_{(n-1)}) \} n^2 \dots\dots\dots \\ \{ (k_m - k_{(m-1)}) + (\Delta k_m - \Delta k_{(m-1)}) \} \\ &= \{ (K_{(m-1)}) - (K_m) \} n^2 \dots\dots\dots \\ \{ (k_2 - k_1) + (\Delta k_2 - \Delta k_1) \} \\ &= \{ (K_1) - (K_2) \} n^2 \end{aligned} \dots\dots\dots (23)$$

If each branch pipe is the same, then:

$$k_1 = k_2 = \dots = k_m = \dots = k_{(n-1)} = k_n \equiv k$$

Accordingly, resistance coefficient to be inserted is as follows:

$$\Delta k_m - \Delta k_{(m-1)} = \{ (K_{(m-1)}) - (K_m) \} n^2 \dots (24)$$

$$(\zeta_m) = \sum_{i=1}^m (\Delta k_m - \Delta k_{(m-1)}) \dots\dots\dots (25)$$

$$\Delta P_m = (\zeta_m) q^2 = \zeta_m \frac{1}{2g\gamma} \left(\frac{1}{f_m} \right)^2 q^2 \dots\dots\dots (26)$$

Also, if we use an orifice or nozzle as the controlling resistance body^[5]

$$\zeta_m = \left(\frac{1}{m'} - 1 \right) \left(\frac{2.75}{m'} - 1.56 \right) \dots (\text{orifice})$$

$$\zeta_m = \left(\frac{1}{m'} - 1 \right) \left(\frac{1.04}{m'} - 1 \right) \dots\dots\dots (\text{nozzle}) \dots\dots\dots (27)$$

where:

$$m' = \left(\frac{d}{D}\right)^2, d = \text{diameter of orifice or nozzle.} \\ D = \text{diameter of branch pipe.}$$

Generally, (K_m) is the function of $r = (q_m/Q_m)$. By Taylor expansion based on the present condition 1, (K_m) may be expressed thus:

$$(K_m) = f(r_1) + \left(\frac{df}{dr}\right)_{r=r_1} (r-r_1) \\ + \frac{1}{2!} \left(\frac{d^2f}{dr^2}\right)_{r=r_1} (r-r_1)^2 + \dots \dots \dots (28) \\ + \frac{1}{3!} \left(\frac{d^3f}{dr^3}\right)_{r=r_1} (r-r_1)^3 + \dots \dots \dots$$

In the condition of uniform distribution of flow, the value of $r = q_m/Q_m$ equals $1/n$, therefore we have:

$$(K_m)_{r=\frac{1}{n}} = f(r_1) + \left(\frac{df}{dr}\right)_{r=r_1} \left(\frac{1}{n} - r_1\right) \\ + \frac{1}{2!} \left(\frac{d^2f}{dr^2}\right)_{r=r_1} \left(\frac{1}{n} - r_1\right)^2 \\ + \frac{1}{3!} \left(\frac{d^3f}{dr^3}\right)_{r=r_1} \left(\frac{1}{n} - r_1\right)^3 + \dots \dots \dots (29)$$

(A) First Approximation

$$(K_m)_{r=\frac{1}{n}} = f(r_1) + \left(\frac{df}{dr}\right)_{r=r_1} \left(\frac{1}{n} - r_1\right) \quad (30)$$

$$P_2 = (K_m)_2 Q_{n2}^2 + (k_m + k'_{m2}) q_m^2 \dots \dots \dots (31)$$

$$(K_m)_2 = f(r_1) + \left(\frac{df}{dr}\right)_{r=r_1} (r_2 - r_1)$$

$$\therefore \left(\frac{df}{dr}\right)_{r=r_1} = \frac{(K_m)_2 - f(r_1)}{(r_2 - r_1)} \dots \dots \dots (32)$$

With the value of (K_m) known as above, the controlling resistance value can be figured out.

(B) Second Approximation

$$(K_m)_{r=\frac{1}{n}} = f(r_1) + \left(\frac{df}{dr}\right)_{r=r_1} \left(\frac{1}{n} - r_1\right) \\ + \frac{1}{2!} \left(\frac{d^2f}{dr^2}\right)_{r=r_1} \left(\frac{1}{n} - r_1\right)^2 \dots \dots \dots (33)$$

$$P_3 = (K_m)_3 Q_{n3}^2 + (k_m + k''_{m3}) q_m^2 \dots \dots \dots (34)$$

$$(K_m)_3 = f(r_1) + \left(\frac{df}{dr}\right)_{r=r_1} (r_3 - r_1) \\ + \frac{1}{2!} \left(\frac{d^2f}{dr^2}\right)_{r=r_1} (r_3 - r_1)^2 \dots \dots \dots (35)$$

In the same manner as above-mentioned, $(df/dr)_{r=r_1}$, $(d^2f/dr^2)_{r=r_1}$ are known; therefore, K_m can be figured out.

Figure 2 given below shows the test piping which consists of a main pipe having 43 mm diameter and a branch pipe 25 mm in diameter and 508 mm long with suction hood and 1007 mm pitch.

This pipe is suction type system as a whole and connects with 30 HP. vacuum pump. There is a measuring nozzle between the pipe and the vacuum pump. A controlling resistance orifice can be inserted in the flange in the middle of each branch pipe. The flow of each branch pipe is figured out from the pressure difference at a distance of 120 mm from the top of the branch pipe. The results of this experiment are shown in Tables 1 and 2. The results of calculation by this method of adjust-

Table 1

Condi- tion	Q_5 kg/s	q_m/Q_5 m					$k_m Q_m^2$ m				
		1	2	3	4	5	1	2	3	4	5
1	5.235×10^{-2}	0.115	0.137	0.171	0.240	0.337	9.3	13.0	20.4	40.2	79.2
	5.550	0.116	0.137	0.173	0.239	0.335	10.5	14.8	23.4	44.9	88.0
	5.744	0.115	0.136	0.174	0.240	0.334	11.8	15.5	25.5	48.4	93.7
	5.972	0.117	0.139	0.174	0.238	0.333	12.4	17.4	27.3	51.3	100.7
	6.196	0.117	0.138	0.174	0.236	0.336	13.4	18.5	29.4	54.3	109.9
2	5.244×10^{-2}	0.131	0.156	0.199	0.271	0.244	11.9	17.0	27.6	51.5	99.1
	5.583	0.132	0.156	0.198	0.271	0.243	13.7	19.4	31.2	58.1	111.8
	5.854	0.132	0.155	0.198	0.271	0.244	15.2	20.9	34.2	63.9	123.9
	6.039	0.133	0.155	0.199	0.269	0.244	16.5	22.2	36.8	67.2	131.7
	6.452	0.133	0.156	0.198	0.270	0.242	18.8	25.9	41.5	77.2	148.5

Condi- tion	Q_5 kg/s	$(K_m) Q_5^2$ m					$(K_m)/\frac{1}{2g\gamma} \left(\frac{1}{F}\right)^2$ m				
		1	2	3	4	5	1	2	3	4	5
1	5.235×10^{-2}	124.2	120.5	113.1	93.3	54.3	2.355	2.284	2.144	1.769	1.029
	5.550	139.5	135.2	126.6	105.1	62.0	2.353	2.281	2.136	1.773	1.046
	5.744	148.2	144.5	134.5	111.6	66.3	2.333	2.275	2.118	1.757	1.044
	5.972	159.6	154.6	144.7	120.7	71.3	2.325	2.253	2.108	1.759	1.039
	6.196	171.6	166.5	155.6	130.7	75.1	2.323	2.254	2.106	1.769	1.017
2	5.244×10^{-2}	130.6	125.5	114.9	91.0	43.4	2.467	2.371	2.171	1.719	0.820
	5.583	148.8	143.1	131.3	104.4	50.7	2.433	2.340	2.147	1.707	0.829
	5.854	159.8	154.1	140.8	111.1	51.1	2.423	2.337	2.135	1.685	0.775
	6.039	169.0	163.1	148.7	118.3	53.8	2.408	2.326	2.118	1.685	0.766
	6.452	193.2	186.1	170.5	134.8	63.5	2.412	2.323	2.128	1.683	0.793

Table 2

Condi- tion	$(K_m)/\frac{1}{2g\gamma}\left(\frac{1}{F}\right)^2$					q_m/Q_0				
	1	2	3	4	5	1	2	3	4	5
1	2.338	2.269	2.122	1.765	1.035	0.116	0.137	0.173	0.239	0.335
2	2.429	2.339	2.140	1.696	0.797	0.132	0.156	0.198	0.270	0.243

Table 3

No. of branch pipe	$(K_m)r^{-\frac{1}{5}}/\frac{1}{2g\gamma}\left(\frac{1}{F}\right)^2$	ζ_m	Diameter of orifice d mm		
			Calculation	Theoretical	Experimental
1	2.816	0	—	—	—
2	2.501	0.899	21.2	22.6	21.10
3	2.141	1.926	19.5	20.1	19.27
4	1.852	2.750	18.6	18.2	18.80
5	0.686	6.076	16.5	16.6	16.65

Fig. 2

ment (Method No. 2) are shown in Table 3. This table also includes values theoretically obtainable.

4. Flow Equation of Duct System with Parallel Blowers

In the typical duct system as shown in Figure 3, we have the following equations:

Fig. 3

$$P_n + \frac{\gamma}{2g} w_n^2 = P_n + \zeta_{bn} \frac{\gamma}{2g} W_n^2 + \lambda \frac{L_n}{D_n} \frac{\gamma}{2g} W_n^2 + \frac{\gamma}{2g} W_n^2 \dots (36)$$

$$P_n + \frac{\gamma}{2g} W_n^2 = P_0 + \zeta_0 \frac{\gamma}{2g} W_n^2 + \frac{\gamma}{2g} W_n^2 \dots (37)$$

$$Q_n = F_n W_n \gamma \quad \therefore W_n = \frac{Q_n}{F_n \gamma} \dots (38)$$

$$q_n = f_n w_n \gamma \quad \therefore w_n = \frac{q_n}{f_n \gamma}$$

By the above relation we have

$$P_n - P_0 = \left(\zeta_{bn} + \lambda \frac{L_n}{D_n} + \zeta_0 + 1 \right) \frac{1}{2g\gamma} \left(\frac{1}{F_n} \right)^2 Q_n^2 - \frac{1}{2g\gamma} \left(\frac{1}{f_n} \right)^2 q_n^2$$

Assuming $P_n - P_0$ make (P_n) , we have:

$$(P_n) = K_n Q_n^2 - k_n q_n^2 \dots (39)$$

where

$$K_n = \left(\zeta_{bn} + \lambda \frac{L_n}{D_n} + \zeta_0 + 1 \right) \frac{1}{2g\gamma} \left(\frac{1}{F_n} \right)^2$$

$$k_n = \frac{1}{2g\gamma} \left(\frac{1}{f_n} \right)^2$$

In the same manner, for the flow of branch $(n-1)$ we have

$$P_{(n-1)} + \frac{\gamma}{2g} w_{n-1}^2 = P_n + \zeta_{b(n-1)} \frac{\gamma}{2g} W_{(n-1)}^2 + \lambda \frac{L_{(n-1)}}{D_{(n-1)}} \frac{\gamma}{2g} W_{(n-1)}^2 + \lambda \frac{L_n}{D_n} \frac{\gamma}{2g} W_n^2 + \frac{\gamma}{2g} W_n^2 \dots (40)$$

$$P_{(n-1)} - P_0 = \left(\zeta_{b(n-1)} + \lambda \frac{L_{(n-1)}}{D_{(n-1)}} \right) \frac{1}{2g\gamma} \left(\frac{1}{F_{(n-1)}} \right)^2 Q_{(n-1)}^2 + \left(\zeta_{dn} + \lambda \frac{L_n}{D_n} + \zeta_0 + 1 \right) \frac{1}{2g\gamma} \left(\frac{1}{F_n} \right)^2 Q_n^2 - \frac{1}{2g\gamma} \left(\frac{1}{f_{(n-1)}} \right)^2 q_{n-1}^2$$

$$(P_{(n-1)}) = K_{(n-1)} Q_{(n-1)}^2 + K'_n Q_n^2 - k_{(n-1)} q_{(n-1)}^2 \dots (39)'$$

where

$$\begin{cases} K_{(n-1)} = \left(\zeta_{b(n-1)} + \lambda \frac{L_{(n-1)}}{D_{(n-1)}} \right) \frac{1}{2g\gamma} \left(\frac{1}{F_{(n-1)}} \right)^2 \\ K'_n = \left(\zeta_{dn} + \lambda \frac{L_n}{D_n} + \zeta_0 + 1 \right) \frac{1}{2g\gamma} \left(\frac{1}{F_n} \right)^2 \\ k_{(n-1)} = \frac{1}{2g\gamma} \left(\frac{1}{f_{(n-1)}} \right)^2 \end{cases}$$

Therefore, generally

$$(P_m) = \sum_{m+1}^n K'_m Q_m^2 + K_m Q_m^2 - k_m q_m^2 \dots \dots \dots (39)''$$

where

$$\begin{cases} K_m = \left(\zeta_{bm} + \lambda \frac{L_m}{D_m} \right) \frac{1}{2g\gamma} \left(\frac{1}{F_m} \right)^2 \\ K'_m = \left(\zeta_{dm} + \lambda \frac{L_m}{D_m} \right) \frac{1}{2g\gamma} \left(\frac{1}{F_m} \right)^2 \\ k_m = \frac{1}{2g\gamma} \left(\frac{1}{f_m} \right)^2 \end{cases}$$

By using the following relation in the manner described above, general flow equations may be simplified as follows:

$$\sum_{m+1}^n K'_m Q_m^2 + K_m Q_m^2 = (K_m) Q_n^2 \dots \dots \dots (41)$$

$$\therefore \left. \begin{aligned} (P_n) &= (K_n) Q_n^2 - k_n q_n^2 \\ (P_{(n-1)}) &= (K_{(n-1)}) Q_n^2 - k_{(n-1)} q_{(n-1)}^2 \dots \dots \dots \\ (P_m) &= (K_m) Q_n^2 - k_m q_m^2 \dots \dots \dots \\ (P_2) &= (K_2) Q_n^2 - k_2 q_2^2 \\ (P_1) &= (K_1) Q_n^2 - k_1 q_1^2 \end{aligned} \right\} \dots \dots \dots (12)$$

5. Uniform Distribution of Duct System with Parallel Blowers

If each branch pipe keeps a uniform flow, then:

$$q_n = q_{(n-1)} = \dots = q_m = \dots = q_2 = q_1 \equiv q \dots \dots (43)$$

Therefore, the flow ratio to the total quantity of flow is equal to $1/n$, that is, $r = q/Q = 1/n$.

$$\left. \begin{aligned} (P_n)_{r=\frac{1}{n}} &= (K_n)_{r=\frac{1}{n}} Q_n^2 - k_n q^2 \\ (P_{(n-1)})_{r=\frac{1}{n}} &= (K_{(n-1)})_{r=\frac{1}{n}} Q_n^2 - k_{(n-1)} q^2 \dots \dots \dots \\ (P_m)_{r=\frac{1}{n}} &= (K_m)_{r=\frac{1}{n}} Q_n^2 - k_m q^2 \dots \dots \dots \\ (P_2)_{r=\frac{1}{n}} &= (K_2)_{r=\frac{1}{n}} Q_n^2 - k_2 q^2 \\ (P_1)_{r=\frac{1}{n}} &= (K_1)_{r=\frac{1}{n}} Q_n^2 - k_1 q^2 \end{aligned} \right\} \dots \dots \dots (44)$$

Now by Taylor expansion up to the second order, (K_m) will be about as follows:

$$\begin{aligned} (K_m)_{r=\frac{1}{n}} &= f(r_1) + \left(\frac{df}{dr} \right)_{r=r_1} \left(\frac{1}{n} - r_1 \right) \\ &+ \frac{1}{2!} \left(\frac{d^2f}{dr^2} \right)_{r=r_1} \left(\frac{1}{n} - r_1 \right)^2 \dots \dots \dots (45) \end{aligned}$$

For the condition of 2, 3

$$\begin{aligned} (K_m)_2 &= f(r_1) + \left(\frac{df}{dr} \right)_{r=r_1} (r_2 - r_1) \\ &+ \frac{1}{2!} \left(\frac{d^2f}{dr^2} \right)_{r=r_1} (r_2 - r_1)^2 \\ (K_m)_3 &= f(r_1) + \left(\frac{df}{dr} \right)_{r=r_1} (r_3 - r_1) \\ &+ \frac{1}{2!} \left(\frac{d^2f}{dr^2} \right)_{r=r_1} (r_3 - r_1)^2 \end{aligned}$$

Therefore, $(df/dr)_{r=r_1}$, $(d^2f/dr^2)_{r=r_1}$ are calculated from the value of $(K_m)_2$, $(K_m)_3$, so we know the value of $(K_m)_{r=\frac{1}{n}}$.

$$Q_n = nq \quad \left(\frac{Q_n}{q} \right)^2 = n^2 \dots \dots \dots (46)$$

Accordingly

$$\left. \begin{aligned} (P_n)_{r=\frac{1}{n}} / q^2 &= (K_n)_{r=\frac{1}{n}} n^2 - k_n \\ (P_{(n-1)})_{r=\frac{1}{n}} / q^2 &= (K_{(n-1)})_{r=\frac{1}{n}} n^2 - k_{(n-1)} \dots \dots \dots \\ (P_m)_{r=\frac{1}{n}} / q^2 &= (K_m)_{r=\frac{1}{n}} n^2 - k_m \dots \dots \dots \\ (P_2)_{r=\frac{1}{n}} / q^2 &= (K_2)_{r=\frac{1}{n}} n^2 - k_2 \\ (P_1)_{r=\frac{1}{n}} / q^2 &= (K_1)_{r=\frac{1}{n}} n^2 - k_1 \end{aligned} \right\} (47)$$

However, blower pressure and quantity of flow are related by the performance curve of the blower. Therefore, generally:

$$\begin{aligned} (P_m) &= f(q_m) \\ (P_m)_{r=\frac{1}{n}} / q^2 &= f(q) / q^2 = A_m \dots \dots \dots (48) \end{aligned}$$

When the branch pipe and blowers are the same in the duct system

$$\left. \begin{aligned} k_1 &= k_2 = \dots = k_{(n-1)} = k_n \equiv k \\ f(q_1) &= f(q_2) = \dots = f(q_{(n-1)}) \\ &= f(q_n) \equiv f(q) \\ A_1 &= A_2 = \dots = A_{(n-1)} = A_n \equiv A \end{aligned} \right\} \dots \dots \dots (49)$$

Therefore,

$$\begin{aligned} (K_n)_{r=\frac{1}{n}} &= (K_{(n-1)})_{r=\frac{1}{n}} = \dots \dots \dots \\ &= (K_2)_{r=\frac{1}{n}} = (K_1)_{r=\frac{1}{n}} \dots \dots \dots (50) \end{aligned}$$

In this case, blower pressure is the function of q , s ; therefore:

$$\begin{aligned} (P_m) &= f(q_m, s_m) \quad \therefore (P_m)_{r=\frac{1}{n}} = f(q, s_m) \\ f(q, s_n) / q^2 &= (K_n)_{r=\frac{1}{n}} n^2 - k \\ f(q, s_{(n-1)}) / q^2 &= (K_{(n-1)})_{r=\frac{1}{n}} n^2 - k \dots \dots \dots \\ f(q, s_m) / q^2 &= (K_m)_{r=\frac{1}{n}} n^2 - k \dots \dots \dots \dots (51) \\ f(q, s_2) / q^2 &= (K_2)_{r=\frac{1}{n}} n^2 - k \\ f(q, s_1) / q^2 &= (K_1)_{r=\frac{1}{n}} n^2 - k \end{aligned}$$

$$\begin{aligned}
\therefore \frac{f(q, s_n) - f(q, s_{(n-1)})}{q^2} &= \left\{ (K_n)_{r=\frac{1}{n}} - (K_{(n-1)})_{r=\frac{1}{n}} \right\} n^2 \\
\frac{f(q, s_{(n-1)}) - f(q, s_{(n-2)})}{q^2} &= \left\{ (K_{(n-1)})_{r=\frac{1}{n}} - (K_{(n-2)})_{r=\frac{1}{n}} \right\} n^2 \dots \dots \\
\frac{f(q, s_3) - f(q, s_2)}{q^2} &= \left\{ (K_3)_{r=\frac{1}{n}} - (K_2)_{r=\frac{1}{n}} \right\} n^2 \\
\frac{f(q, s_2) - f(q, s_1)}{q^2} &= \left\{ (K_2)_{r=\frac{1}{n}} - (K_1)_{r=\frac{1}{n}} \right\} n^2
\end{aligned} \quad \dots \dots \dots (52)$$

Now, $f(q, s)$ may be written approximately by Taylor expansion up to the second order and by keeping q equal to constant at the condition of $s = s_n$.

$$f(q, s) = f(q, s_n) + \left(\frac{df}{ds} \right)_{s=s_n} (s - s_n) + \frac{1}{2!} \left(\frac{d^2f}{ds^2} \right)_{s=s_n} (s - s_n)^2 \dots \dots \dots (53)$$

In the same way as above mentioned, for the condition of 2, 3 we have

$$\begin{cases} f_2(q, s) = f(q, s_n) + \left(\frac{df}{ds} \right)_{s=s_n} (s_2 - s_n) + \frac{1}{2!} \left(\frac{d^2f}{ds^2} \right)_{s=s_n} (s_2 - s_n)^2 \\ f_3(q, s) = f(q, s_n) + \left(\frac{df}{ds} \right)_{s=s_n} (s_3 - s_n) + \frac{1}{2!} \left(\frac{d^2f}{ds^2} \right)_{s=s_n} (s_3 - s_n)^2 \end{cases}$$

By the above equation, $(df/ds)_{s=s_n}$, $(d^2f/ds^2)_{s=s_n}$ are evaluated, so we have the following equation for $s - s_n = \Delta s$

$$\begin{aligned}
\frac{\left(\frac{df}{ds} \right)_{s=s_n} \Delta s_{(n-1)} + \frac{1}{2!} \left(\frac{d^2f}{ds^2} \right)_{s=s_n} (\Delta s_{(n-1)})^2}{q^2} &= \left\{ K_{(n-1)r=\frac{1}{n}} - (K_n)_{r=\frac{1}{n}} \right\} n^2 \\
\frac{\left(\frac{df}{ds} \right)_{s=s_n} \Delta s_{(n-2)} + \frac{1}{2!} \left(\frac{d^2f}{ds^2} \right)_{s=s_n} (\Delta s_{(n-2)})^2}{q^2} &= \left\{ (K_{(n-2)})_{r=\frac{1}{n}} - (K_{(n-1)})_{r=\frac{1}{n}} \right\} n^2 \dots \dots \\
\frac{\left(\frac{df}{ds} \right)_{s=s_n} \Delta s_2 + \frac{1}{2!} \left(\frac{d^2f}{ds^2} \right)_{s=s_n} (\Delta s_2)^2}{q^2} &= \left\{ (K_2)_{r=\frac{1}{n}} - (K_3)_{r=\frac{1}{n}} \right\} n^2 \\
\frac{\left(\frac{df}{ds} \right)_{s=s_n} \Delta s_1 + \frac{1}{2!} \left(\frac{d^2f}{ds^2} \right)_{s=s_n} (\Delta s_1)^2}{q^2} &= \left\{ (K_1)_{r=\frac{1}{n}} - (K_2)_{r=\frac{1}{n}} \right\} n^2
\end{aligned} \quad \dots \dots \dots (54)$$

By these relations, the rate of increase in the revolutions of a blower can be figured out. If the rate to which the revolutions of a blower must be increased is an excessive rate which will make the fan unusable, the alternative is to use two fans in series and increase their revolutions to a safe maximum rate. The combined increased revolutions of the two blowers will equal the excessive rate but will not make the fans unusable.

6. Conclusions

- (1) Our experiments have shown that the general flow equation, by both the pressure and suction duct systems, is the same in form. This discovery has led to the discovery of Method No. 1 by which the unknown factors can be controlled in order to obtain a uniform distribution.
- (2) Method No. 2 is more practical and con-

venient. It is based on the principle that blower or fan pressure P is equal to the sum of equivalent pressure term of branch flow q and equal to the sum of equivalent pressure term of main flow Q nearest the blower or fan.

- (3) Drag control method is not so effective as duct system of single blower.
- (4) The effective method of flow control is to distribute the flow to each branch pipe by adjusting the revolutions of blowers so as to balance them with the duct pressure equation.

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