Numerical Analysis of Two-Dimensional Viscoelastic Flow

By Kiyoji Nakamura and Akira Horikawa, Member, TMSJ

Department of Mechanical Engineering, Osaka University, Suita, Osaka

Based on the Journal of the Textile Machinery Society of Japan, Transaction, Vol. 29, No. 7, T92-99 (1976)

Abstract

The numerical analysis using the vorticity and the stream function is so useful to study two-dimensional viscous liquid flows that many problems have been solved by this method. In the present paper, this method has been expanded to two-dimensional viscoelastic liquids, and, as an example, the flow of a viscoelastic liquid into a slit has been calculated by using the Maxwell model.

Comparing the results of numerical calculations with those of experiments, we found that the former agreed well with the latter. From this fact, it is clear that this numerical analysis is valid for the study of viscoelastic liquid flows.

As it is easy to change the constitutive equations in this method, this method is applicable to the analysis of many viscoelastic flows.

1. Introduction

In analysing theoretically the flow of fluids, it is important and instructive to understand the characteristics of the mathematical equations governing the flows. Such equations are generally expressed in non-linear partial differential equations which cannot be solved analytically in many cases, and so the numerical analysis is the only way effective to solve flow problems successfully, many of which are Newtonian flows governed by the Navier-Stokes equation. However, there are many liquids not governed by the Navier-Stokes equation. These are called non-Newtonian liquids, which are more complicated than the Newtonian flow in expressing their characteristics mathematically. Hence, the numerical analysis is expected suitable in such studies.

Unfortunately, there has been very little literature available for the numerical analysis of non-Newtonian flows in spite of the urgent necessity. So we try in this paper a numerical study of a two-dimensional non-Newtonian flow, particularly of a two-dimensional viscoelastic flow into a slit.

2. Fundamental Equations

2.1 Equations involving vorticity and stress function

Fundamental equations for the flow of incompressible fluid are:

The equation of continuity;

$$v_{i,i} = 0 \quad \dots\dots\dots (1)$$

The equation of motion;

$$\rho \left(\frac{\partial v_i}{\partial t} + v_j v_{i,j} \right) = -p_{,i} + T_{ij,j} \quad \dots\dots\dots (2)$$

where v_i is a component of velocity vector, T_{ij} a deviatoric

component of stress tensor, p an isotropic component of stress tensor, ρ the density and t the time.

If we assume the flow to be two-dimensional, we can introduce the stream function ψ by

$$\left. \begin{aligned} v_1 &= \frac{\partial \psi}{\partial x_2} \\ v_2 &= -\frac{\partial \psi}{\partial x_1} \end{aligned} \right\} \dots\dots\dots (3)$$

Using this stream function ψ , we can rewrite eqs. (1) and (2) in the form

$$\begin{aligned} -\frac{\partial \Delta \psi}{\partial t} + \frac{\partial \psi}{\partial x_1} \frac{\partial \Delta \psi}{\partial x_2} - \frac{\partial \psi}{\partial x_2} \frac{\partial \Delta \psi}{\partial x_1} \\ = \frac{1}{\rho} \left\{ \frac{\partial^2 (T_{22} - T_{11})}{\partial x_1 \partial x_2} + \left(\frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} \right) T_{12} \right\} \dots\dots\dots (4) \end{aligned}$$

Now, when a relation between stress tensor T_{ij} and velocity vector v_i is given, a stream function ψ is calculated from eq. (4) to obtain a flow field. It is difficult, however, to calculate eq. (4) numerically because this equation is non-linear in stream function ψ . Then, introducing vorticity ω as follows

$$\omega = \frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2} \quad \dots\dots\dots (5)$$

we obtain

$$\omega = -\Delta \psi \quad \dots\dots\dots (6)$$

and can rewrite eq. (4) in the form

$$\begin{aligned} \frac{\partial \omega}{\partial t} + \frac{\partial (v_1 \omega)}{\partial x_1} + \frac{\partial (v_2 \omega)}{\partial x_2} \\ = \frac{1}{\rho} \left\{ \frac{\partial^2 (T_{22} - T_{11})}{\partial x_1 \partial x_2} + \left(\frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} \right) T_{12} \right\} \dots\dots\dots (7) \end{aligned}$$

These eqs. (6) and (7) are fundamental equations for numerical analysis.

2.2 Constitutive equation for viscoelastic fluids

While many different constitutive equations have been proposed for viscoelastic fluids, they may be divided into two broad groups; the rate type and the integral type. The present study will be restricted to the former type.

The rate type constitutive equation for fluids is given by the form^[1]

$$\hat{T}_{ij}^{(s)} = F_{ij}(T_{pq}, \hat{T}_{pq}, \dots, \hat{T}_{pq}^{(s-1)}; D_{km}, \dots, D_{km}^{(r)}; \rho) \quad (8)$$

where Note 1)

$$\hat{T}_{ij} = \frac{\partial T_{ij}}{\partial t} + v_k T_{ij,k} - W_{ik} T_{kj} + T_{ik} W_{kj} \quad (9)$$

$$D_{km} = \frac{1}{2} (v_{k,m} + v_{m,k}) \quad (10)$$

$$W_{km} = \frac{1}{2} (v_{k,m} - v_{m,k}) \quad (11)$$

Since eq. (8) is in a functional expression, it must be reduced to a specific function before a numerical calculation is carried out. Such specific models are proposed as Maxwell model, Oldroyd model^[2,3], Denn model^[4] etc. Substituting the relation between stress and velocity into eqs.(3), (6) and (7), we will obtain the flow field.

3. Finite Difference Approximation of Fundamental Equations

The equations relating to the flow field are eqs. (3), (6), (7) and (8). Let us solve these equations by the finite difference method.

When the flow field $v_i(t)$ at time $t \leq t_0$ is given, we can calculate the flow field by the following procedure;

- calculating $T_{ij}(t_0)$ using eq. (8),
- calculating $\omega(t_0)$ using eq. (5),
- calculating $\omega(t_0 + \Delta t)$ using eq. (7),
- calculating $\phi(t_0 + \Delta t)$ using Poisson's equation (6),
- calculating $v_i(t_0 + \Delta t)$ using eq. (3).

Once $v_i(t_0 + \Delta t)$ is obtained in the above manner, we can calculate $v_i(t_0 + 2\Delta t)$ by repeating i)–v). The iterative process is terminated when v_i converges, and this result is considered to be the stable flow field.

3.1 Location of the finite difference points of the variables relative to the mesh

In the fluid we specify a mesh of points at which the variables v_1, v_2, ω, T_{ij} and ϕ may be defined^[5]. A representation of this is given in Fig. 1; the stream function ϕ is located on the mesh point and the velocity v_i is located halfway between two adjacent mesh points. Namely,

$$(v_1)_{I, J+1/2} = \frac{\phi_{I, J+1} - \phi_{I, J}}{\Delta x_2} \quad (12)$$

$$(v_2)_{I+1/2, J} = \frac{\phi_{I, J} - \phi_{I+1, J}}{\Delta x_1} \quad (13)$$

Fig. 1 Location of the finite difference points of the variables (ϕ, ω, T_{ij})

$$(v_1)_{I, J} = \frac{(v_1)_{I, J+1/2} + (v_1)_{I, J-1/2}}{2} \quad (14)$$

$$(v_2)_{I, J} = \frac{(v_2)_{I+1/2, J} + (v_2)_{I-1/2, J}}{2} \quad (15)$$

Vorticity ω and stress T_{ij} are located on the mesh point as in the stream function ϕ .

3.2 Finite difference approximation of Poisson's equation (6)

The value of stream function $\phi_{I, J}$ at the (I, J) mesh point is computed using the accelerated Liepmann method.

$$\begin{aligned} \phi_{I, J}^{(k+1)} = & \phi_{I, J}^{(k)} + \frac{\xi}{2(\Delta x_1^2 + \Delta x_2^2)} \left\{ \Delta x_2^2 (\phi_{I+1, J}^{(k)} \right. \\ & + \phi_{I-1, J}^{(k+1)} + \Delta x_1^2 (\phi_{I, J+1}^{(k)} + \phi_{I, J-1}^{(k+1)}) \\ & \left. - 2(\Delta x_1^2 + \Delta x_2^2) \phi_{I, J}^{(k)} + \Delta x_1^2 \Delta x_2^2 \omega_{I, J} \right\} \quad (16) \end{aligned}$$

where k is the number of iteration step and ξ is the over-relaxation factor. Criteria of the convergence of the stream function ϕ may be, for example,,

$$\frac{|\phi_{I, J}^{(k+1)} - \phi_{I, J}^{(k)}|_{\max}}{\phi_r} < 0.0002 \quad (17)$$

and

$$\begin{aligned} & \sum_I \sum_J |\Delta x_1^2 \Delta x_2^2 \omega_{I, J}| \\ & \frac{\sum_I \sum_J |2(\Delta x_1^2 + \Delta x_2^2) \phi_{I, J} - \Delta x_2^2 (\phi_{I+1, J} + \phi_{I-1, J}) \\ & - \Delta x_1^2 (\phi_{I, J+1} + \phi_{I, J-1})|}{\phi_r} - 1 < 0.0002 \quad (18) \end{aligned}$$

where ϕ_r is the value of the reference stream function.

3.3 Finite difference approximation of the vorticity equation (7)

Assume that the entire set of values of vorticity ω , velocity v_i and stress T_{ij} at some time t . Then we may approximate ω at a small increment of time later, i.e. at $t + \Delta t$, by a finite difference approximation of eq. (7):

$$\begin{aligned} \omega_{I,J}(t+\Delta t) = & \omega_{I,J}(t) - \Delta t \left\{ \frac{(v_1 \omega)_{I+1,J}(t)}{\Delta x_1} \right. \\ & - \frac{(v_1 \omega)_{I,J}(t)}{\Delta x_2} + \frac{(v_2 \omega)_{I,J+1}(t) - (v_2 \omega)_{I,J}(t)}{\Delta x_2} \left. \right\} \\ & + \frac{\Delta t}{\rho} \left\{ \frac{(T_{22} - T_{11})_{I+1,J+1}(t) - (T_{22} - T_{11})_{I,J+1}(t)}{\Delta x_1 \Delta x_2} \right. \\ & + \frac{(T_{22} - T_{11})_{I,J}(t) - (T_{22} - T_{11})_{I+1,J}(t)}{\Delta x_1^2} \\ & + \frac{(T_{12})_{I+1,J}(t) - 2(T_{12})_{I,J}(t) + (T_{12})_{I-1,J}(t)}{\Delta x_1^2} \\ & \left. - \frac{(T_{12})_{I,J+1}(t) - 2(T_{12})_{I,J}(t) + (T_{12})_{I,J-1}(t)}{\Delta x_2^2} \right\} \end{aligned} \quad (19)$$

3.4 Finite difference approximation of constitutive equation (8)

Assuming that the constitutive equation (8) is in linear relation to T_{ij} , $\hat{T}_{ij}$, ..., $\hat{T}_{ij}^{(s-1)}$, the time derivative (9) is written in the form

$$\begin{aligned} (\hat{T}_{ij})_{I,J}(t) = & \frac{(T_{ij})_{I,J}(t) - (T_{ij})_{I,J}(t - \Delta t)}{\Delta t} \\ & + (v_k T_{ij,k})_{I,J}(t - \Delta t) - (W_{ik} T_{kj})_{I,J}(t - \Delta t) \\ & + (T_{ik} W_{kj})_{I,J}(t - \Delta t) \end{aligned} \quad (20)$$

Then, we can calculate $(\hat{T}_{ij})_{I,J}(t)$ from the values at time t and $t - \Delta t$. Thus, eq. (8) may be written in the following form:

$$\begin{aligned} (T_{ij})_{I,J}(t) = & G_{ij} \{ (T_{pq})_{I,J}(t - \Delta t), \dots, \\ & (T_{pq})_{I,J}(t - s\Delta t); (D_{km})_{I,J}(t), \dots, \\ & (D_{km})_{I,J}(t - r\Delta t); \rho \} \end{aligned} \quad (21)$$

and stress values at time t are obtained from both the stress and the deformation velocity at the past time.

4. Application to a Viscoelastic Flow into a Slit

Consider a two-dimensional viscoelastic converging flow shown in Fig. 2 as an example of the application of the

Fig. 2 Converging flow geometry

$$\begin{aligned} (T_{22}^* - T_{11}^*) + W_e \left\{ \frac{\partial (T_{22}^* - T_{11}^*)}{\partial t^*} + \frac{\partial \{v_1^* (T_{22}^* - T_{11}^*)\}}{\partial x_1^*} + \frac{\partial \{v_2^* (T_{22}^* - T_{11}^*)\}}{\partial x_2^*} - 2\omega^* T_{12}^* \right\} \\ = 2(D_{22}^* - D_{11}^*) \end{aligned} \quad (28)$$

$$T_{12}^* + W_e \left\{ \frac{\partial T_{12}^*}{\partial t^*} + \frac{\partial (v_1^* T_{12}^*)}{\partial x_1^*} + \frac{\partial (v_2^* T_{12}^*)}{\partial x_2^*} + \frac{1}{2} \omega^* (T_{22}^* - T_{11}^*) \right\} = 2D_{12}^* \quad (29)$$

above proposed numerical method.

We first of all assume as follows:

- No body force,
- Incompressible and isothermal flow,
- A constant velocity in x_1 direction and zero velocity in x_2 direction at the exit.
- A fully-developed two-dimensional plane flow at the entrance $x_1=0$.

4.1 Fundamental flow equations

Introducing dimensionless quantities

$$\left. \begin{aligned} t^* &= \frac{V}{H} t \\ x_i^* &= \frac{x_i}{H} \\ v_i^* &= \frac{v_i}{V} \\ T_{ij}^* &= \frac{H}{\eta V} T_{ij} \\ p^* &= \frac{H}{\eta V} p \\ \phi^* &= \frac{\phi}{HV} \\ \omega^* &= \frac{\omega}{HV} \end{aligned} \right\} \quad (22)$$

where V and H are the characteristic velocity and length respectively, we may rewrite eqs. (3), (5), (6) and (7) in the following forms;

$$\left. \begin{aligned} v_1^* &= \frac{\partial \phi^*}{\partial x_2^*} \\ v_2^* &= -\frac{\partial \phi^*}{\partial x_1^*} \end{aligned} \right\} \quad (23)$$

$$\omega^* = \frac{\partial v_2^*}{\partial x_1^*} - \frac{\partial v_1^*}{\partial x_2^*} \quad (24)$$

$$\omega^* = -\Delta \phi^* \quad (25)$$

$$\begin{aligned} \frac{\partial \omega^*}{\partial t^*} + \frac{\partial (v_1^* \omega^*)}{\partial x_1^*} + \frac{\partial (v_2^* \omega^*)}{\partial x_2^*} \\ = \frac{1}{Re} \left\{ \frac{\partial^2 (T_{22}^* - T_{11}^*)}{\partial x_1^{*2} \partial x_2^{*2}} + \left(\frac{\partial^2}{\partial x_1^{*2}} - \frac{\partial^2}{\partial x_2^{*2}} \right) T_{12}^* \right\} \end{aligned} \quad (26)$$

where $Re = \rho V H / \eta$.

Here, we further restrict the constitutive equation to the Maxwell model:

$$T_{ij} + \lambda \hat{T}_{ij} = 2\eta D_{ij} \quad (27)$$

This equation may be written by eq. (22) in the dimensionless form

where λ is relaxation time, η is coefficient of viscosity and $We = \lambda V/H$.

The finite difference equations of (28) and (29) become

$$\left(1 + \frac{We}{\Delta t}\right) (T_{22}^* - T_{11}^*)_{I,J}(t) = 2(D_{22}^* - D_{11}^*)_{I,J}(t) + We \left\{ \frac{(T_{22}^* - T_{11}^*)_{I,J}(t - \Delta t)}{\Delta t} - \frac{1}{\Delta x} \left[\left\{ v_1^* (T_{22}^* - T_{11}^*) \right\}_{I+1,J}(t - \Delta t) - \left\{ v_1^* (T_{22}^* - T_{11}^*) \right\}_{I,J}(t - \Delta t) \right] + 2(\omega^* T_{12}^*)_{I,J}(t - \Delta t) \right\} \dots (30)$$

$$\left(1 + \frac{We}{\Delta t}\right) (T_{12}^*)_{I,J}(t) = 2(D_{12}^*)_{I,J}(t) + We \left\{ \frac{(T_{12}^*)_{I,J}(t - \Delta t)}{\Delta t} - \frac{1}{\Delta x} \left[(v_1^* T_{12}^*)_{I+1,J}(t - \Delta t) - (v_1^* T_{12}^*)_{I,J}(t - \Delta t) + (v_2^* T_{12}^*)_{I,J+1}(t - \Delta t) - (v_2^* T_{12}^*)_{I,J}(t - \Delta t) \right] - \frac{1}{2} \left[\omega^* (T_{22}^* - T_{11}^*) \right]_{I,J}(t - \Delta t) \right\} \dots (31)$$

where $\Delta x_1^* = \Delta x_2^* = \Delta x$ is further assumed.

4.2 Boundary conditions

A. Boundary conditions of v_1^* and v_2^*

$$\left. \begin{aligned} v_1^* &= -\frac{3}{2} (1 - x_2^{*2}) \\ v_2^* &= 0 \end{aligned} \right\} \dots (32)$$

at $x_1^* = 0$ Note 2)

ii) As the flow is symmetric with respect to x_1^* axis,

$$\left. \begin{aligned} \frac{\partial v_1^*}{\partial x_2^*} &= 0 \\ v_2^* &= 0 \end{aligned} \right\} \dots (33)$$

at $x_2^* = 0$.

iii)

$$\left. \begin{aligned} v_1^* &= -\frac{1}{\kappa} \\ v_2^* &= 0 \end{aligned} \right\} \dots (34)$$

at $x_1^* = \alpha, 0 \leq x_2^* \leq \kappa$

iv)

$$v_1^* = v_2^* = 0 \dots (35)$$

at $0 \leq x_1^* \leq \beta, x_2^* = 1$

$$v_1^* = v_2^* = 0 \dots (36)$$

at

$$\beta \leq x_1^* \leq \alpha, x_2^* = \frac{\kappa - 1}{\alpha - \beta} x_1^* + 1 - \frac{\kappa - 1}{\alpha - \beta} \beta$$

B. Boundary conditions of ϕ^*

Boundary conditions of ϕ^* are obtained from boundary conditions of v_1^* and v_2^* ;

i)

$$\phi^* = -\frac{3}{2} \left(x_2^* - \frac{1}{3} x_2^{*3} \right) \dots (37)$$

at $x_1^* = 0$

ii)

$$\phi^* = 0 \dots (38)$$

at $x_2^* = 0$

iii)

$$\phi^* = -\frac{1}{\kappa} x_2^* \dots (39)$$

at $x_1^* = \alpha, 0 \leq x_2^* \leq \kappa$

iv)

$$\phi^* = 1 \dots (40)$$

at the wall.

C. Hypothetical points of v_1^* and v_2^*

In order to calculate the values of $T_{22}^* - T_{11}^*$, T_{12}^* and ω^* at the wall, the values of $\partial v_1^* / \partial x_2^*$ and $\partial v_2^* / \partial x_1^*$ etc. are necessary. Since the values of v_1^* and v_2^* are not specified at the wall, hypothetical points are introduced such that, in the case of the boundary shown in Fig. 3(a),

$$(v_1^*)_{-1/2} = (v_1^*)_0 \dots (41)$$

That is to say, $(v_1^*)_{-1/2}$ outside of the wall is made equal to zero to satisfy the boundary conditions (35) and (36). Similar rule applies to v_2^* . In the case of Fig. 3(b), the value outside of the wall is assumed to be equal to $(v_1^*)_0$.

4.3 Initial velocity field

To start the numerical calculation, an initial velocity field has to be assumed. As the initial velocity field is desirable to be as close as possible to the final velocity field, we assume such an initial velocity field as is given by Stokes' approximation of the Navier-Stokes equation under the boundary conditions of eqs. (43) and (44). (This solution does not satisfy $v_2^* = 0$ at the wall.) This solution

Fig. 3 Hypothetical point of v_1

is briefly described as follows: When the wall is oblique, the initial velocity field is identical to that in which velocities are zero in the region outside of the oblique wall.

In the Stokes' approximation, the stream function satisfies the biharmonic equation

$$\nabla^2 \nabla^2 \phi^* = 0 \quad (42)$$

Boundary conditions are assumed as shown in Fig. 4:

i)

$$v_1^* = \frac{1}{\kappa} \left\{ H(x_2^* - 1 + \kappa) - H(x_2^* - 1 - \kappa) \right\} \quad (43)$$

$$v_2^* = 0$$

at

$$x_1^* = -\frac{\alpha}{2}, \quad 0 \leq x_2^* \leq 2$$

where $H(x)$ is Heaviside's step function.

$$\phi^* = \sum_{n=1}^{\infty} \{ A_n \cosh \mu_n x_1^* + B_n x_1^* \cosh \mu_n x_1^* + C_n \sinh \mu_n x_1^* + D_n x_1^* \sinh \mu_n x_1^* \} \cos \mu_n x_2^* \quad (48)$$

where $\mu_n = n\pi/2$. A_n , B_n , C_n and D_n are derived from the boundary conditions as follows:

$$A_n = - \frac{\left(\sinh \frac{\alpha \mu_n}{2} + \frac{\alpha \mu_n}{2} \cosh \frac{\alpha \mu_n}{2} \right) \left\{ \frac{2}{\mu_n \kappa} \sinh \mu_n \sin \mu_n \kappa + \frac{3}{\mu_n^3} (1 + (-1)^{n-1}) \right\}}{\mu_n \left\{ \alpha \mu_n + 2 \sinh \frac{\alpha \mu_n}{2} \cosh \frac{\alpha \mu_n}{2} \right\}} \quad (49)$$

$$B_n = \frac{\cosh \frac{\mu_n \alpha}{2} \left\{ \frac{2}{\mu_n \kappa} \sin \mu_n \sin \mu_n \kappa - \frac{3}{\mu_n^3} (1 + (-1)^{n-1}) \right\}}{-\alpha \mu_n + 2 \sinh \frac{\alpha \mu_n}{2} \cosh \frac{\alpha \mu_n}{2}} \quad (50)$$

$$C_n = \frac{\left(\cosh \frac{\alpha \mu_n}{2} + \frac{\alpha \mu_n}{2} \sinh \frac{\alpha \mu_n}{2} \right) \left\{ \frac{2}{\mu_n \kappa} \sin \mu_n \sin \mu_n \kappa - \frac{3}{\mu_n^3} (1 + (-1)^{n-1}) \right\}}{\left\{ \alpha \mu_n - 2 \sinh \frac{\alpha \mu_n}{2} \cosh \frac{\alpha \mu_n}{2} \right\} \mu_n} \quad (51)$$

$$D_n = \frac{\sinh \frac{\alpha \mu_n}{2} \left\{ \frac{2}{\mu_n \kappa} \sin \mu_n \sin \mu_n \kappa + \frac{3}{\mu_n^3} (1 + (-1)^{n-1}) \right\}}{\alpha \mu_n + 2 \sinh \frac{\alpha \mu_n}{2} \cosh \frac{\alpha \mu_n}{2}} \quad (52)$$

ii)

$$\left. \begin{aligned} v_1^* &= 3x_2^* - \frac{3}{2}x_2^* \\ v_2^* &= 0 \end{aligned} \right\} \quad (44)$$

at

$$x_1^* = -\frac{\alpha}{2}, \quad 0 \leq x_2^* \leq 2$$

Eqs. (43) and (44) can be expanded in Fourier series respectively as follows^[8]:

$$v_1^* = \sum_{n=1}^{\infty} \frac{4}{n\pi\kappa} \sin \frac{n\pi}{2} \sin \frac{n\pi\kappa}{2} \sin \frac{n\pi x_2^*}{2} \quad (45)$$

$$v_1^* = \sum_{n=1}^{\infty} \frac{24}{(n\pi)^3} \{ 1 + (-1)^{n-1} \} \sin \frac{n\pi x_2^*}{2} \quad (46)$$

Now, assuming the solution in the form

$$\phi^* = \sum_{n=1}^{\infty} \phi_n(\mu_n x_1^*) \cos \mu_n x_2^* \quad (47)$$

and substituting eq. (47) into eq. (42), we obtain

4.4 Results of numerical calculation and its discussion

Figs. 5 and 6 show calculated results for two different Weissenberg numbers, 5×10^{-5} and 5×10^{-4} , under the same Reynolds number 2×10^{-6} . In this calculation, Δx

was equal to $1/20$, Δt was equal to $Re We$ and the number of time steps for convergence was about 4500. Eq. (17) was used as the criterion of convergence. Since Reynolds number was held constant in this calculation, the characteristic velocity V may be considered not to change. Hence, the change of Weissenberg number is equivalent to change the relaxation time of the viscoelastic fluid. Figs. 5 and 6 show that the circulation flow enlarges as Weissenberg number increases, i.e. as the relaxation time increases. Fig. 7 shows the velocity field v_1^* at $Re = 2 \times 10^{-6}$ and $We = 5 \times 10^{-4}$. Moreover, Figs. 5 and 6 show that the velocity of circulating flow is much smaller than that of main flow. In comparison of the above numerical results with the experimental results^[7], tendencies agree well.

Fig. 4 Converging flow geometry for initial velocity

Fig. 5 Stream function ψ^*
(Reynolds number 2×10^{-6} ,
Weissenberg number 5×10^{-5})

Fig. 6 Stream function ψ^*
(Reynolds number 2×10^{-6} ,
Weissenberg number 5×10^{-4})

Fig. 8 shows experimental results of flow patterns, which coincide with stream lines except near a U-shaped central region; calculated results are similar to the experimental results. Fig. 8(b) shows a flow pattern having large circulating flows. In this case the shape of the main flow lines looks like a wine-glass, but we failed to obtain this shape by the numerical calculation.

Fig. 9 shows the calculated stream line for the case of the oblique exit wall angle 60° under the same Reynolds number and Weissenberg number as Fig. 6. In comparison

Fig. 9 Stream function ψ^* (Reynolds number 2×10^{-6} ,
Weissenberg number 5×10^{-4} and oblique wall angle 60°)

of Figs. 6 and 9, it is seen that the circulating region becomes small in proportion to this oblique angle, and this result agrees with experimental results.

5. Conclusions

We proposed a numerical method for the two-dimensional viscoelastic flow, and calculated a flow into a slit using Maxwell model. In comparison of numerical results with experimental results the tendency agrees well. From this fact, this numerical method will be effective for the study of viscoelastic flow. Moreover, we can easily change the constitutive equation in this method.

Note 1) $\hat{T}_{ij}$ is called Jaumann derivative. Other derivatives may be used if they satisfy the material objectivity. Material derivative is used for D_{km} .

Note 2) If the flow channel is long in x_1^* direction, the influence of velocity distribution at $x_1^*=0$ in the inlet region of the slit becomes small.

Fig. 7 Velocity distribution
(Reynolds number 2×10^{-6} , Weissenberg number 5×10^{-4})

Fig. 8 Flow pattern obtained by experiment

References

- [1] C. Truesdell and W. Noll; The Non-Linear Field Theories of Mechanics, Springer-Verlag, Berlin (1965).
- [2] J. G. Oldroyd, *Proc. Roy. Soc. Lond.*, A200, 523(1950).
- [3] J. G. Oldroyd, *Proc. Roy. Soc. Lond.*, A245, 278(1958).
- [4] M. M. Denn, *Chem. Eng. Sci.*, 22, 395(1967).
- [5] J. Fromm, *Method in Computational Physics*, Academic Press, New York (1964).
- [6] T. Akasaka, Suchikeisan (in Japanese), Korona-sha (1967).
- [7] A. Horikawa, K. Nakamura and S. Umegaki, *Sen'i-kikai-gakkai Ronbunshu* (in Japanese), 29, No. 5, 41 (1976).
- [8] S. Moriguchi, K. Udagawa and S. Hitotsumatsu, *Sugakukoshiki II* (in Japanese), Iwanami-shoten (1957).