Axisymmetric Slow Flow of a Viscoelastic Liquid into a Capillary

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Abstract

The elastic influence of a viscoelastic liquid on the three-dimensional axisymmetric slow flow into a capillary was numerically investigated for the Maxwell model by the perturbation method. Comparing the behaviour of the three-dimensional axisymmetric flow with that of the two-dimensional plane one, it was confirmed that there was a large difference between their values of Weissenberg number at which the viscoelastic effect on flow began to appear. In addition, differences in shear and normal stress of three-dimensional axisymmetric flow were studied and compared with those of two-dimensional ones.

1. Introduction

The flow phenomenon of high viscous polymer melts from a large reservoir into a slit was investigated; the flow and the stress field were numerically calculated for the Maxwell type. From these numerical results, large circulating flows not found in the Newtonian flow were obtained near the slit on account of the elastic effect.

Now, the flow just cited is a two-dimensional plane flow. On the other hand, the flow into a capillary is a three-dimensional flow on which Tordella and Ballenger et al experimentally investigated. In the present paper, we calculate the three-dimensional flow and the three-dimensional stress field using Maxwell's constitutive equation. Furthermore, these results are compared with two-dimensional results obtained by the authors.

2. Fundamental equations

Motions of incompressible continua obey the following equations.

Equation of continuity;

\[ \rho \frac{Dv^i}{Dt} = 0 \] ................................................ (1)

Cauchy's law of motion;

\[ \rho \dot{v}^i + \frac{\partial p}{\partial x^i} + T^{ik}_{\cdot \cdot} = 0 \] ................................................ (2)

where \( g_{ik} \) is a metric tensor and \( T^{ik}_{\cdot \cdot} \) is a covariant differential of \( T^{ik} \) given by

\[ T^{ik}_{\cdot \cdot} = \frac{\partial T^{ik}}{\partial x^i} + \Gamma^{ik}_{lm} T^{lm} + \Gamma^{ik}_{m,1} T^{lm}. \]

\( \Gamma^{ik}_{lm} \) is a coefficient of connection on curvilinear coordinates \( \{ k \} \). Here, we consider

\[ \Gamma^{ik}_{lm} = \{ i \} \text{ of } \{ km \}. \]

\( \{ i \} \text{ of } \{ km \} \) is Christoffel's symbol and expresses Riemannian connection as

\[ \{ i \text{ of } \{ km \} \} = \frac{1}{2} g^{ik} \left( \frac{\partial g_{lm}}{\partial x^i} + \frac{\partial g_{im}}{\partial x^l} - \frac{\partial g_{il}}{\partial x^m} \right). \]

The viscoelastic constitutive equation of Maxwell's type is

\[ T^{ii} + \lambda \frac{D_f T^{ii}}{D_t} = -2 \eta W^{ii} \] ................................................ (3)

where \( \lambda \) is the relaxation time, \( \eta \) the coefficient of viscosity and \( D_f \) the Jaumann derivative defined by

\[ D_f T^{ii} = \frac{\partial T^{ii}}{\partial t} + v^i T^{ii} - W^{ik} T^{ki} + T^{ik} W^{ik}. \]

and

\[ D_k = \frac{1}{2} (v_{ik} + v_{ki}) \text{ and } W_{km} = \frac{1}{2} (v_{km} - v_{mk}). \]

Consider a three-dimensional axisymmetric flow and introduce a cylindrical coordinate; \( x^1 = r \), \( x^2 = \theta \) and \( x^3 = z \). Then, metric tensor \( g_{ij} \) is

\[ g_{11} = g_{22} = 1 \]
\[ g_{22} = r^2 \]
\[ g_{33} = 0 \]

And Christoffel's symbols \( \{ i \text{ of } \{ km \} \} \) are

\[ \{ 1 \text{ of } 22 \} = -r \]
\[ \{ 2 \text{ of } 22 \} = -\frac{1}{r} \]

and the other symbols are zeros. Take the \( z \)-axis along the flow direction. For the axisymmetric flow, the velocity field is

\[ v_r = v_r(z, r), \quad v_\theta = 0, \quad v_z = v_z(z, r) \] ................................................ (5)
and the derivative in the $\theta$-direction is zero.

Next, we rewrite all vectors and tensors as physical components on cylindrical coordinates and introduce dimensionless quantities as follows;

$$ v_r^* = \frac{v_r}{V}, \quad v_\theta^* = \frac{v_\theta}{V}, \quad v_z^* = \frac{v_z}{V}, \quad \frac{R^*}{R}, \quad \frac{z^*}{z} = \frac{z}{R}, \quad \frac{p^*}{\rho V^2}, \quad \frac{T_{rr}^*}{\rho V^2}, \quad T_{\theta\theta}^* = \frac{R}{\rho V^2}, \quad T_{\theta z}^* = \frac{R}{\rho V^2},$$

$$ T_{zz}^* = \frac{R}{\rho V^2} T_{rr}, \quad T_{\theta z}^* = \frac{R}{\rho V^2} T_{\theta z}, \quad R^* \theta^* = \frac{R}{\rho V^2} T_{\theta z},$$

where $V$ is the mean velocity in a cylindrical reservoir and $R$ is its radius. Considering eq. (5), we can rewrite eqs. (1), (2) and (3) as

$$ \frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* v_r^* \right) + \frac{\partial v_\theta^*}{\partial z^*} = 0 \quad \cdots \quad (7a)$$

$$ R^* \left\{ v_r^* \frac{\partial v_r^*}{\partial r^*} + v_\theta^* \frac{\partial v_\theta^*}{\partial z^*} \right\} - \frac{\partial p^*}{\partial r^*} + \frac{\partial T_{rr}^*}{\partial r^*} + T_{rr}^* + T_{\theta z}^* \quad \cdots \quad (8a)$$

$$ T_{\theta z}^* = \frac{R}{\rho V^2} T_{rr}, \quad T_{\theta z}^* = \frac{R}{\rho V^2} T_{\theta z},$$

$$ \frac{\partial p^*}{\partial r^*} = \frac{R^*}{R} \frac{\partial p^*}{\partial r^*}, \quad T_{\theta z}^* = \frac{R}{\rho V^2} T_{\theta z}, \quad T_{\theta z}^* = \frac{R}{\rho V^2} T_{\theta z},$$

Boundary conditions are

$$ v_r^* = 2(1 - r^*), \quad v_\theta^* = 0 \quad \text{at} \quad z^* = -\infty \cdots \quad (11a)$$

$$ v_r^* = v_r^* \left( r^* \right), \quad v_\theta^* = v_\theta^* \left( r^* \right), \quad \text{when} \quad z^* = 0, \quad 0 \leq r^* < 1 \cdots \quad (11b)$$

$$ v_r^* = v_r^* = 0 \quad \text{when} \quad z^* = 0, \quad I^* \leq r^* \leq 1 \cdots \quad (11c)$$

$$ v_r^* = v_r^* = 0 \quad \text{at} \quad r^* = 1 \cdots \quad (11d)$$

$$ \frac{\partial v_r^*}{\partial r^*} = v_r^* = 0 \quad \text{at} \quad r^* = 0 \cdots \quad (11e)$$

3. Development of fundamental equations

In order to obtain the flow and the stress field, we must solve eqs. (7), (8) and (9) simultaneously under the boundary condition (11). As the constitutive equation of Maxwell's type expressed by eq. (9) is not explicit on the stress, it is difficult to solve these systems. The purpose of this paper, however, is to analyze the flow when the viscoelastic effect begins to appear, that is to say, when the wine-glass flow begins to appear near a slit by the viscoelastic effect. Hence, under the assumption that the Weissenberg number $W_e$ is smaller than 1, we develop the velocity, the stress and the stream function in power series of the Weissenberg number as follows;

$$ v_r^* = v_r^* (0) + W_e v_r^* (1) + W_e^2 v_r^* (2) + \cdots$$

$$ p^* = p^* (0) + W_e p^* (1) + W_e^2 p^* (2) + \cdots$$

$$ T_{rr}^* = T_{rr}^* (0) + W_e T_{rr}^* (1) + W_e^2 T_{rr}^* (2) + \cdots$$

Substitute eq. (12) into eqs. (7), (8), (9), (10) and (11), and rearrange them on the Weissenberg number to get the following.

3.1 Terms having no Weissenberg number

$$ \frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* v_r^* (0) \right) + \frac{\partial v_\theta^* (0)}{\partial z^*} = 0 \quad \cdots \quad (13a)$$

$$ \frac{\partial T_{rr}^* (0)}{\partial r^*} + \frac{\partial T_{rr}^* (0)}{\partial r^*} + T_{rr}^* (0) = \frac{\partial p^* (0)}{\partial r^*} \quad \cdots \quad (13b)$$

$$ \frac{\partial T_{rr}^* (0)}{\partial r^*} + \frac{\partial T_{rr}^* (0)}{\partial r^*} + T_{rr}^* (0) - T_{\theta z} (0) = \frac{\partial \phi (0)}{\partial r^*} \quad \cdots \quad (13c)$$

$$ T_{\theta z} (0) = 2 \frac{\partial v_r^* (1)}{\partial z^*}, \quad T_{\theta z} (0) = 2 \frac{\partial v_r^* (1)}{\partial z^*} \quad \cdots \quad (13d)$$

$$ T_{\theta z} (0) = 2 \frac{\partial v_r^* (1)}{\partial z^*}, \quad \frac{\partial \phi (0)}{\partial z^*} \quad \cdots \quad (13e)$$

Boundary conditions are

$$ v_r^* (0) = 2(1 - r^*), \quad v_r^* (0) = 0 \quad \text{at} \quad z^* = -\infty \cdots \quad (14a)$$

$$ v_r^* (0) = v_r^* (0), \quad \phi (0) = 0 \quad \text{at} \quad r^* = 1 \cdots \quad (14b)$$

$$ \frac{\partial v_r^* (0)}{\partial r^*} = v_r^* (0) = 0 \quad \text{at} \quad r^* = 0 \cdots \quad (14c)$$

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\( v_1^{(0)} = v_2^{(0)} = 0 \) when \( z^* = 0, \ l^* \leq r^* \leq 1 \) \hspace{1cm} (14c)
\( v_1^{(0)} = v_2^{(0)} = 0 \) at \( r^* = 1 \) \hspace{1cm} (14d)
\[ \frac{\partial v_1^{(0)}}{\partial r^*} = v_1^{(0)} = 0 \] at \( r^* = 0 \) \hspace{1cm} (14e)

Eliminating \( \partial p^{(0)}/\partial z^* \) and \( \partial p^{(0)}/\partial r^* \) from eq. (13), we obtain
\[ \left[ \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right] \psi^{(0)} = 0 \] \hspace{1cm} (15)

The solution \( \psi^{(0)} = \psi^{(0)}(z^*, r^*) \) of eq. (15) under the boundary condition (14) shows the Newtonian flow, and this solution is of the zero's order approximate term of the viscoelastic flow.

3.2 First order terms of Weissenberg number
Substituting eq. (12) into eq. (9), and equating the first order terms of the Weissenberg number, following equations are obtained;
\[ T_{zz}(1) = 2 \frac{\partial v_1^{(1)}}{\partial z^*} - v_1^{(1)} \frac{\partial T_{zz}(0)}{\partial r^*} - v_{zz}(0) \frac{\partial T_{zz}(0)}{\partial z^*} \] \hspace{1cm} (16a)
\[ T_{rr}(1) = 2 \frac{\partial v_1^{(1)}}{\partial r^*} - v_1^{(1)} \frac{\partial T_{rr}(0)}{\partial r^*} - v_{rr}(0) \frac{\partial T_{rr}(0)}{\partial r^*} \] \hspace{1cm} (16b)
\[ T_{\theta \theta}(1) = 2 \frac{\partial v_1^{(1)}}{\partial r^*} - v_1^{(1)} \frac{\partial T_{\theta \theta}(0)}{\partial r^*} - v_{\theta \theta}(0) \frac{\partial T_{\theta \theta}(0)}{\partial r^*} \] \hspace{1cm} (16c)
\[ T_{rz}(1) = 2 \frac{\partial v_1^{(1)}}{\partial r^*} - v_1^{(1)} \frac{\partial T_{rz}(0)}{\partial r^*} - v_{rz}(0) \frac{\partial T_{rz}(0)}{\partial r^*} \] \hspace{1cm} (16d)

\( v_1^{(0)} \) and \( T_{ij}^{(0)} \) are components of the velocity and the stress in the zero's order of the Weissenberg number and are known. The equation of continuity, the equation of motion and the boundary condition are obtained by the same way;
\[ \frac{1}{r^*} \frac{\partial}{\partial r^*} (r^* v_1^{(1)}) + \frac{\partial v_1^{(1)}}{\partial z^*} = 0 \] \hspace{1cm} (17)
\[ \frac{\partial p^{(1)}/\partial z^*}{\partial r^*} = v_1^{(1)} + \frac{\partial T_{zz}(1)}{\partial z^*} + \frac{\partial T_{\theta \theta}(1)}{\partial r^*} + T_{rz}(1) \] \hspace{1cm} (18a)
\[ \frac{\partial p^{(1)}/\partial r^*}{\partial z^*} = v_1^{(1)} + \frac{\partial T_{\theta \theta}(1)}{\partial r^*} + \frac{\partial T_{rr}(1)}{\partial r^*} + \frac{\partial T_{rr}(1)}{\partial \theta^*} + T_{\theta \theta}(1) - T_{\theta \theta}(1) \] \hspace{1cm} (18b)
\[ z^* = -\infty \hspace{1cm} \text{c.} \hspace{1cm} v_1^{(1)} = v_1^{(1)} = 0 \] \hspace{1cm} (19a)
\[ v_1^{(1)} = 0 \] at \( z^* = 0 \) \hspace{1cm} (19b)
\[ v_1^{(1)} = v_1^{(1)} = 0 \] at \( r^* = 1 \) \hspace{1cm} (19c)
\[ \frac{\partial v_1^{(1)}}{\partial r^*} = v_i^{(1)} = 0 \] at \( r^* = 0 \) \hspace{1cm} (19d)

The Stream function \( \psi^{(1)} \) is
\[ \psi^{(1)}(1) = \frac{1}{r^*} \frac{\partial \psi^{(1)}}{\partial r^*} - v_1^{(1)} - \frac{1}{r^*} \frac{\partial \psi^{(1)}}{\partial z^*} \] \hspace{1cm} (19)

Eliminate \( \psi^{(1)} \) from eq. (18) using eq. (20), and
\[ A_1 = \frac{\partial^2 p^{(1)}}{\partial z^2} + \frac{\partial^2 p^{(1)}}{\partial r^2} - \frac{1}{r^*} \frac{\partial p^{(1)}}{\partial r^*} + \frac{\partial^2 p^{(1)}}{\partial z^2} + \frac{\partial^2 p^{(1)}}{\partial r^2} + \frac{\partial^2 p^{(1)}}{\partial z^2} + \frac{\partial^2 p^{(1)}}{\partial r^2} + \frac{\partial^2 p^{(1)}}{\partial z^2} + \frac{\partial^2 p^{(1)}}{\partial r^2} + \frac{\partial^2 p^{(1)}}{\partial z^2} \] \hspace{1cm} (20)
\[ B_k = r\left\{ \left( \frac{\partial v_k^{(0)}}{\partial r^*} \right) + 1 - \frac{1}{2r^*} \left( \frac{\partial v_k^{(0)}}{\partial r^*} - \frac{\partial v_k^{(1)}}{\partial z^*} \right) \right\} \]

\[ B_1 = -\frac{\partial v_1^{(0)}}{\partial z^*} \]

\[ B_2 = -r \left\{ \left( \frac{1}{2} - \frac{\partial^2}{\partial z^*} \right) \left( \frac{\partial v_1^{(0)}}{\partial r^*} - \frac{\partial v_1^{(1)}}{\partial z^*} \right) \right\} \]

\[ B_3 = -\frac{1}{2r^2} \left( \frac{\partial^2 v_1^{(0)}}{\partial r^*} - \frac{\partial v_1^{(1)}}{\partial z^*} \right) \]

As the right term of eq. (21) is known, we can obtain the first order term of the stream function \( \phi^{(1)} \) in power series of the Weissenberg number. The elastic effect on velocity profiles begins to appear already at the first order term of the Weissenberg number. On the contrary, in the two-dimensional plane flow, the elastic effect appears at the second order term of the Weissenberg number. Under the condition that the Weissenberg number is smaller than 1, the elastic effect appears at smaller Weissenberg number in the three-dimensional axisymmetric flow than in the two-dimensional plane flow. This fact supports that Ballenger et al.\cite{5,6} observed easily the circulating flow of a wine-glass shape in the axisymmetric flow but Han\cite{10} had the difficulty to observe this flow in the plane flow.

### 3.3 Second order terms of Weissenberg number

Substituting eq. (12) into eq. (9), and equating the second order terms of the Weissenberg number, we obtain

\[ T^{(2)} = 2 \frac{\partial v_{r_1}^{(2)}}{\partial z^*} - \nu^{(0)} \frac{\partial T_{r_1}^{(1)}}{\partial z^*} - \nu^{(0)} \frac{\partial T_{r_1}^{(1)}}{\partial z^*} + \left( \frac{\partial v_{r_1}^{(2)}}{\partial r^*} - \frac{\partial v_{r_1}^{(1)}}{\partial z^*} \right) T_{r_1}^{(1)} + \left( \frac{\partial v_{r_1}^{(1)}}{\partial r^*} - \frac{\partial v_{r_1}^{(1)}}{\partial z^*} \right) T_{r_1}^{(1)} + \left( \frac{\partial v_{r_1}^{(1)}}{\partial r^*} - \frac{\partial v_{r_1}^{(1)}}{\partial z^*} \right) T_{r_1}^{(1)} \]

\[ T^{(2)} = 2 \frac{\partial v_{r_1}^{(2)}}{\partial r^*} - \nu^{(0)} \frac{\partial T_{r_1}^{(1)}}{\partial r^*} - \nu^{(0)} \frac{\partial T_{r_1}^{(1)}}{\partial z^*} + \left( \frac{\partial v_{r_1}^{(2)}}{\partial r^*} - \frac{\partial v_{r_1}^{(1)}}{\partial z^*} \right) T_{r_1}^{(1)} + \left( \frac{\partial v_{r_1}^{(1)}}{\partial r^*} - \frac{\partial v_{r_1}^{(1)}}{\partial z^*} \right) T_{r_1}^{(1)} + \left( \frac{\partial v_{r_1}^{(1)}}{\partial r^*} - \frac{\partial v_{r_1}^{(1)}}{\partial z^*} \right) T_{r_1}^{(1)} \]

Eliminating \( p^{(2)} \) from eq. (24) by using eq. (26), we obtain

\[ \frac{\partial v_{r_1}^{(2)}}{\partial r^*} + \frac{\partial v_{r_1}^{(2)}}{\partial z^*} = \frac{\partial T_{r_1}^{(1)}}{\partial r^*} + \frac{\partial T_{r_1}^{(1)}}{\partial z^*} \]

\[ \frac{\partial v_{r_1}^{(2)}}{\partial r^*} + \frac{\partial v_{r_1}^{(2)}}{\partial z^*} = \frac{\partial T_{r_1}^{(1)}}{\partial r^*} + \frac{\partial T_{r_1}^{(1)}}{\partial z^*} \]

\[ \frac{\partial v_{r_1}^{(2)}}{\partial r^*} + \frac{\partial v_{r_1}^{(2)}}{\partial z^*} = \frac{\partial T_{r_1}^{(1)}}{\partial r^*} + \frac{\partial T_{r_1}^{(1)}}{\partial z^*} \]
where $A_1, \ldots, A_5, B_1, \ldots, B_6, C_1$ and $C_2$ are given by eq. (21). Others are:

$$D_1 = -2r \frac{\partial \psi}{\partial z^2}$$

$$D_2 = 2r \frac{\partial v}{\partial z^2}$$

$$D_3 = -r \left( \frac{\partial v}{\partial z^2} + \frac{\partial v}{\partial r^2} - 2 \frac{\partial v}{\partial z^2} \frac{\partial v}{\partial z^2} + \frac{1}{r} \frac{\partial v}{\partial z} \frac{\partial v}{\partial z} \right)$$

$$D_4 = r \left( \frac{\partial v}{\partial z^2} + \frac{\partial v}{\partial r^2} - 2 \frac{\partial v}{\partial z^2} \frac{\partial v}{\partial z^2} + \frac{1}{r} \frac{\partial v}{\partial z} \frac{\partial v}{\partial z} \right)$$

$$D_5 = \left( 2 \frac{\partial v}{\partial z^2} + 2 \frac{\partial v}{\partial r^2} - 2 \frac{\partial v}{\partial z^2} \frac{\partial v}{\partial z^2} + \frac{1}{r} \frac{\partial v}{\partial z} \frac{\partial v}{\partial z} \right)$$

$$D_6 = -\frac{\partial v}{\partial z^2} + 2 \frac{\partial v}{\partial r^2}$$

$$D_7 = \frac{1}{r} \frac{\partial v}{\partial z}$$

$$D_8 = \frac{1}{2r^2}$$

$$D_9 = -\frac{1}{2r}$$

$$D_{10} = -\frac{1}{2r^2}$$

As the right terms of eq. (27) are known, we can get the second order term of the stream function $\psi^{(2)}$ in power series of the Weissenberg number.

4. Velocity profiles

Fig. 1 shows a comparison between the three-dimensional axisymmetric flow and the two-dimensional plane flow. They are calculated on the assumption that the exit velocity profile is uniform. Stream functions $\psi$ are obtained from $\psi = \psi^{(0)} + W_1 \psi^{(1)} + W_2 \psi^{(2)}$ in the three-dimensional axisymmetric flow, and from $\psi = \psi^{(0)} + W_3 \psi^{(1)}$ in the two-dimensional plane flow. For reference, let us show some values of $\psi^{(0)}$, $W_1 \psi^{(1)}$ and $W_2 \psi^{(2)}$ at $\psi = 1.001$ in Fig. 1 (a); these values are 0.9982, 0.0026 and 0.0002, respectively. In Fig. 1(b), values of $\psi^{(0)}$ and $W_2 \psi^{(2)}$ are 0.991 and 0.0019, respectively. Values of the second order term are much smaller than those of the zero's order term. This may show that terms over the third order become smaller and hardly influence the value of the stream function. In Fig. 1(a) is shown a three-dimensional axisymmetric flow under the condition that the Weissenberg number ($W_e = \lambda V/R$) is equal to $5 \times 10^{-8}$. In Fig. 1(b) is shown a two-dimensional plane flow under the condition that the Weissenberg number ($W_e = \lambda V/H$) is equal to $5 \times 10^{-2}$. The method for these calculations is shown in reference[1]. In both figures, the stream line of $\psi = 1$ is convex downwards, and the influence of the elastic effect is small. But there is a large difference between these Weissenberg numbers. Now, if we consider that mean velocities and characteristic lengths are equal between two flows cited above, the ratio of relaxation times $\lambda_p/\lambda_a$ is equal to $10^6$, where $\lambda_a$ is the relaxation in the three-dimensional axisymmetric flow and $\lambda_p$ in the two-dimensional plane flow. This means that the value of the relaxation time, at which the elastic effect begins to appear, is considerably different between these flows; in the polymer processing, the elastic effect influences the three-dimensional axisymmetric flow, even if a polymer

![Fig. 1 Stream function (a) three-dimensional axisymmetric flow, Weissenberg number $5 \times 10^{-8}$, (b) two-dimensional plane flow$^{(1)}$, Weissenberg number $5 \times 10^{-2}$)](image)
melt has a small relaxation time. However, in the two-
dimensional plane flow, the elastic effect influences only the
flow having a large relaxation time.

In the axisymmetric flow, as the Weissenberg number be-
comes larger, circulating flows appear near a capillary, and
they are growing larger as shown in Fig. 2. The main flow
is shaped into a wine-glass as in the two-dimensional
plane flow. Weissenberg numbers in Fig. 2 are $1 \times 10^{-7}$ in (a)
and $3 \times 10^{-7}$ in (b). Weissenberg numbers in the two-dimen-
sional flow having such circulating flows as large as those
in Fig. 2 are about $8 \times 10^{-2}$ and $1.5 \times 10^{-1}$, respectively. In
these cases, the difference between Weissenberg numbers
is nearly $10^{-6}$.

Figure 3 shows a flow field using the velocity vector when
the Weissenberg number is $3 \times 10^{-7}$. The starting point of
each arrow shows the position. In Fig. 3(a) is shown the whole
flow field, and the velocity of circulating flows near a capil-
lary is much smaller than that of the main flow. In Fig. 3(b)
is shown a circulating flow, enlarged ten times as large as the
size of the velocity vector. The velocity difference between
the main and the circulating flow is larger in the three-dimen-
sional axisymmetric flow than in the two-dimensional
plane flow.

5. Stress distributions

Stress distributions are shown in Figs. 4 and 5 in the three-
dimensional axisymmetric flow. Fig. 4 shows the dimension-
less shear stress and the dimensionless normal stress differ-
ence when the Weissenberg number is $5 \times 10^{-8}$. Fig. 5 shows
the stress when the Weissenberg number is $3 \times 10^{-7}$ and has
large circulating flows. As the Weissenberg number becomes
large, two equivalent stress lines extend to the upstream
similarly to the two-dimensional plane flow. The shape of the
stress distribution is similar to that of two-dimensional flow.

Fig. 2 Stream function in the three-dimensional axisymmetric
flow (Weissenberg number (a) $1 \times 10^{-7}$, (b) $3 \times 10^{-7}$)

Fig. 3 Velocity profile (Weissenberg number $3 \times 10^{-7}$) (a) whole
flow, (b) circulating flow

Fig. 4 Stress distribution (Weissenberg number $5 \times 10^{-8}$) (a)
dimensionless shear stress, (b) dimensionless normal stress
difference

Fig. 5 Stress distribution (Weissenberg number $3 \times 10^{-7}$) (a)
dimensionless shear stress, (b) dimensionless normal stress
difference
and the zero's line of the normal stress difference extends from the wall of a capillary to the upstream. Values of dimensionless stresses, however, are much larger in the three-dimensional axisymmetric flow than in the two-dimensional plane flow, and their increase rate is remarkable as the flow approaches the capillary's inlet. For example, comparing the values of dimensionless normal stress difference between Fig. 1(a) and Fig. 1(b), i.e. between that in the three-dimensional axisymmetric flow and that in the two-dimensional plane flow, they differ as much as five times at R/2 from the exit on the centre line. The shear stress also has about the same difference near a capillary's inlet. Hence, in the polymer processing, we must take care of the three-dimensional axisymmetric flow, considering that the high stress may produce a bad effect on the outside appearance of products.

6. Conclusions
The three-dimensional axisymmetric flow of viscoelastic liquids into a capillary was calculated by the perturbation method to find a flow resembling a wine-glass peculiar to the viscoelastic flow. The flow pattern is similar to that of the two-dimensional flow, but there is a large difference between Weissenberg numbers. Therefore, even if polymer melts have smaller relaxation times than in the two-dimensional plane flow, circulating flows are observed in the three-dimensional axisymmetric flow. Moreover, the velocity difference between the main and the circulating flows, shear stresses and normal stress differences are larger in the three-dimensional axisymmetric flow than in the two-dimensional plane flow.

Reference